

Eureka Math™

Grade 2, Module 6

Teacher Edition

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Table of Contents

GRADE 2 • MODULE 6

Foundations of Multiplication and Division

Module Overview	2
Topic A: Formation of Equal Groups	9
Topic B: Arrays and Equal Groups	61
Mid-Module Assessment and Rubric	122
Topic C: Rectangular Arrays as a Foundation for Multiplication and Division.....	133
Topic D: The Meaning of Even and Odd Numbers.....	222
End-of-Module Assessment and Rubric	269
Answer Key	278

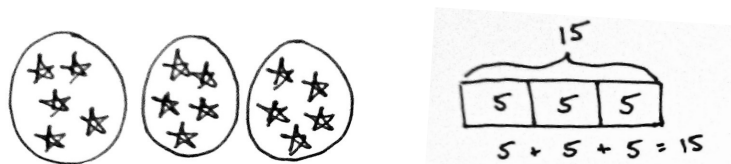
Grade 2 • Module 6

Foundations of Multiplication and Division

OVERVIEW

Grade 2 Module 6 lays the conceptual foundation for multiplication and division in Grade 3 and for the idea that numbers other than 1, 10, and 100 can serve as units.

In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups (e.g., given 15 objects, they create 3 groups of 5 or 5 groups of 3), and progress to pictorial representations where they may begin by circling a group of 5 stars, adding 5 more, and then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition equation (pictured below). Students calculate the repeated addition sums by adding on to the previous addends, step-by-step, or by grouping the addends into pairs and adding. By the end of Topic A, students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit (pictured below). Hence, they begin their experience toward understanding that any unit may be counted (e.g., 3 dogs, 3 tens, or even 3 fives). This is the bridge between Grades 2 and 3. Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.



In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time and express the total via repeated addition equations (**2.OA.4**). For example, students might arrange one column of 5 counters, then another, and then another to compose an array of 3 columns of 5, or 15 counters. As they compose and decompose arrays, students create different number sentences yielding the same total (e.g., $5 + 5 + 5 = 15$ and $3 + 3 + 3 + 3 + 3 = 15$). They find the total number of objects in each array by counting on from left to right. "Three plus 3 is 6. Six plus 3 is 9. Nine plus 3 is 12." As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically (e.g., 3 columns of 5 or 5 rows of 3). Then, they use same-size square tiles, moving them closer together in preparation for composing rectangles in Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems.

In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they need in preparation for Grade 3's area content. They use same-size squares to tile a rectangle with no gaps or overlaps and then count to find the total number of squares that make up the rectangle (**2.G.2**).

After composing rectangles, students partition, or decompose, rectangles. First, they decompose rectangles made of square tiles. Next, they use scissors to cut apart paper rectangles. Finally, they draw and iterate a square unit. In doing so, students begin to see the row or the column as a composite of multiple squares or as a single entity, or unit, which is, in turn, part of the larger rectangle. Students further develop spatial structuring skills by copying and creating drawings on grid paper. Note that the concept of a square unit begins in Grade 3 and is not assessed in Grade 2. Throughout the topic, students relate repeated addition to the model. They are encouraged to think flexibly and to consider the many ways to construct or partition a given array. Students are not multiplying or dividing in Grade 2; rather, this topic lays the foundation for the relationship between the two operations. As equal parts can be composed to form a whole, likewise, a whole can be decomposed into equal parts.

Topic D focuses on doubles and even numbers (**2.OA.3**), thus setting the stage for the multiplication table of two in Grade 3. As students progress through the lessons, they learn the following interpretations of even numbers:

1. A number that occurs when skip-counting by twos is even: 2, 4, 6, 8, ...
2. When objects are paired up with none left unpaired, the number is even.
3. A number that is twice a whole number (doubles) is even.
4. A number whose last digit is 0, 2, 4, 6, or 8 is even.

Armed with an understanding of the term *even*, students learn that any whole number that is not even is called *odd* and that when 1 is added to or subtracted from an even number, the resulting number is odd.¹

Initially, students arrange pairs into two rows and realize that an even number is the sum of two equal addends, or a repeated sum of twos. They then write number sentences to express the even number (e.g., 2 rows of 7 can be expressed as $7 + 7 = 14$ or as $2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$) (**2.OA.3**). Next, students pair objects to make groups of two with none left over, thus discovering one means of determining whether a group of objects (up to 20) has an even or odd number of members. Finally, students learn that any number up to 20 whose last digit is 0, 2, 4, 6, or 8 is even. After gaining a firm understanding of even numbers, students learn that all other whole numbers are odd. They use the previously learned rules and patterns to identify larger numbers as even or odd and to defend their reasoning. The module concludes with an investigation of what happens when we add two even numbers, two odd numbers, or an odd number with an even number, and the relationship of these pairings to repeated addition (e.g., $3 + 3$ is even, but $3 + 3 + 3$ is odd).

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Notes on Pacing for Differentiation

If pacing is a challenge, consider consolidating Lessons 1 and 2. Omit Lessons 3, 8, and 11. Use Lesson 3's Problem Set and Homework as a center activity for early finishers or for a future date when additional review homework is needed. Consider moving Lesson 16, which guides students through a tessellation project with 1-inch tiles, to art class.

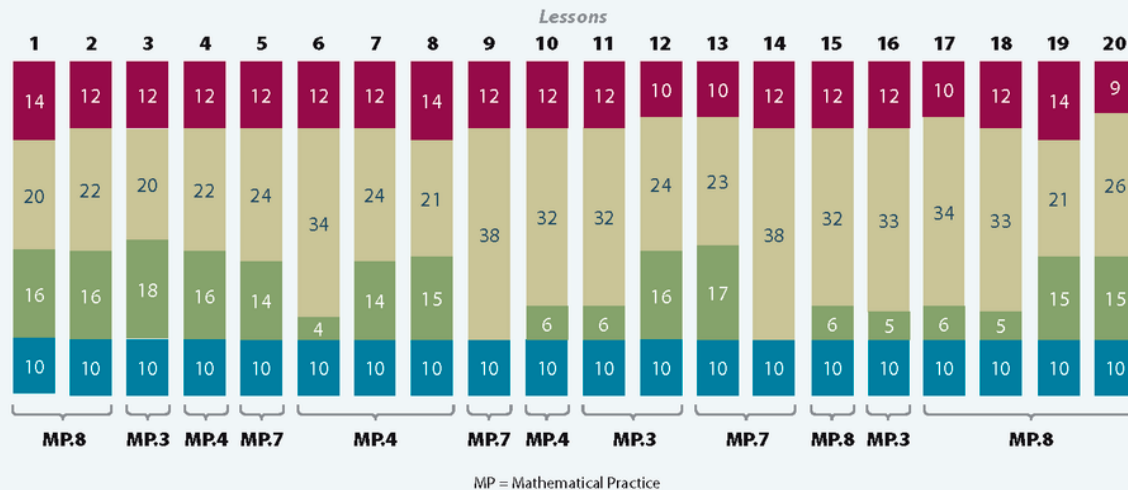
¹See *Elementary Mathematics for Teachers* by Scott Baldridge and Thomas Parker.



Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief



Focus Grade Level Standards

Work with equal groups of objects to gain foundations for multiplication.

- 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Reason with shapes and their attributes.²

- 2.G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

²2.G.2 is included in this module because the array model is so important to the foundation for multiplication. The balance of this cluster is addressed in Module 8.

Foundational Standards

- 1.OA.7** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.*
- 2.NBT.2** Count within 1000; skip-count by 5s, 10s, and 100s.
- 2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations.

Focus Standards for Mathematical Practice

- MP.3** **Construct viable arguments and critique the reasoning of others.** Students explain their thinking using drawings, models, and equations to lay the conceptual foundation for multiplication and division. “If I build an array with 3 columns of 4 objects, then I must have twelve objects because $4 + 4 + 4 = 12$. Likewise, if I partition my rectangle into twelve equally sized tiles, I can make 3 equal groups of 4 tiles, or I can make 4 equal groups of 3 tiles.” Students defend their reasoning as they prove that a number is even or odd, making connections to the previous concepts of counting by twos, adding on, equal groups, and doubles.
- MP.4** **Model with mathematics.** Students learn to organize a set of objects into equal groups and then into rows and columns, or rectangular arrays. They use math drawings to analyze the relationship between rows and columns (e.g., 3 rows of 4 or 4 columns of 3) and to model the array as the sum of equal addends (e.g., $4 + 4 + 4 = 12$).
- MP.7** **Look for and make use of structure.** As students compose and decompose arrays, they recognize that the array structure is a collection of rows or columns and that either can be seen as a unit. Students match repeated addition to both the structure of the rows and columns (e.g., $5 + 5 + 5$ can be 3 rows or columns of 5, or 3 fives).
- MP.8** **Look for and express regularity in repeated reasoning.** As students create equal groups using objects, they recognize that they are repeatedly adding the same number; for example, 3 groups of 4 bears can be expressed as $4 + 4 + 4$. Students also discover patterns in odd and even numbers, recognizing the repetition of 0, 2, 4, 6, and 8 in the ones place.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives	Days
2.OA.4 2.NBT.2 2.NBT.6	A Formation of Equal Groups Lesson 1: Use manipulatives to create equal groups. Lessons 2–3: Use math drawings to represent equal groups, and relate to repeated addition. Lesson 4: Represent equal groups with tape diagrams, and relate to repeated addition.	4
2.OA.4 2.NBT.2	B Arrays and Equal Groups Lesson 5: Compose arrays from rows and columns, and count to find the total using objects. Lesson 6: Decompose arrays into rows and columns, and relate to repeated addition. Lesson 7: Represent arrays and distinguish rows and columns using math drawings. Lesson 8: Create arrays using square tiles with gaps. Lesson 9: Solve word problems involving addition of equal groups in rows and columns.	5
	Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
2.OA.4 2.G.2	C Rectangular Arrays as a Foundation for Multiplication and Division Lessons 10–11: Use square tiles to compose a rectangle, and relate to the array model. Lesson 12: Use math drawings to compose a rectangle with square tiles. Lesson 13: Use square tiles to decompose a rectangle. Lesson 14: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares. Lesson 15: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition. Lesson 16: Use grid paper to create designs to develop spatial structuring.	7



Standards		Topics and Objectives	Days
2.OA.3	D	The Meaning of Even and Odd Numbers	4
		Lesson 17: Relate doubles to even numbers, and write number sentences to express the sums.	
		Lesson 18: Pair objects and skip-count to relate to even numbers.	
		Lesson 19: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.	
		Lesson 20: Use rectangular arrays to investigate odd and even numbers.	
		End-of-Module Assessment: Topics A–D (assessment $\frac{1}{2}$ day, return $\frac{1}{2}$ day, remediation or further applications 1 day)	2
Total Number of Instructional Days			24

Terminology

New or Recently Introduced Terms

- Array (an arrangement of objects in rows and columns)
- Columns (the vertical groups in a rectangular array)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Odd number (any number that is not even)
- Repeated addition (e.g., $2 + 2 + 2$)
- Rows (the horizontal groups in a rectangular array)
- Tessellation (tiling of a plane using one or more geometric shapes with no overlaps and no gaps)
- Whole number (e.g., 0, 1, 2, 3, ...)

Familiar Terms and Symbols³

- Addends
- Doubles
- Equation
- Number path
- Number sentence
- Pair
- Rectangle
- Skip-counting

³These are terms and symbols students have seen previously.

- Square
- Sum
- Tape diagram
- Total
- Unit

Suggested Tools and Representations

- Counters
- Number bond
- Number path
- Personal white board
- Rectangular array
- Square tiles

Scaffolds⁴

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	2.OA.4
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	2.OA.3 2.OA.4 2.G.2

⁴Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.



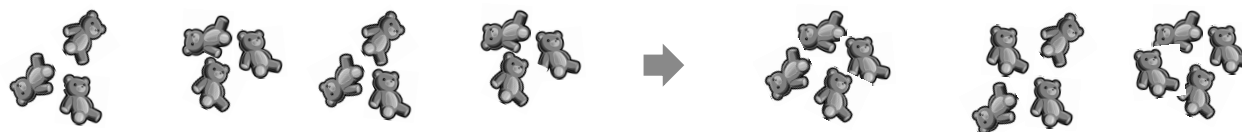
Topic A

Formation of Equal Groups

2.OA.4, 2.NBT.2, 2.NBT.6

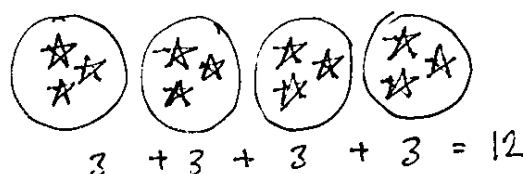
Focus Standard:	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Instructional Days:	4	
Coherence	-Links from: G2–M3	Place Value, Counting, and Comparison of Numbers to 1,000
	-Links to: G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

Topic A begins at the concrete level as students use objects to create equal groups, providing a foundation for the construction of arrays in Topic B. In Lesson 1, for example, students are given 12 counters, such as teddy bears, pebbles, or beans, and they are asked to put them into groups of 3, thereby creating 4 equal groups of 3 objects.



Students then see that they can manipulate the same number of counters to make 3 equal groups of 4 objects. Finally, they are presented with unequal groups and challenged to make them equal.

Lessons 2 and 3 move to the pictorial level, introducing math drawings to represent equal groups. In Lesson 2, students are asked to show groups: “Show me 3 stars, now 3 more. Add 3 more, now 3 more than that.” They then determine the total number of stars and write the corresponding repeated addition number sentence as shown to the right (2.OA.4).



Lesson 3 extends this understanding as students look for and practice a more efficient way to add by bundling. They calculate repeated addition sums by grouping the addends into pairs and then adding. For example, for 4 groups of 3, students might say, “I bundled 2 pairs of three to make sixes, so $6 + 6 = 12$.” If there is an odd number of addends (e.g., 5 groups of 3), students group them into pairs and then add on the remaining quantity such that $(3 + 3) + (3 + 3) = 6 + 6 = 12$, and then, $12 + 3 = 15$. As students work with equal groups, they begin to see that they are adding units of 3.

This concept transitions into Lesson 4, where students understand that numbers other than 1, 10, and 100 can serve as units. At a more abstract level than Lesson 3, students represent the total of a given number of units with tape diagrams or using repeated addition (e.g., $2 + 2 + 2 + 2 = 8$). This concept serves as a bridge to Topic B, wherein either a row or column of an array can be seen as the unit being counted—the foundation for building rectangular arrays (**2.OA.4**).

A Teaching Sequence Toward Mastery of Formation of Equal Groups

Objective 1: Use manipulatives to create equal groups.
(Lesson 1)

Objective 2: Use math drawings to represent equal groups, and relate to repeated addition.
(Lessons 2–3)

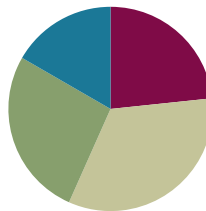
Objective 3: Represent equal groups with tape diagrams, and relate to repeated addition.
(Lesson 4)

Lesson 1

Objective: Use manipulatives to create equal groups.

Suggested Lesson Structure

Fluency Practice	(14 minutes)
Concept Development	(20 minutes)
Application Problem	(16 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (14 minutes)

- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)
- Get the Ten Out and Subtract **2.NBT.5** (5 minutes)
- Subtract Common Units **2.NBT.5, 2.NBT.7** (4 minutes)

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets

Note: During Topic A and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Five options are provided in this lesson for the Core Fluency Practice Set, with Set A being the simplest and Set E the most complex. Start all students on Set A.

Students complete as many problems as they can in 120 seconds. One hundred percent accuracy and completion is recommended before moving to the next level. Collect any Practice Sets that have been completed within the 120 seconds, and check the answers. The next time Core Fluency Practice Sets are used, students who have successfully completed their set can move on to the next level. Keep a record of student progress.

Consider assigning early finishers a counting pattern and start number (e.g., count by fives from 195). Celebrate improvement as well as advancement. Students should be encouraged to compete with themselves rather than their peers. Discuss with students possible strategies to solve. Notify caring adults of each student's progress.

Get the Ten Out and Subtract (5 minutes)

Note: Students practice taking out the ten and subtracting.

T: For every expression I give, subtract the ones from ten. When I say, “ $12 - 4$,” you say, $10 - 4 = 6$. Ready?

T: $12 - 4$.

S: $10 - 4 = 6$.

T: $13 - 7$.

S: $10 - 7 = 3$.

Students practice taking the ten out of expressions fluently before adding the ones back.

T: Now, let’s add back the ones.

T: $12 - 4$. Take from ten.

S: $10 - 4 = 6$.

T: Now, add back the ones.

S: $6 + 2 = 8$.

Continue with the following possible sequence: $13 - 7$, $11 - 8$, $13 - 9$, $15 - 7$, and $14 - 8$.

Subtract Common Units (4 minutes)

Materials: (S) Personal white board

Note: This activity reminds students of the importance of the subtraction algorithm.

T: (Project 88.) Say the number in unit form.

S: 8 tens 8 ones.

T: (Write $88 - 22 = \underline{\quad}$.) Say the subtraction sentence and answer in unit form.

S: 8 tens 8 ones – 2 tens 2 ones = 6 tens 6 ones.

T: Write the subtraction sentence in standard form on your personal white board.

Continue with the following possible sequence: $66 - 33$, $99 - 22$, $299 - 22$, $77 - 33$, and $777 - 33$.

Concept Development (20 minutes)

Materials: (T) Sentence frame: There are ____ groups of ____ counters. (S) 12 counters

This lesson prepares students to express equal groups as equal addends in Lesson 2. Throughout the lesson, circulate to check for understanding as students move their counters.

T: (Show 6 counters separated into 3 groups of 2.) Talk with your partner: Are these groups equal or unequal, and how do you know?

S: They’re equal because there’s the same number of counters in each group. → They’re equal because there are 2 in each group.

- T: (Rearrange the counters to show 4 in one group and 2 in another.) Talk again. Equal or unequal?
- S: Unequal. There are 4 in one group and 2 in the other. → They're unequal because they don't have the same amount in each group.
- T: So, for groups to be equal, they need to have the same number in each. True?
- S: True!
- T: Take 8 counters, and move them into groups of 2.
(Give students time to organize their groups.)
- T: How many groups of 2 are there? Use the sentence frame: There are ____ groups of ____ counters.
- S: There are 4 groups of 2 counters.
- T: Now, make groups of 4 counters. (Pause.) How many groups of 4 are there?
- S: There are 2 groups of 4 counters.
- T: Now, use all of your counters, and find a way to arrange them into equal groups. (Give students time to organize their counters.)
- T: Who would like to share how they organized their counters?
- S: I made 2 groups and put 6 in each. → I put 4 in each group, and there are 3 groups.
→ I made 6 groups of 2. → I made 4 groups of 3 counters.
- T: So, there can be more than one way to make equal groups. Try arranging your counters another way. (Give students time to try another way.)
- T: Use the sentence frame to tell your partner how many counters are in each group.
- S: (Turn and talk.) There are ____ groups of ____ counters.
- T: Set 2 counters aside. Make groups of 5.
- S: (Move counters.)
- T: How many groups did you make? Use the sentence frame.
- S: There are 2 groups of 5 counters.
- T: Now, make groups of 2. Use the sentence frame.
- S: There are 5 groups of 2 counters.
- T: Now, let's try something different. (Draw or project 3 groups with 3, 5, and 4 counters, respectively.)
- T: Arrange all of your counters to look like mine.
- T: Are these groups equal?
- S: No!
- T: Move your counters to make the 3 groups equal.
- T: How did you make the groups equal?
- S: I moved one counter from the group that has 5 to the group that has 3. Now there are 4 counters in each group.
→ I looked for the group with the most, and I moved one counter to the group with the least. Now there are equal groups of 4.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Have students show equal groups by grouping themselves into groups of 4, 5, and so on. This activity can be done outdoors as well.



T: Yes! We made 3 groups of 4.

T: Move your counters to form 2 equal groups. (Pause.)

T: How many counters are in the 2 groups? Use the sentence frame.

S: There are 2 groups of 6 counters.

T: How did you figure that out?

MP.8

S: (Demonstrate.) I moved one counter here and one counter here, and I kept doing that until all the counters were in 2 groups. → I made rows and matched the counters, so there was the same number in each row. → I split the counters in half.

T: Ooh! I like that! What addition fact helped you know that?

S: $6 + 6 = 12$.

T: Nice mental work!



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students may put all 12 counters into one group. Support the fact that this is one way to make a group. Then, encourage these students to show equal groups, too.

Application Problem (16 minutes)

Julisa has 12 stuffed animals. She wants to put the same number of animals in each of her 3 baskets.

- Draw a picture to show how she can put the animals into 3 equal groups.
- Complete the sentence.

Julisa put ____ animals in each basket.



Julisa put 4 animals
in each basket.

Note: This problem is intended for independent practice. It follows the Concept Development so students can apply the day's learning in a real world context. The allotted time period includes 6 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide the selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Use manipulatives to create equal groups.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

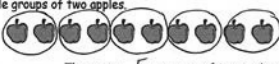
- For Problem 1, how many groups of 2 did you circle? How many apples are there altogether? What counting strategies did you use?
- For Problem 2, how many groups of 3 did you circle? If you were circling groups of 5 balls, would there be more or fewer groups?
- For Problems 3 and 4, what steps did you take to redraw the oranges into 4 equal groups? When you drew the oranges into 3 equal groups, did you put more or fewer oranges in each group? Could you make 5 equal groups? Six equal groups?
- For Problem 5, how did you go about making the three groups equal?
- Make a prediction: How are these equal groups related to addition?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

1. Circle groups of two apples.



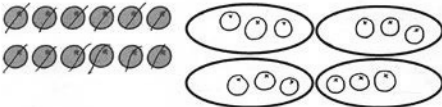
There are 5 groups of two apples.

2. Circle groups of three balls.



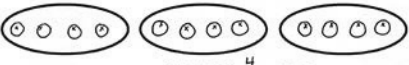
There are 5 groups of three balls.

3. Redraw the 12 oranges into 4 equal groups.



4 groups of 3 oranges

4. Redraw the 12 oranges into 3 equal groups.



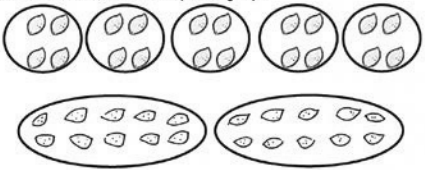
3 groups of 4 oranges

5. Redraw the flowers to make each of the 3 groups have an equal number.



3 groups of 4 flowers = 12 flowers

6. Redraw the lemons to make 2 equal size groups.



2 groups of 10 lemons = 20 lemons

Name _____

Date _____

1.	$10 + 3 =$	21.	$7 + 9 =$
2.	$10 + 6 =$	22.	$4 + 8 =$
3.	$10 + 4 =$	23.	$5 + 9 =$
4.	$5 + 10 =$	24.	$8 + 6 =$
5.	$8 + 10 =$	25.	$7 + 5 =$
6.	$10 + 9 =$	26.	$5 + 8 =$
7.	$12 + 2 =$	27.	$8 + 3 =$
8.	$13 + 4 =$	28.	$9 + 8 =$
9.	$16 + 3 =$	29.	$6 + 5 =$
10.	$2 + 17 =$	30.	$7 + 6 =$
11.	$5 + 14 =$	31.	$4 + 6 =$
12.	$7 + 12 =$	32.	$8 + 7 =$
13.	$16 + 3 =$	33.	$7 + 7 =$
14.	$11 + 5 =$	34.	$8 + 6 =$
15.	$9 + 2 =$	35.	$6 + 9 =$
16.	$5 + 9 =$	36.	$8 + 5 =$
17.	$7 + 9 =$	37.	$4 + 7 =$
18.	$9 + 4 =$	38.	$3 + 9 =$
19.	$7 + 8 =$	39.	$6 + 6 =$
20.	$8 + 8 =$	40.	$4 + 9 =$

Name _____

Date _____

1.	$10 + 4 =$	21.	$4 + 8 =$
2.	$10 + 9 =$	22.	$7 + 6 =$
3.	$5 + 10 =$	23.	$\underline{\hspace{1cm}} + 4 = 11$
4.	$2 + 10 =$	24.	$\underline{\hspace{1cm}} + 8 = 13$
5.	$11 + 4 =$	25.	$6 + \underline{\hspace{1cm}} = 14$
6.	$12 + 5 =$	26.	$8 + \underline{\hspace{1cm}} = 15$
7.	$16 + 2 =$	27.	$\underline{\hspace{1cm}} = 9 + 8$
8.	$13 + \underline{\hspace{1cm}} = 18$	28.	$\underline{\hspace{1cm}} = 4 + 7$
9.	$11 + \underline{\hspace{1cm}} = 20$	29.	$\underline{\hspace{1cm}} = 7 + 8$
10.	$14 + 3 =$	30.	$3 + 9 =$
11.	$\underline{\hspace{1cm}} = 3 + 16$	31.	$6 + 7 =$
12.	$\underline{\hspace{1cm}} = 7 + 12$	32.	$8 + \underline{\hspace{1cm}} = 13$
13.	$\underline{\hspace{1cm}} = 15 + 4$	33.	$\underline{\hspace{1cm}} = 7 + 9$
14.	$9 + 2 =$	34.	$6 + 5 =$
15.	$6 + 9 =$	35.	$\underline{\hspace{1cm}} = 5 + 7$
16.	$\underline{\hspace{1cm}} + 4 = 11$	36.	$\underline{\hspace{1cm}} = 8 + 4$
17.	$\underline{\hspace{1cm}} + 6 = 13$	37.	$15 = 8 + \underline{\hspace{1cm}}$
18.	$\underline{\hspace{1cm}} + 5 = 12$	38.	$17 = \underline{\hspace{1cm}} + 9$
19.	$8 + 8 =$	39.	$14 = \underline{\hspace{1cm}} + 7$
20.	$6 + 6 =$	40.	$19 = 8 + \underline{\hspace{1cm}}$

Name _____

Date _____

1.	$12 - 2 =$	21.	$16 - 9 =$
2.	$18 - 8 =$	22.	$14 - 6 =$
3.	$19 - 10 =$	23.	$16 - 8 =$
4.	$14 - 10 =$	24.	$15 - 6 =$
5.	$16 - 6 =$	25.	$17 - 8 =$
6.	$11 - 10 =$	26.	$18 - 9 =$
7.	$17 - 12 =$	27.	$15 - 7 =$
8.	$20 - 10 =$	28.	$13 - 8 =$
9.	$13 - 11 =$	29.	$11 - 3 =$
10.	$18 - 13 =$	30.	$12 - 5 =$
11.	$12 - 3 =$	31.	$11 - 2 =$
12.	$11 - 2 =$	32.	$13 - 6 =$
13.	$14 - 2 =$	33.	$16 - 7 =$
14.	$13 - 4 =$	34.	$12 - 8 =$
15.	$11 - 3 =$	35.	$16 - 13 =$
16.	$13 - 2 =$	36.	$15 - 14 =$
17.	$12 - 4 =$	37.	$17 - 12 =$
18.	$14 - 5 =$	38.	$19 - 16 =$
19.	$11 - 4 =$	39.	$18 - 11 =$
20.	$12 - 5 =$	40.	$20 - 16 =$

Name _____

Date _____

1.	$19 - 9 =$	21.	$16 - 7 =$
2.	$12 - 10 =$	22.	$17 - 8 =$
3.	$18 - 11 =$	23.	$16 - 7 =$
4.	$15 - 10 =$	24.	$14 - 8 =$
5.	$17 - 12 =$	25.	$17 - 9 =$
6.	$16 - 13 =$	26.	$12 - 9 =$
7.	$12 - 2 =$	27.	$16 - 8 =$
8.	$20 - 10 =$	28.	$15 - 7 =$
9.	$14 - 11 =$	29.	$13 - 8 =$
10.	$13 - 3 =$	30.	$14 - 7 =$
11.	$\underline{\hspace{1cm}} = 11 - 3$	31.	$13 - 9 =$
12.	$\underline{\hspace{1cm}} = 14 - 4$	32.	$15 - 9 =$
13.	$\underline{\hspace{1cm}} = 13 - 4$	33.	$14 - 6 =$
14.	$\underline{\hspace{1cm}} = 11 - 4$	34.	$\underline{\hspace{1cm}} = 13 - 5$
15.	$\underline{\hspace{1cm}} = 12 - 3$	35.	$\underline{\hspace{1cm}} = 15 - 8$
16.	$\underline{\hspace{1cm}} = 13 - 2$	36.	$\underline{\hspace{1cm}} = 18 - 9$
17.	$\underline{\hspace{1cm}} = 11 - 2$	37.	$\underline{\hspace{1cm}} = 20 - 4$
18.	$16 - 8 =$	38.	$\underline{\hspace{1cm}} = 20 - 17$
19.	$15 - 6 =$	39.	$\underline{\hspace{1cm}} = 20 - 11$
20.	$12 - 5 =$	40.	$\underline{\hspace{1cm}} = 20 - 3$

Name _____

Date _____

1.	$13 + 3 =$	21.	$11 - 8 =$
2.	$12 + 8 =$	22.	$13 - 7 =$
3.	$16 + 2 =$	23.	$15 - 8 =$
4.	$11 + 7 =$	24.	$12 + 6 =$
5.	$6 + 9 =$	25.	$13 + 2 =$
6.	$7 + 8 =$	26.	$9 + 11 =$
7.	$4 + 7 =$	27.	$6 + 8 =$
8.	$13 - 5 =$	28.	$8 + 9 =$
9.	$16 - 6 =$	29.	$7 + 5 =$
10.	$17 - 9 =$	30.	$13 - 7 =$
11.	$14 - 6 =$	31.	$15 - 8 =$
12.	$18 - 7 =$	32.	$11 - 9 =$
13.	$8 + 8 =$	33.	$12 - 3 =$
14.	$7 + 6 =$	34.	$14 - 5 =$
15.	$4 + 9 =$	35.	$13 + 6 =$
16.	$5 + 7 =$	36.	$8 + 5 =$
17.	$6 + 5 =$	37.	$4 + 7 =$
18.	$13 - 8 =$	38.	$7 + 8 =$
19.	$16 - 9 =$	39.	$4 + 9 =$
20.	$14 - 8 =$	40.	$20 - 12 =$

Name _____

Date _____

1. Circle groups of two apples.



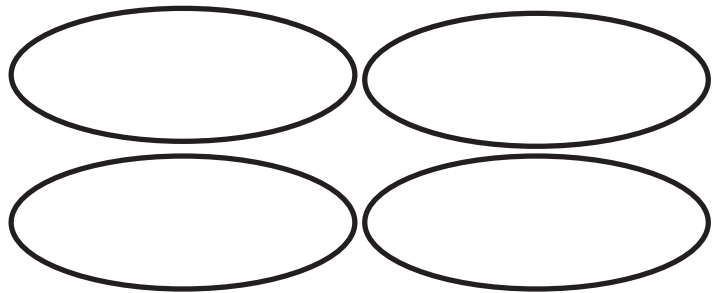
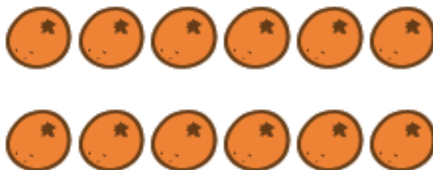
There are _____ groups of two apples.

2. Circle groups of three balls.



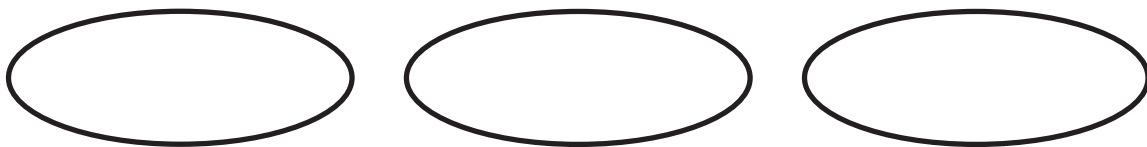
There are _____ groups of three balls.

3. Redraw the 12 oranges into 4 equal groups.



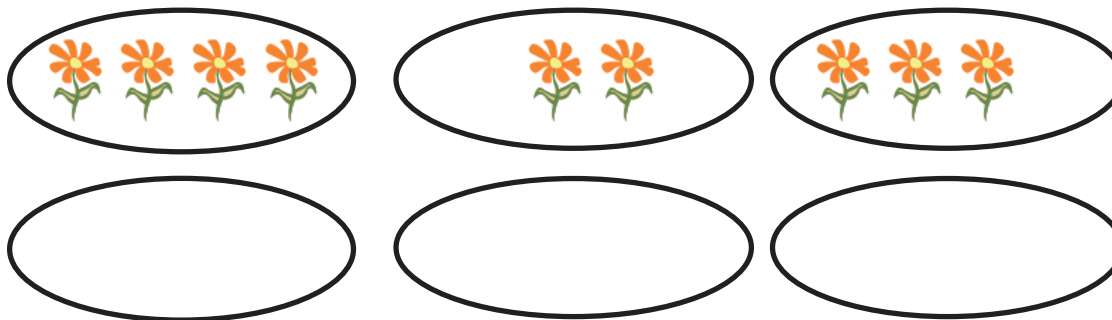
4 groups of _____ oranges

4. Redraw the 12 oranges into 3 equal groups.



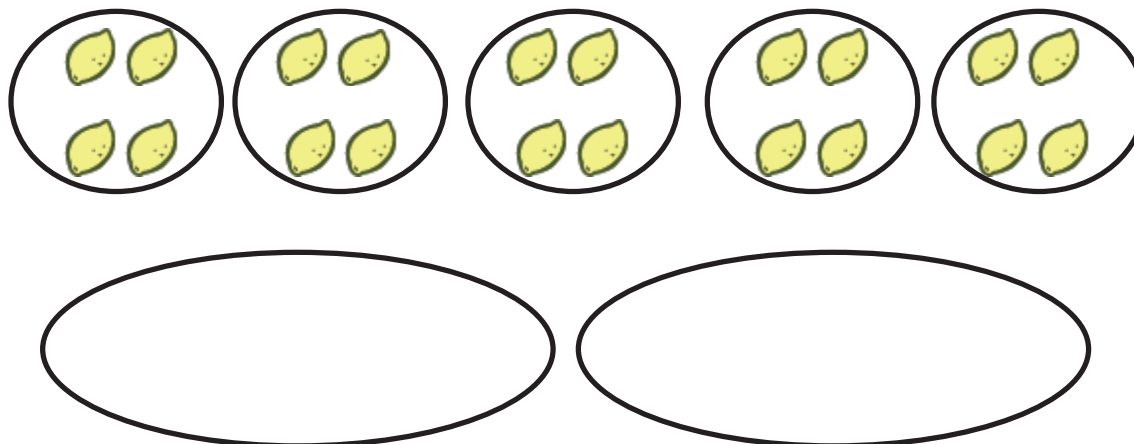
3 groups of _____ oranges

5. Redraw the flowers to make each of the 3 groups have an equal number.



3 groups of _____ flowers = _____ flowers.

6. Redraw the lemons to make 2 equal size groups.



2 groups of _____ lemons = _____ lemons.

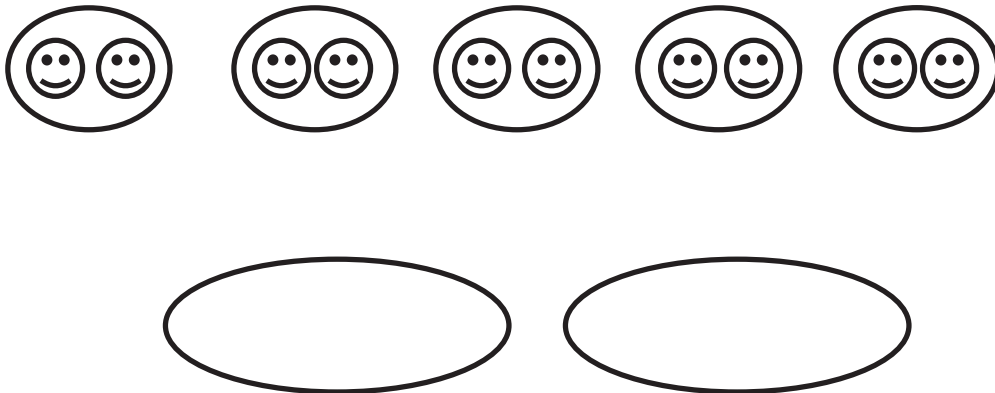
Name _____

Date _____

1. Circle groups of 4 hats.



2. Redraw the smiley faces into 2 equal groups.



2 groups of _____ = _____.

Name _____

Date _____

1. Circle groups of two shirts.



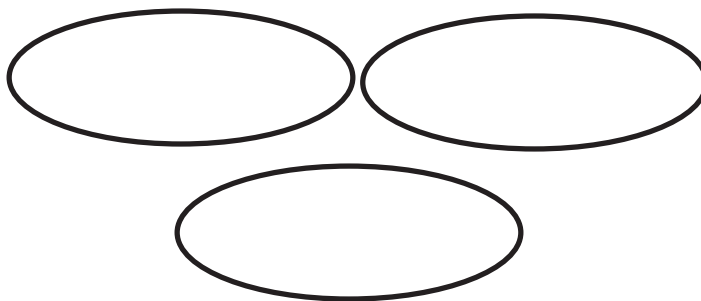
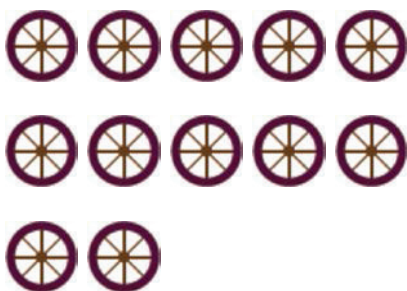
There are _____ groups of two shirts.

2. Circle groups of three pants.



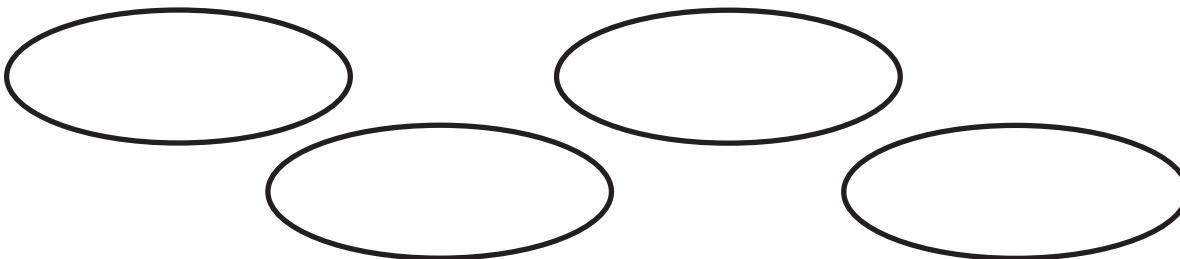
There are _____ groups of three pants.

3. Redraw the 12 wheels into 3 equal groups.



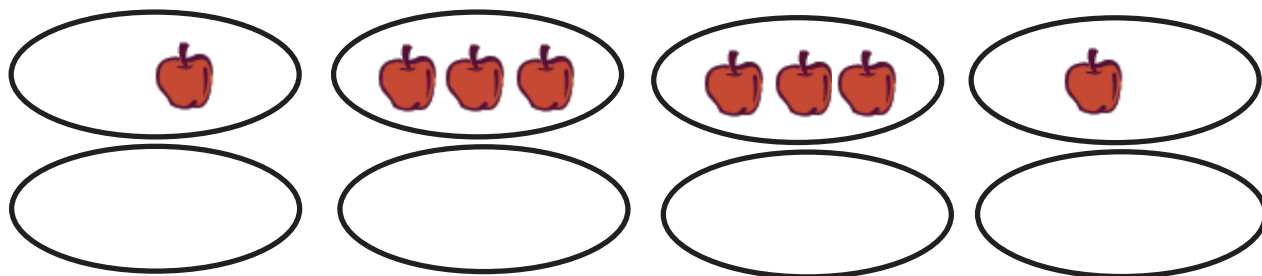
3 groups of _____ wheels

4. Redraw the 12 wheels into 4 equal groups.



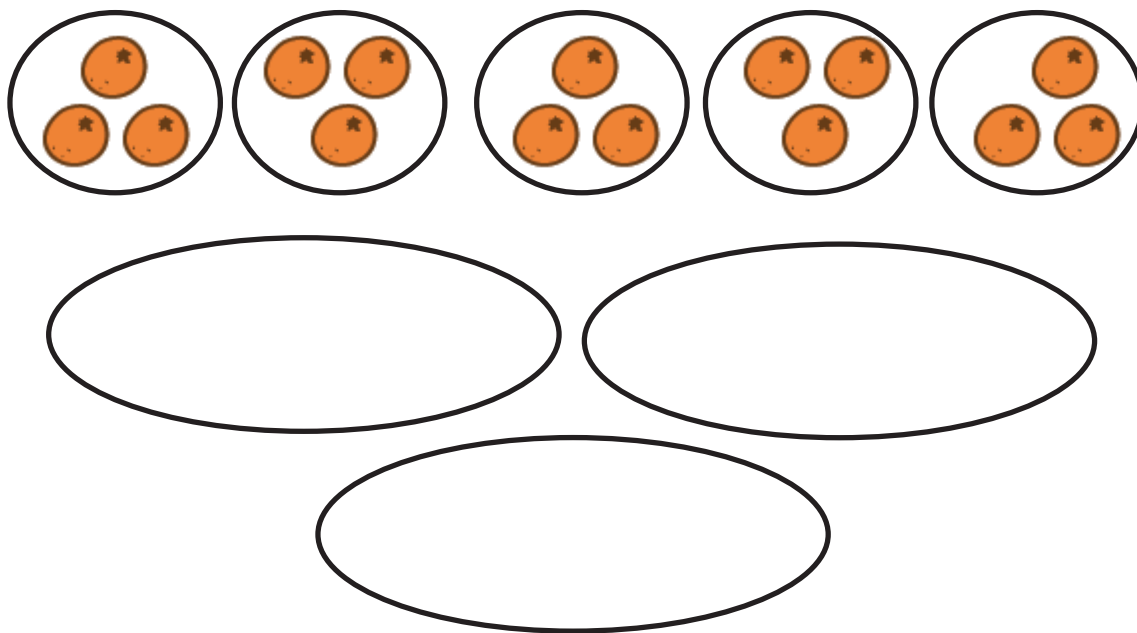
4 groups of _____ wheels

5. Redraw the apples to make each of the 4 groups have an equal amount.



4 groups of _____ apples = _____ apples.

6. Redraw the oranges to make 3 equal groups.



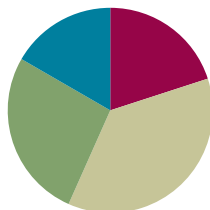
3 groups of _____ oranges = _____ oranges.

Lesson 2

Objective: Use math drawings to represent equal groups, and relate to repeated addition.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(22 minutes)
Application Problem	(16 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)
- Using the Nearest Ten to Subtract **2.NBT.5** (5 minutes)
- Subtracting Multiples of Hundreds and Tens **2.NBT.7** (2 minutes)

Grade 2 Core Fluency Practice Sets (5 minutes)

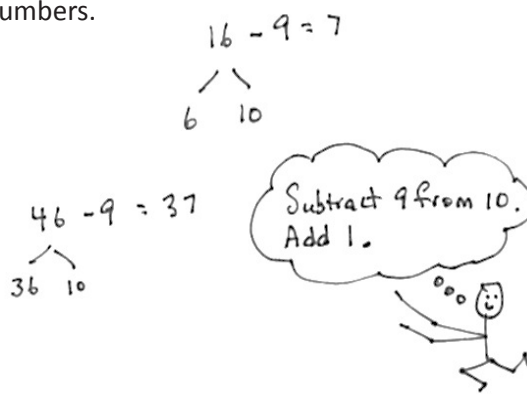
Materials: (S) Core Fluency Practice Sets (Lesson 1 Core Fluency Practice Sets)

Note: During Topic A and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 1.

Using the Nearest Ten to Subtract (5 minutes)

Note: Students apply bonds of 10 to subtracting with larger numbers.

- T: (Post $16 - 9$ on the board.) Raise your hand when you know the answer.
- S: 7.
- T: (Write the bond of 16 as 6 and 10.) $10 - 9$ is...?
- S: 1.
- T: $1 + 6$ is...?
- S: 7.



- T: $16 - 9$ is...?
S: 7.
T: $46 - 9$ is...? (Pause.)
S: 37.
T: (Write the bond of 46 as 36 and 10.) $10 - 9$ is...?
S: 1.
T: $36 + 1$ is...?
S: 37.
T: $46 - 9$ is...?
S: 37.

Continue with the following possible sequence: $15 - 9$ and $45 - 9$, $13 - 8$ and $33 - 8$, $15 - 7$ and $35 - 7$, $12 - 9$ and $42 - 9$, $13 - 7$ and $53 - 7$, $16 - 9$ and $46 - 9$, $14 - 8$ and $54 - 8$, $13 - 5$ and $43 - 5$, and $16 - 9$ and $36 - 9$.

Subtracting Multiples of Hundreds and Tens (2 minutes)

Note: Students review subtracting multiples of hundreds and tens to maintain their ability to isolate and manipulate place value units.

- T: What is 2 tens less than 130?
S: 110.
T: Give the subtraction number sentence.
S: $130 - 20 = 110$.
T: What is 2 hundreds less than 350?
S: 150.
T: Give the subtraction number sentence.
S: $350 - 200 = 150$.

Continue with the following possible sequence: 6 tens less than 150, 3 hundreds less than 550, and 7 tens less than 250.

Concept Development (22 minutes)

Materials: (T) Counters (S) Personal white board

- T: (Display 4 groups with 3 counters in each group.) How can we find the total number of counters?
S: Add them all up.
T: Excellent! Let's do that. (Draw a line below each group. Point to the counters in the first group.) How many in this group?
S: 3.
T: (Write 3 below the first group, followed by the plus sign.)
T: (Point to the next group, and repeat the process for the remaining groups.)

T: So, what's our number sentence to find the total?

S: $3 + 3 + 3 + 3 = \underline{\quad}$.

T: We just used **repeated addition**! (Point to each 3 from left to right.) $3 + 3$ is...?

S: 6.

T: $6 + 3$ is...?

S: 9.

T: $9 + 3$ is...?

S: 12.

T: Four groups of 3 equals...? Say the complete sentence.

S: Four groups of 3 equals 12.

T: Talk with your partner about how the number sentence relates to the model.

S: The 3 stands for how many are in each group.
 → There are 4 groups of 3, so that's why we added 3 four times. → There are 4 threes in the model, and there are 4 threes in the addition sentence.

T: Yes! And, since the groups are equal, instead of counting one by one, we can use repeated addition. We add the same addend over and over.

T: Now it's your turn! Since it's math, not art, we want to be quick and efficient, so you're going to draw groups of circles. You can pretend they're stars or donuts, whatever you want, but when we model, we'll be drawing circles!

T: Draw 5 large circles to represent the groups. (Model as students draw.)

T: Draw 2 circles in each group. (Model as students draw.)

T: Draw a line beneath each group. (Model as students draw.)

T: Tell your partner what number you're going to write on each line and why.

S: 2, because there are 2 in each group.

T: What number is repeating?

S: 2.

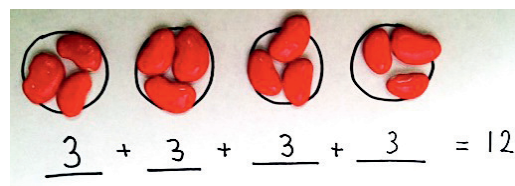
T: Let's write our repeated addition equation. (Write $2 + 2 + 2 + 2 + 2 = \underline{\quad}$ as students do the same.)

T: Let's read the equation.

S: $2 + 2 + 2 + 2 + 2 = \underline{\quad}$.

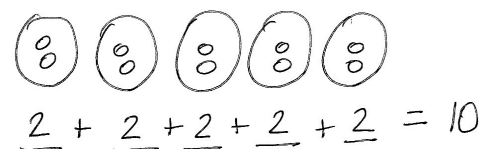
T: $2 + 2$ equals...?

S: 4.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Have manipulatives available for students who need to return to the concrete experience to master this concept. Allow them to progress at their own pace through the pictorial and abstract levels of this concept.



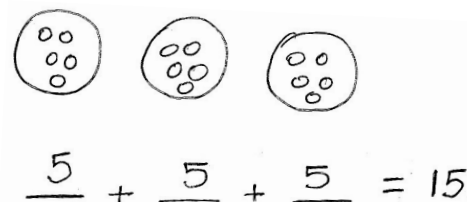
MP.8

- T: $4 + 2$ equals...?
 S: 6.
 T: $6 + 2$ equals...?
 S: 8.
 T: $8 + 2$ equals...?
 S: 10.
 T: (Point to each group of circles.) So, 5 groups of 2 equals 10. Repeat that with me.
 S: Five groups of 2 equals 10.
 T: Erase your personal white board. This time, let's draw groups of 5. Draw one group of 5. (Model as students do the same.)
 T: Now, add another group of 5.
 T: Now, draw 5 more than that.
 T: How many groups do we have?
 S: 3 groups.
 T: How many in each group?
 S: 5 in each group.
 T: What is the repeated addition expression?
 S: $5 + 5 + 5$.
 T: Let's write that as an equation. (Write $5 + 5 + 5 = \underline{\quad}$ as students do the same.)
 T: $5 + 5$ equals...?
 S: 10.
 T: $10 + 5$ equals...?
 S: 15.
 T: Three groups of 5 equals...? Say the complete sentence.
 S: Three groups of 5 equals 15.
 T: We can also say 3 fives equal 15. Say that with me.
 S: 3 fives equals 15.
 T: Now you're going to show what you know on an Application Problem!



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Periodically inform parents of new vocabulary, such as *addend* and *repeated addition*, so parents can help their children with the homework and understand the math vocabulary they are learning. Send home handouts or update the vocabulary on a class website.



Application Problem (16 minutes)

Mayra sorts her socks by color. She has 4 purple socks, 4 yellow socks, 4 pink socks, and 4 orange socks.

- Draw groups to show how Mayra sorts her socks.
- Write a repeated addition equation to match.

- c. How many socks does Mayra have in all?

Note: This problem is intended to give students independent practice drawing equal groups and writing the corresponding repeated addition equation to solve. The allotted time period includes 6 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use math drawings to represent equal groups, and relate to repeated addition.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1(a), what **repeated addition** equation matches the picture? How did you find the total?
- For Problem 1(b), what repeated addition equation matches the picture? Why are there 4 addends?
- For Problem 2, how many equal groups are there? What repeated addition equation matches the picture? What does the number 4 represent? How did you find the total?
- For Problem 3, share your drawing and your equation with a partner. There are 4 equal groups. Why didn't you add $4 + 4 + 4 + 4$?
- For Problem 4, share your drawing and your equation with a partner. How many groups of hearts are there altogether? How did you find the total? Compare this problem to Problem 1(a).

Name Roberto Date _____

1. Write a repeated addition equation to show the number of objects in each group. Then find the total.

a. 
 $2 + 2 + 2 = 6$
 3 groups of $2 = 6$

b. 
 $3 + 3 + 3 + 3 = 12$
 4 groups of $3 = 12$

2. Draw 1 more group of four. Then write a repeated addition equation to match.


 $4 + 4 + 4 + 4 + 4 = 20$
 5 groups of $4 = 20$

- Fill in the blank: “When writing a repeated addition equation, the repeated number shows _____.”

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students’ understanding of the concepts that were presented in today’s lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

3. Draw 1 more group of three. Then write a repeated addition equation to match.



$$\underline{3} + \underline{3} + \underline{3} + \underline{3} = \underline{12}$$

$$\underline{4} \text{ groups of } 3 = \underline{12}$$

4. Draw 2 more equal groups. Then write a repeated addition equation to match.



$$\underline{2} + \underline{2} + \underline{2} + \underline{2} + \underline{2} = \underline{10}$$

$$\underline{5} \text{ groups of } 2 = \underline{10}$$

5. Draw 3 groups of 5 stars. Then write a repeated addition equation to match.



$$5 + 5 + 5 = 15$$

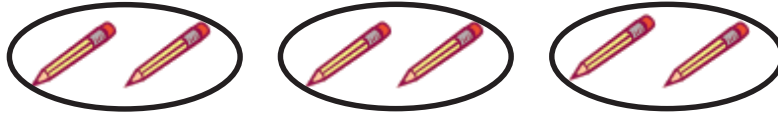
$$3 \text{ groups of } 5 = 15$$

Name _____

Date _____

1. Write a repeated addition equation to show the number of objects in each group. Then, find the total.

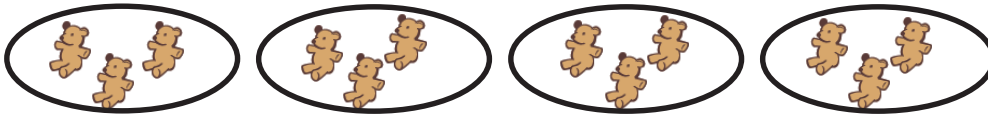
a.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$3 \text{ groups of } \underline{\quad} = \underline{\quad}$$

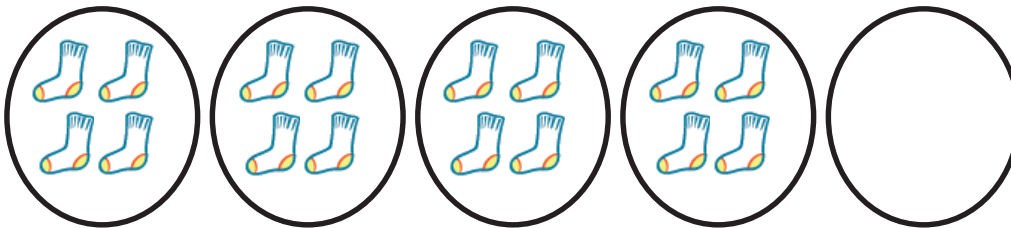
b.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = \underline{\quad}$$

2. Draw 1 more group of four. Then, write a repeated addition equation to match.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$5 \text{ groups of } \underline{\quad} = \underline{\quad}$$

3. Draw 1 more group of three. Then, write a repeated addition equation to match.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \text{ groups of } 3 = \underline{\quad}$$

4. Draw 2 more equal groups. Then, write a repeated addition equation to match.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

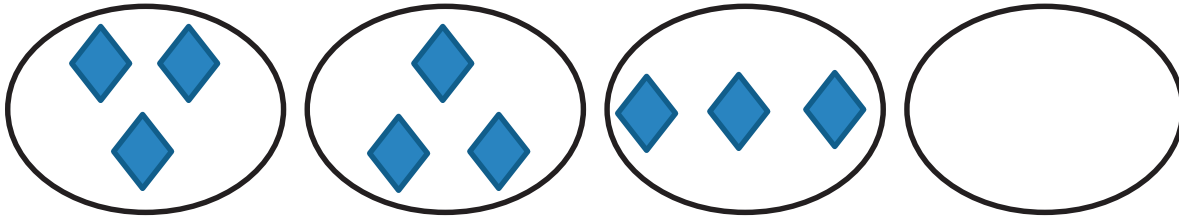
$$\underline{\quad} \text{ groups of } 2 = \underline{\quad}$$

5. Draw 3 groups of 5 stars. Then, write a repeated addition equation to match.

Name _____

Date _____

1. Draw 1 more equal group.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = \underline{\quad}$$

2. Draw 2 groups of 3 stars. Then, write a repeated addition equation to match.

Name _____

Date _____

1. Write a repeated addition equation to show the number of objects in each group. Then, find the total.

a.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$3 \text{ groups of } \underline{\quad} = \underline{\quad}$$

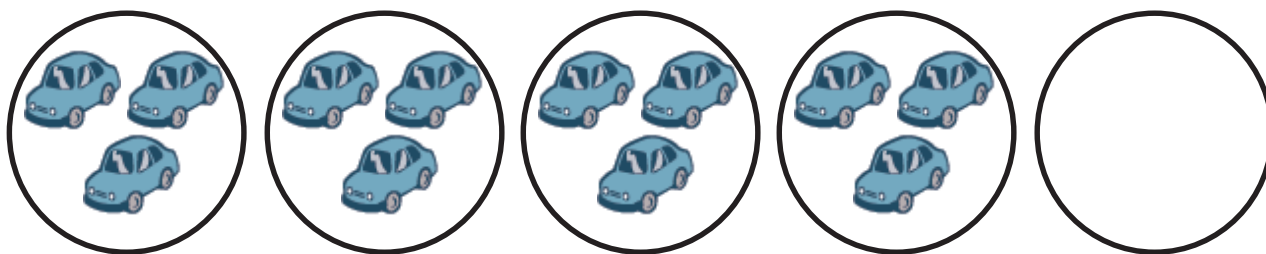
b.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = \underline{\quad}$$

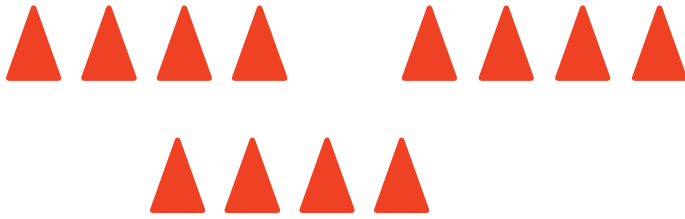
2. Draw 1 more equal group.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$5 \text{ groups of } \underline{\quad} = \underline{\quad}$$

3. Draw 1 more group of four. Then, write a repeated addition equation to match.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \text{ groups of } 4 = \underline{\quad}$$

4. Draw 2 more equal groups. Then, write a repeated addition equation to match.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \text{ groups of } 4 = \underline{\quad}$$

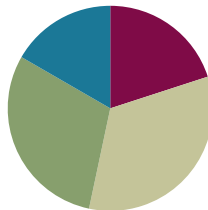
5. Draw 4 groups of 3 circles. Then, write a repeated addition equation to match.

Lesson 3

Objective: Use math drawings to represent equal groups, and relate to repeated addition.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(20 minutes)
Application Problem	(18 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Happy Counting by Fives **2.NBT.2** (3 minutes)
- Sprint: Subtraction Within 20 **2.OA.2** (9 minutes)

Happy Counting by Fives (3 minutes)

Note: This activity promotes the grade level fluency standard of skip-counting by 5s and anticipates telling time to the nearest five minutes in Module 7.

- T: Let's count by fives, starting at 0. Ready? (Point up rhythmically until a change is desired. Close hand to indicate a stopping point. Point down to count in the opposite direction. Continue, periodically changing direction.)
- S: 0, 5, 10, 15, 20 (switch) 15, 10 (switch) 15, 20, 25, 30, 35, 40 (switch) 35, 30, 25 (switch) 30, 35, 40, 45 (switch) 40, 35, 30 (switch) 35, 40, 45, 50 (switch) 45, 40, 35 (switch) 40, 45, 50 (switch) 45, 40, 35, 30, 25, 20, 15.
- T: Excellent! Try it for 30 seconds with your partner, starting at 0. Partner A, you are the teacher today.

Sprint: Subtraction Within 20 (9 minutes)

Materials: (S) Subtraction Within 20 Sprint

Note: This fluency activity promotes mastery of subtracting from teen numbers to address the grade level fluency standard.

Concept Development (20 minutes)

Materials: (T) Counters (S) Personal white board

In this lesson, students continue working at the pictorial level, using math drawings to represent equal groups and relating those groups to repeated addition. They also use addition strategies, such as doubles, to add more efficiently.

T: (Display counters showing 4 groups of 4.) What repeated addition equation matches this model?

S: $4 + 4 + 4 + 4 = \underline{\quad}$.

T: Yes! (Point to each 4.) To find the total, I can think $4 + 4$ is 8, $8 + 4$ is 12, and $12 + 4$ is 16.

T: Can anyone think of a faster way to solve?

S: You can use doubles!

T: Can you explain what you mean?

S: I know $4 + 4$ is 8, and there's another $4 + 4$, which is 8. And $8 + 8$ is 16.

T: (Move the counters to show how the pairs of 4 make 2 groups of 8.) You used a known doubles fact, $4 + 4$, to be efficient.

T: Let me show what I just did in writing. (Draw the 4 groups of 4 on the board with a blank line beneath each group.) What repeated addition equation matches this picture? (Record as they speak.)

S: $4 + 4 + 4 + 4 = \underline{\quad}$.

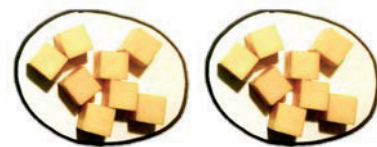
T: (Draw the number bond to show the bundling.)

T: Use the picture to talk with your partner about this question: How are 4 groups of 4 the same as 2 groups of 8?

S: If we draw a big circle around the first 2 groups of 4 and a big circle around the other 2 groups of 4, we'd have 2 groups of 8. → There are 2 fours inside of each 8. → Together, they equal 16.

T: Let's try another one. This time, let's draw it on our personal white boards. Draw a group of 5 circles. I like to circle mine so it's easy to see each group. (Model as students do the same.)

T: Now, show 5 more. (Model, and continue in this way until students have drawn 4 groups of 5.)



$$\begin{array}{ccccccc}
 \textcircled{0000} & \textcircled{0000} & \textcircled{0000} & \textcircled{0000} & & & \\
 \hline
 4 & + & 4 & + & 4 & + & 4 & = & 16 \\
 & & \swarrow & & \swarrow & & & & \\
 & & 8 & + & 8 & = & 16
 \end{array}$$



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Use a Rekenrek as an alternative way to show repeated addition. Show the same number of beads along the left side, and then show the repeated addition equation that goes with the beads. For example, show 3 rows of 4 beads, and then write $4 + 4 + 4 = 12$ to show the addition.

- T: Tell your partner the repeated addition equation that matches your model, and then explain how they relate to each other.
- S: $5 + 5 + 5 + 5 = \underline{\quad}$. → The 5 stands for how many are in each group. → There are 4 groups of 5, so we add 4 fives.
- T: Correct! Tell your partner two different ways you could add to find the total.
- S: $5 + 5 = 10$. $10 + 5 = 15$. $15 + 5 = 20$. → We can use doubles. $5 + 5 = 10$ and $10 + 10 = 20$. → We could skip-count: 5, 10, 15, 20.
- T: I like the way you made the connection between repeated addition and skip-counting!
- T: Let's think about 4 groups of 5 and 2 groups of 10. How are they the same?
- S: They both equal 20. → They're the same. You're just grouping the circles differently. → There are 2 fives in each group of 10.
- T: That's a clever way to look at it!
- T: Now, show me 6 groups of 3. (Model as students do the same.)
- T: Let's write the repeated addition equation. Say it with me as you write. (Model as students do the same.)
- S: $3 + 3 + 3 + 3 + 3 + 3 = \underline{\quad}$.
- T: How can we group the addends to find the total?
- S: Use doubles! → $3 + 3 = 6$.
- T: Okay. So, let's add all our doubles. What is the new repeated addition equation?
- S: $6 + 6 + 6 = \underline{\quad}$.
- T: What doubles fact can we use now?
- S: $6 + 6 = 12$.
- T: Yes! And $12 + 6$ is...?
- S: 18.
- T: So, we can group addends into pairs and use doubles to add quickly. If there's an extra addend, we just add on that amount.
- T: Let's do one more before you work on the Problem Set.
- T: Draw 5 groups of 2 circles. (Model as students do the same.)
- T: Write the repeated addition equation as I do the same. Say it with me as you write. (Model.)
- S: $2 + 2 + 2 + 2 + 2 = \underline{\quad}$.

$$\begin{array}{r} 5 + 5 + 5 + 5 = 20 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 10 + 10 = 20 \end{array}$$

$$\begin{array}{r} 3 + 3 + 3 + 3 + 3 + 3 = 18 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 6 + 6 + 6 = 18 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 12 + 6 = 18 \end{array}$$



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

At this point, some students may make the connection between repeated addition and multiplication. Praise their observation, but keep the focus on repeated addition for the lessons and assessments. Multiplication is taught in Grade 3.

MP.3

- T: Group the addends. $2 + 2$ is...?
 S: 4.
 T: And $2 + 2$ is...?
 S: 4.
 T: And we have 2 more. Now we have $4 + 4 + 2$.
 T: Can we group another pair of addends?
 S: Yes! $4 + 4 = 8$.
 T: Plus 2 more?
 S: 10.
 T: Excellent work!

$$2 + 2 + 2 + 2 + 2 = 10$$

$$4 + 4 + 2 = 10$$

$$8 + 2 = 10$$

Application Problem (18 minutes)

Markers come in packs of 2. If Jessie has 6 packs of markers, how many markers does she have in all?

- Draw groups to show Jessie's packs of markers.
- Write a repeated addition equation to match your drawing.
- Group addends into pairs, and add to find the total.

Note: This problem is intended for independent practice, giving students a context in which to practice drawing equal groups. Students write the corresponding repeated addition equation and use doubles as a strategy to add efficiently. The allotted time period includes 8 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

$$2 + 2 + 2 + 2 + 2 + 2 = 12$$

$$4 + 4 + 4 = 12$$

$$8 + 4 = 12$$

Jessie has 12 markers.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use math drawings to represent equal groups, and relate to repeated addition.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1(a), how did you show a more efficient way to add? How do you know that 4 groups of 3 and 2 groups of 6 are equal?
- For Problem 1(b), how did you bundle the addends into new groups? What was your new equation? Why didn't the total change?
- For Problem 1(c), how did you make fewer groups? Which equation enabled you to add more efficiently? (Note: Students might answer that the longer one was more efficient because they were able to skip-count by twos.)
- For Problem 2(a), how was this problem different from the previous ones? Does every group have a partner? How did you find the total?
- For Problem 2(b), how many pairs did you find? How many new groups did you make? Why did you add on 3?
- What strategies did we use today to add more efficiently?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

1. Write a repeated addition equation to match the picture. Then, group the addends into pairs to show a more efficient way to add.



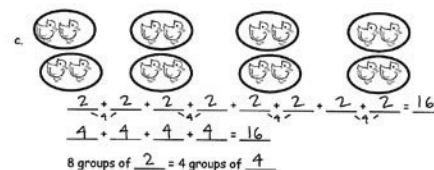
$$\begin{array}{r} 3 + 3 + 3 + 3 = 12 \\ \hline 6 + 6 = 12 \end{array}$$

4 groups of 3 = 2 groups of 6



$$\begin{array}{r} 2 + 2 + 2 + 2 = 8 \\ \hline 4 + 4 = 8 \end{array}$$

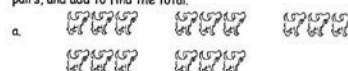
4 groups of 2 = 2 groups of 4



$$\begin{array}{r} 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16 \\ \hline 4 + 4 + 4 + 4 = 16 \end{array}$$

8 groups of 2 = 4 groups of 4

2. Write a repeated addition equation to match the picture. Then, group addends into pairs, and add to find the total.



$$\begin{array}{r} 3 + 3 + 3 + 3 + 3 = 15 \\ \hline 6 + 6 + 3 = 15 \end{array}$$

12 + 3 = 15



$$\begin{array}{r} 3 + 3 + 3 = 9 \\ \hline 6 + 3 = 9 \end{array}$$

A

Number Correct: _____

Subtraction Within 20

1.	$11 - 10 =$	
2.	$12 - 10 =$	
3.	$13 - 10 =$	
4.	$19 - 10 =$	
5.	$11 - 1 =$	
6.	$12 - 2 =$	
7.	$13 - 3 =$	
8.	$17 - 7 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 8 =$	
13.	$18 - 8 =$	
14.	$13 - 4 =$	
15.	$13 - 5 =$	
16.	$13 - 6 =$	
17.	$13 - 8 =$	
18.	$16 - 6 =$	
19.	$12 - 3 =$	
20.	$12 - 4 =$	
21.	$12 - 5 =$	
22.	$12 - 9 =$	

23.	$19 - 9 =$	
24.	$15 - 6 =$	
25.	$15 - 7 =$	
26.	$15 - 9 =$	
27.	$20 - 10 =$	
28.	$14 - 5 =$	
29.	$14 - 6 =$	
30.	$14 - 7 =$	
31.	$14 - 9 =$	
32.	$15 - 5 =$	
33.	$17 - 8 =$	
34.	$17 - 9 =$	
35.	$18 - 8 =$	
36.	$16 - 7 =$	
37.	$16 - 8 =$	
38.	$16 - 9 =$	
39.	$17 - 10 =$	
40.	$12 - 8 =$	
41.	$18 - 9 =$	
42.	$11 - 9 =$	
43.	$15 - 8 =$	
44.	$13 - 7 =$	

B

Subtraction Within 20

Number Correct: _____

Improvement: _____

1.	$11 - 1 =$	
2.	$12 - 2 =$	
3.	$13 - 3 =$	
4.	$18 - 8 =$	
5.	$11 - 10 =$	
6.	$12 - 10 =$	
7.	$13 - 10 =$	
8.	$18 - 10 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 7 =$	
13.	$19 - 9 =$	
14.	$12 - 3 =$	
15.	$12 - 4 =$	
16.	$12 - 5 =$	
17.	$12 - 8 =$	
18.	$17 - 7 =$	
19.	$13 - 4 =$	
20.	$13 - 5 =$	
21.	$13 - 6 =$	
22.	$13 - 9 =$	

23.	$16 - 6 =$	
24.	$14 - 5 =$	
25.	$14 - 6 =$	
26.	$14 - 7 =$	
27.	$14 - 9 =$	
28.	$20 - 10 =$	
29.	$15 - 6 =$	
30.	$15 - 7 =$	
31.	$15 - 9 =$	
32.	$14 - 4 =$	
33.	$16 - 7 =$	
34.	$16 - 8 =$	
35.	$16 - 9 =$	
36.	$20 - 10 =$	
37.	$17 - 8 =$	
38.	$17 - 9 =$	
39.	$16 - 10 =$	
40.	$18 - 9 =$	
41.	$12 - 9 =$	
42.	$13 - 7 =$	
43.	$11 - 8 =$	
44.	$15 - 8 =$	

c.



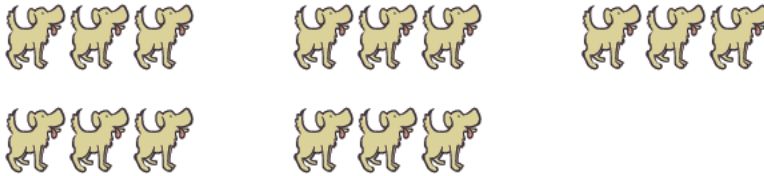
$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

8 groups of $\underline{\quad}$ = 4 groups of $\underline{\quad}$

2. Write a repeated addition equation to match the picture. Then, group addends into pairs, and add to find the total.

a.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} + 3 = \underline{\quad}$$

$$\underline{\quad} + 3 = \underline{\quad}$$

b.



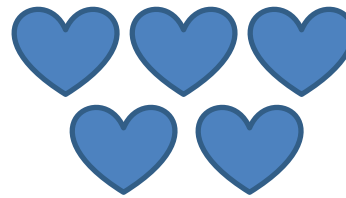
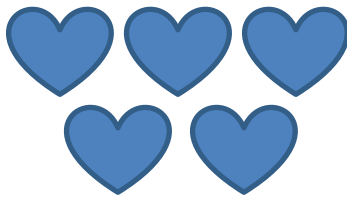
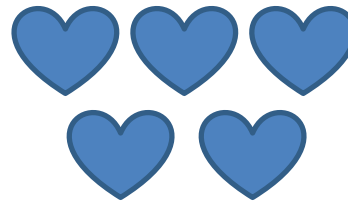
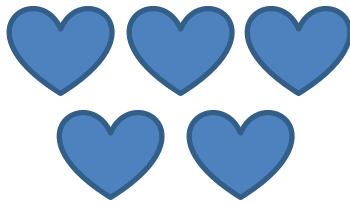
$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + 3 = \underline{\quad}$$

Name _____

Date _____

Write a repeated addition equation to match the picture. Then, group the addends into pairs to show a more efficient way to add.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

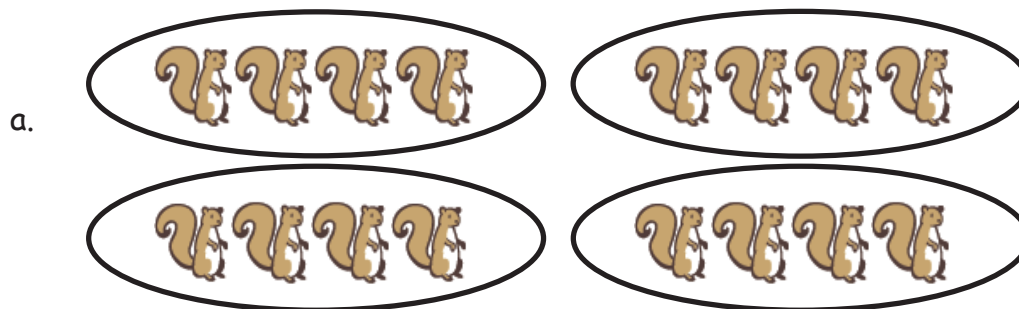
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = 2 \text{ groups of } \underline{\quad}$$

Name _____

Date _____

1. Write a repeated addition equation to match the picture. Then, group the addends into pairs to show a more efficient way to add.

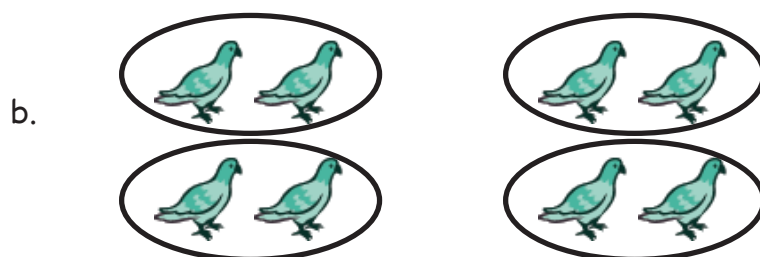


$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

\ / \ /

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = 2 \text{ groups of } \underline{\quad}$$

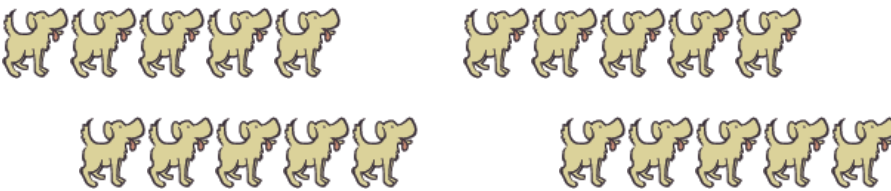


$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$4 \text{ groups of } \underline{\quad} = 2 \text{ groups of } \underline{\quad}$$

c.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

4 groups of $\underline{\quad}$ = 2 groups of $\underline{\quad}$

2. Write a repeated addition equation to match the picture. Then, group addends into pairs, and add to find the total.

a.

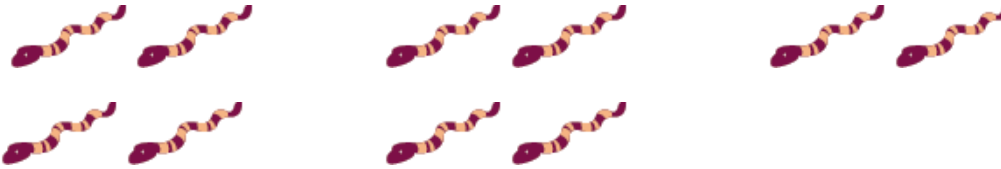


$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} + 3 = \underline{\quad}$$

$$\underline{\quad} + 3 = \underline{\quad}$$

b.



$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} + 2 = \underline{\quad}$$

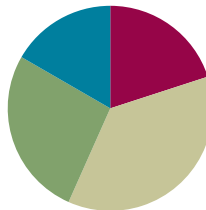
$$\underline{\quad} + 2 = \underline{\quad}$$

Lesson 4

Objective: Represent equal groups with tape diagrams, and relate to repeated addition.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(22 minutes)
■ Application Problem	(16 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Happy Counting by Fives **2.NBT.2** (3 minutes)
- Sprint: Adding Crossing Ten **2.OA.2** (9 minutes)

Happy Counting by Fives (3 minutes)

Note: Students fluently skip-count by fives with an emphasis on crossing one hundred in anticipation of counting coins in Module 7 and telling time in Module 8.

- T: Let's count by fives, starting at 50. Ready? (Point up rhythmically until a change is desired. Close hand to indicate a stopping point. Point down to count in the opposite direction. Continue, periodically changing direction.)
- S: 50, 55, 60, 65, 70 (switch) 65, 60 (switch) 65, 70, 75, 80, 85, 90 (switch) 85, 80, 75 (switch) 80, 85, 90, 95, 100 (switch) 95, 90, 85 (switch) 90, 95, 100, 105 (switch) 100, 95, 90 (switch) 95, 100, 105 (switch) 100, 95, 90, 85, 80, 75, 70.
- T: Excellent! Try it for 30 seconds with your partner, starting at 50. Partner B, you are the teacher today.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Invite a student to lead the class in Happy Counting, and participate with students. Rotate this opportunity so students have the chance to lead and, therefore, to think more critically about the patterns involved in the counting.

Sprint: Adding Crossing Ten (9 minutes)

Materials: (S) Adding Crossing Ten Sprint

Note: Students add numbers within 20 to help gain mastery of the grade level fluency standard.

Concept Development (22 minutes)

Materials: (S) Personal white board, counters

T: Let's read this word problem together.

T: (Project or write the problem on the board.) There are 2 apples in Jane's bag, 3 apples in Sam's bag, and 1 apple in Ann's bag. How many apples do the children have in all?

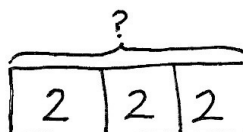
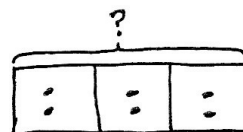
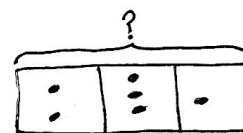
T: Use part-whole language to tell me how to solve.

S: We know the parts, so we add them together. → We add the parts, $2 + 3 + 1$, to get the whole, which is 6.

T: Draw a tape diagram on your personal white board, and use your counters to model the problem. (Model on the board, drawing dots, as students do the same using counters.)

T: Now, talk with your partner. How would this model be different if there were equal groups of 2 apples in each bag? Show the change on your model.

S: You would put 2 counters in each box. → There are still 3 groups, but they are all equal. → Now we have 3 groups of 2.



T: You've noticed that the boxes represent the groups and that the counters inside are the number, or amount, in each group.

T: Now, let's change our model to show numbers instead of counters. What number should we write in each box?

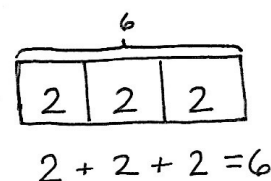
S: 2.

T: Of course! Remove your counters, and write 2 in each box.

T: What do we do when we know the parts?

S: We add to find the whole!

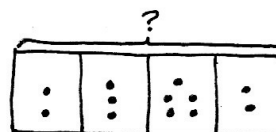
T: It's easy to see the repeated addition, isn't it? Write the repeated addition equation to find the total for this tape diagram. Read the equation.



S: $2 + 2 + 2 = 6$.

T: So, we are adding twos! Just like we have added units of 1 or 10, we can also add units of two.

T: Let's try another one! Draw a tape diagram that has 4 parts. (Model as students do the same.)

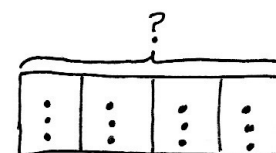


T: Use your counters to show 2 in the first group, 3 in the next group, 5 in the next group, and 2 in the last group. (Model on the board.)

T: Are all of the groups equal?

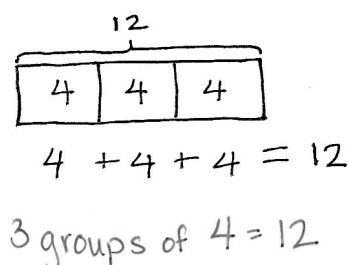
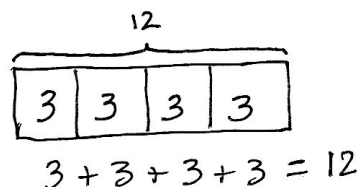
S: No!

T: Move your counters to show equal groups of 3 in each part. (Model with dots as students rearrange their counters.)



MP.4

- T: Say it with me: “We have 4 equal groups of 3.” (Students repeat.)
- T: Remove your counters, and write the number in each group. What number will you write?
- S: 3.
- T: Yes! Write the repeated addition equation that relates to this model, and then solve.
- T: Read the equation.
- S: $3 + 3 + 3 + 3 = 12$.
- T: Tell your partner how you added to find the answer.
- S: $3 + 3$ is 6. $6 + 3$ is 9. $9 + 3$ is 12. → I used doubles, so $3 + 3 = 6$ and $3 + 3 = 6$. Then, $6 + 6 = 12$.
- T: So, 4 groups of 3 is...?
- S: 12.
- T: Talk with your partner: How would the tape diagram change if there were 3 groups of 4? Draw a tape diagram that shows 3 groups of 4 to explain your thinking.



Circulate to check for understanding, and call on students to share.

- S: There are only 3 boxes because there are 3 groups. → We can write 4 in each box. → The repeated addition is $4 + 4 + 4$. Before, it was $3 + 3 + 3 + 3$. But they both equal 12.
- T: Excellent reasoning! Let's do one more before you work on the Application Problem. Draw a tape diagram that shows 4 groups of 5.
- T: Explain to your partner which part of the tape diagram stands for the number of groups and which part represents the number in each group.
- S: The 4 boxes are the 4 groups. → The number 5 is how many are in each group.
- T: What repeated addition equation matches your diagram?
- S: $5 + 5 + 5 + 5 = 20$.
- T: So, you added 4 groups of five, or 4 fives. What new unit did you repeatedly add?
- S: 5.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students may benefit from writing the numerals within the groups and placing the counters on top of the written numeral. Then, have them remove the counters so they see only the abstract number.

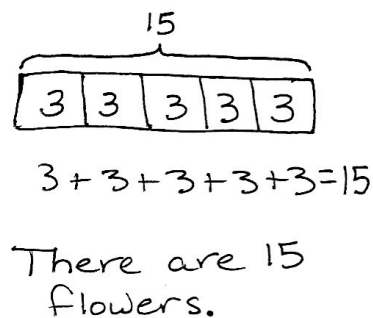
Allow students who show comprehension to move on to the Application Problem. Continue working with students who struggle, using concrete objects such as counters or linking cubes to model the problem and draw the tape diagram.

Application Problem (16 minutes)

The flowers are blooming in Maria's garden. There are 3 roses, 3 buttercups, 3 sunflowers, 3 daisies, and 3 tulips. How many flowers are there in all?

- Draw a tape diagram to match the problem.
- Write a repeated addition equation to solve.

Note: This problem is intended for independent practice. Students may write numbers or draw dots to represent the number of flowers in each group. The allotted time period includes 6 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.



$$3 + 3 + 3 + 3 + 3 = 15$$

There are 15 flowers.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Represent equal groups with tape diagrams, and relate to repeated addition.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1(b), what does the number 4 represent? Why are there 5 addends? What addition strategy did you use to find the total?
- For Problem 1(c), what does the 5 represent in the tape diagram? What strategy did you use to find the total?

Name Roberto Date _____

1. Write a repeated addition equation to find the total of each tape diagram.

a. 

$$\underline{2} + \underline{2} + \underline{2} + \underline{2} = \underline{8}$$
 4 groups of 2 = 8

b. 

$$\underline{4} + \underline{4} + \underline{4} + \underline{4} + \underline{4} = \underline{20}$$
 5 groups of 4 = 20

c. 

$$\underline{5} + \underline{5} + \underline{5} = \underline{15}$$
 3 groups of 5 = 15

d. 

$$\underline{3} + \underline{3} + \underline{3} + \underline{3} + \underline{3} + \underline{3} = \underline{18}$$
 6 groups of 3 = 18

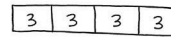
- For Problem 1(d), how many groups are there in the tape diagram? How many are in each group? What addition strategies could you use to find the total?
- For Problems 2(a) and (b), share your tape diagrams with a partner. What do you notice about these two problems? How are they the same and different?
- For Problems 2(c), (d), and (e), share your tape diagrams with a partner. What steps did you take when drawing your tape diagrams? How did you show the number of groups? How did you show the number in each group?

Exit Ticket (3 minutes)

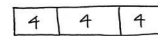
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Draw a tape diagram to find the total.

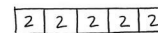
a. $3 + 3 + 3 + 3 = \underline{12}$



b. $4 + 4 + 4 = \underline{12}$

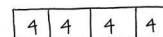


c. 5 groups of 2



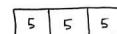
$2 + 2 + 2 + 2 + 2 = 10$

d. 4 groups of 4



$4 + 4 + 4 + 4 = 16$

e. 



$5 + 5 + 5 = 15$

A

Number Correct: _____

Adding Crossing Ten

1.	$9 + 1 =$	
2.	$9 + 2 =$	
3.	$9 + 3 =$	
4.	$9 + 9 =$	
5.	$8 + 2 =$	
6.	$8 + 3 =$	
7.	$8 + 4 =$	
8.	$8 + 9 =$	
9.	$9 + 1 =$	
10.	$9 + 4 =$	
11.	$9 + 5 =$	
12.	$9 + 8 =$	
13.	$8 + 2 =$	
14.	$8 + 5 =$	
15.	$8 + 6 =$	
16.	$8 + 8 =$	
17.	$9 + 1 =$	
18.	$9 + 7 =$	
19.	$8 + 2 =$	
20.	$8 + 7 =$	
21.	$9 + 1 =$	
22.	$9 + 6 =$	

23.	$7 + 3 =$	
24.	$7 + 4 =$	
25.	$7 + 5 =$	
26.	$7 + 9 =$	
27.	$6 + 4 =$	
28.	$6 + 5 =$	
29.	$6 + 6 =$	
30.	$6 + 9 =$	
31.	$5 + 5 =$	
32.	$5 + 6 =$	
33.	$5 + 7 =$	
34.	$5 + 9 =$	
35.	$4 + 6 =$	
36.	$4 + 7 =$	
37.	$4 + 9 =$	
38.	$3 + 7 =$	
39.	$3 + 9 =$	
40.	$5 + 8 =$	
41.	$2 + 8 =$	
42.	$4 + 8 =$	
43.	$1 + 9 =$	
44.	$2 + 9 =$	

B

Adding Crossing Ten

Number Correct: _____

Improvement: _____

1.	$8 + 2 =$	
2.	$8 + 3 =$	
3.	$8 + 4 =$	
4.	$8 + 8 =$	
5.	$9 + 1 =$	
6.	$9 + 2 =$	
7.	$9 + 3 =$	
8.	$9 + 8 =$	
9.	$8 + 2 =$	
10.	$8 + 5 =$	
11.	$8 + 6 =$	
12.	$8 + 9 =$	
13.	$9 + 1 =$	
14.	$9 + 4 =$	
15.	$9 + 5 =$	
16.	$9 + 9 =$	
17.	$9 + 1 =$	
18.	$9 + 7 =$	
19.	$8 + 2 =$	
20.	$8 + 7 =$	
21.	$9 + 1 =$	
22.	$9 + 6 =$	

23.	$7 + 3 =$	
24.	$7 + 4 =$	
25.	$7 + 5 =$	
26.	$7 + 8 =$	
27.	$6 + 4 =$	
28.	$6 + 5 =$	
29.	$6 + 6 =$	
30.	$6 + 8 =$	
31.	$5 + 5 =$	
32.	$5 + 6 =$	
33.	$5 + 7 =$	
34.	$5 + 8 =$	
35.	$4 + 6 =$	
36.	$4 + 7 =$	
37.	$4 + 8 =$	
38.	$3 + 7 =$	
39.	$3 + 9 =$	
40.	$5 + 9 =$	
41.	$2 + 8 =$	
42.	$4 + 9 =$	
43.	$1 + 9 =$	
44.	$2 + 9 =$	

Name _____

Date _____

1. Write a repeated addition equation to find the total of each tape diagram.

a.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$4 \text{ groups of } 2 = \underline{\quad\quad}$$

b.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$5 \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

c.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$3 \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

d.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$\underline{\quad\quad} \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

2. Draw a tape diagram to find the total.

a. $3 + 3 + 3 + 3 = \underline{\hspace{2cm}}$

b. $4 + 4 + 4 = \underline{\hspace{2cm}}$

c. 5 groups of 2

d. 4 groups of 4

e.



Name _____

Date _____

Draw a tape diagram to find the total.

1.   

2. 3 groups of 3

3. $2 + 2 + 2 + 2 + 2$

Name _____

Date _____

1. Write a repeated addition equation to find the total of each tape diagram.

a.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$4 \text{ groups of } 3 = \underline{\quad\quad}$$

b.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$5 \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

c.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$4 \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

d.



$$\underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$$

$$\underline{\quad\quad} \text{ groups of } \underline{\quad\quad} = \underline{\quad\quad}$$

2. Draw a tape diagram to find the total.

a. $5 + 5 + 5 + 5 = \underline{\hspace{2cm}}$

b. $4 + 4 + 4 + 4 + 4 = \underline{\hspace{2cm}}$

c. 4 groups of 2

d. 5 groups of 3

e.





Topic B

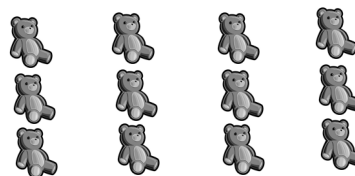
Arrays and Equal Groups

2.OA.4, 2.NBT.2

Focus Standard:	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Instructional Days:	5	
Coherence	-Links from:	G2–M3 Place Value, Counting, and Comparison of Numbers to 1,000
	-Links to:	G2–M8 Time, Shapes, and Fractions as Equal Parts of Shapes
		G3–M1 Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

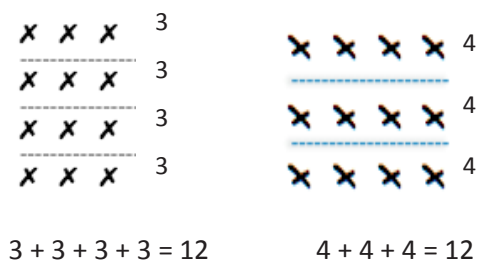
Topic B focuses on spatial relationships and structuring as students organize equal groups (from Topic A) into rectangular arrays. They build small arrays (up to 5 by 5) and use repeated addition of the number in each row or column (i.e., group) to find the total.

In Lesson 5, students compose arrays either one row or one column at a time and count to find the total using the scattered sets from Topic A. For example, they might arrange one row of 3 counters, followed by three more identical rows, to compose a 4 by 3 array of 12 counters. Then, students use the same equal groups to create an array, column by column (shown below). They count to find the total, noticing that each row and each column contain the same number of units. Thus, for 4 rows of 3 or 4 columns of 3, a student might observe, “There are 4 equal groups of 3.” This is foundational to the spatial structuring students need to discern a row or column as a single entity, or unit, when working with tiled arrays without gaps and overlaps in Topic C.



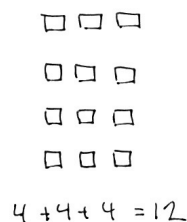
In Lesson 6, students decompose one array by both rows and columns. Thus, an array of 4 rows of 3 teddy bears can be pulled apart to show either 4 rows of 3 or 3 columns of 4. Also, students see that when another row or column is added or removed, so is another group, or unit. As Lesson 6 progresses, students move the objects of the arrays closer together so the gaps are smaller, forcing them to discern the rows and columns without the visual aid of spacing. For example, when decomposing a 4 by 3 array, students see the rows as equal groups of 3. After identifying the number in each row, or group, students realize that they can write a repeated addition sentence to find the total number of objects in the array: $3 + 3 + 3 + 3 = 12$. It may be noted that since there are 4 rows, the equation has 4 addends, or 4 threes. Students add from left to right and write the sum, such that 3 plus 3 equals 6, 6 plus 3 equals 9, and 9 plus 3 equals 12.

In Lesson 7, students move to the pictorial as they use math drawings to represent arrays and relate the drawings to repeated addition. For example, students are asked to draw an array with 4 rows of 3 or 3 rows of 4 on their personal white boards and then use their markers to draw horizontal lines to see the rows within the array (shown below). When counting rows containing 3 or 4 objects, students apply repeated addition strategies once again, adding from left to right to find the sum (e.g., $4 + 4 + 4 = 12$, such that 4 plus 4 equals 8 and 8 plus 4 equals 12). Additionally, when representing arrays with rows of 2 or 5, students may add to find the total and naturally point out a connection to skip-counting by twos or fives (**2.NBT.2**); the focus, however, is on establishing a strong connection between the array and repeated addition.



In Lesson 8, students work with square tiles to create arrays with gaps, composing the arrays from parts to whole, either one row or one column at a time. Seeing arrays as composed of individual, separated tiles provides the foundation for Topic C, where students work with square tiles without gaps. As usual, students relate the arrays to repeated addition.

In Lesson 9, students apply the work of Topic B to word problems involving repeated addition (shown below), interpreting array situations as either rows or columns and using the RDW process, (e.g., “Mrs. Levy moves desks into 3 columns of 4 desks. How many desks does she move?”) In addition to drawing objects, students may also represent the situation with more abstract tape diagrams, just as they did in the final lesson of Topic A.



A Teaching Sequence Toward Mastery of Arrays and Equal Groups

Objective 1: Compose arrays from rows and columns, and count to find the total using objects.
(Lesson 5)

Objective 2: Decompose arrays into rows and columns, and relate to repeated addition.
(Lesson 6)

Objective 3: Represent arrays and distinguish rows and columns using math drawings.
(Lesson 7)

Objective 4: Create arrays using square tiles with gaps.
(Lesson 8)

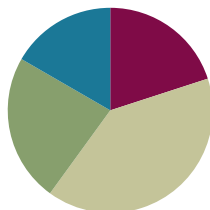
Objective 5: Solve word problems involving addition of equal groups in rows and columns.
(Lesson 9)

Lesson 5

Objective: Compose arrays from rows and columns, and count to find the total using objects.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(24 minutes)
Application Problem	(14 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Making the Next Ten to Add **2.NBT.5** (4 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)
- Happy Counting by Tens: Crossing 100 **2.NBT.2** (3 minutes)

Making the Next Ten to Add (4 minutes)

Note: This fluency activity reviews the make ten addition strategy.

T: When I say “9 + 4,” you say “10 + 3.” Ready? 9 + 4.

S: 10 + 3.

T: Answer.

S: 13.

Post on the board:

$$9 + 4 = \underline{\quad}$$

$$\wedge$$

$$1 \quad 3$$

$$10 + 3 = 13$$

Continue with the following possible sequence, one column at a time:

$19 + 4, 49 + 4$

$8 + 3, 18 + 3, 68 + 3$

$7 + 4, 17 + 4, 87 + 4$

$9 + 6, 19 + 6, 59 + 6$

$8 + 5, 18 + 5, 78 + 5$

$7 + 6, 17 + 6, 97 + 6$

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 1 Core Fluency Practice Sets)

Note: During Topic B and for the remainder of the year, each day’s Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 1.

Happy Counting by Tens: Crossing 100 (3 minutes)

Note: Students skip-count by tens as a foundation for counting rows and columns in today's lesson.

- T: This time, let's play Happy Counting, skip-counting by tens!
- T: Watch my fingers to know whether to count up or down. A closed hand means stop. (Show signals while explaining.)
- T: Let's count by tens, starting at 60. Ready? (Point up rhythmically until a change is desired. Close hand to indicate a stopping point. Point down to count in the opposite direction. Continue, periodically changing direction.)
- S: 60, 70, 80, 90, 100, 110, 120, 130, 140 (switch) 130, 120, 110, 100, 90 (switch) 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220 (switch) 210, 200, 190, 180.
- T: Excellent! Try it for 30 seconds with your partner, starting at 80. Partner A, you are the teacher today.

Concept Development (24 minutes)

Materials: (T/S) 6 counting bears, 12 beans

Call students to the carpet or communal area for Problem 1.

Problem 1: Make arrays with 6 objects.

- T: (Show 2 groups of 3 bears.)
- T: Look at my bears. How many are in each group?
- S: 3 bears!
- T: How many groups?
- S: 2 groups!
- T: How many bears altogether?
- S: 6 bears!
- T: How did you know?
- S: Because I counted 1, 2, 3, 4, 5, 6. $\rightarrow 3 + 3 = 6$.
- T: Turn and talk: How can we arrange these groups of 3 into **rows** of 3?
- S: Make them go straight across instead of in a group. $\rightarrow$ Line them up.
- T: To put them in rows, we can place them in straight horizontal lines.
- T: (Call on a student volunteer to organize the bears into 2 equal rows.)
- T: What do you notice about the rectangular **array** we just created?
- S: The same number is in each row. $\rightarrow$ It looks like a ten-frame, but there are only 3 in each row. $\rightarrow$ There are 2 groups of three.
- T: How many bears are in each row?
- S: 3 bears!



- T: How many rows are there?
 S: 2 rows!
 T: So, there are two equal groups of...?
 S: 3.
 T: How many bears altogether?
 S: 6 bears!
 T: Did the total number of bears change when I organized them into a rectangular array?
 S: No.
 T: (Scatter the counters so they are no longer arranged in an array.)
 T: Turn and talk: What if we want to arrange them into **two columns** of 3? Columns are groups that are arranged vertically, or up and down. Tell your partner what that would look like.
 S: It would have 3 on one side and 3 on the other side. → It would look the same as the other one but facing sideways.
 T: Let's try that. (Model arranging the counters into 2 columns of 3.)
 T: How many columns do you see?
 S: 2 columns!
 T: How many bears are in each column?
 S: 3 bears!
 T: So, there are two equal groups of...?
 S: 3.
 T: And the total number of bears is...?
 S: 6.
 T: Turn and talk: Is there another way I can group the bears other than into 2 groups of 3?
 S: They can stay in 1 group of 6. → You can make 6 groups of 1. → You can put them in 3 groups of 2.
 T: Yes. Let's try that! Organize your bears with your partner into 3 groups of 2.
 T: How many rows did you make?
 S: 3 rows!
 T: How many bears are in each row?
 S: 2 bears!
 T: Let's count together to find the total.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Make the connection with students between the words *array* and *arrange*. Show the spelling and how the roots relate, and discuss how an array is an orderly arrangement of things. This connection especially supports English language learners.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

The concept of *rows* and *columns* is introduced here to support the understanding of repeated addition rather than as a goal unto itself. Students should not be assessed on their understanding of rows and columns. Encourage correct answers regardless of directionality.



T: Count with me.

S: 1, 2, 3, 4, 5, 6.

T: Now, let's count them row by row. Count the top row with me.

S: 1, 2.

T: Move down, and count the next row with me.

S: 1, 2.

T: Let's count the last row together.

S: 1, 2.

T: How many groups of 2 was that?

S: 3 groups of 2.

T: This time, let's count the columns together. Start with the column on the left, and move from top to bottom with me.

S: 1, 2, 3.

MP.7

T: Count the other column.

S: 1, 2, 3.

T: How many bears are in each column?

S: 3 bears!

T: How many columns?

S: 2 columns!

T: How many bears are in the array altogether? How do you know?

S: There are 6 bears altogether. There are 3 in each column, and $3 + 3 = 6$. → I know that $2 + 2 + 2 = 6$.
→ I counted 2, 4, 6.

T: Did the number of bears change when we reorganized them into rows of two?

S: No!

Problem 2: Make arrays with 12 objects.

T: Let's try another one. (Take out 12 beans, and tell students to do the same.)

T: How can we put these beans into equal groups?

S: You can make groups of 2. → You can make groups of 3. → Put them into groups of 4.

T: Let's start with groups of 2. (Call on a volunteer to separate the beans into groups of two.)

T: How many groups of two beans did we make?

S: 6 groups.

T: Turn and talk: How can we arrange the 6 groups of two beans into a rectangular array?

S: You can make 6 rows of two. → You can make 6 columns of two. → Make 2 rows of six.

T: Let's start with 2 rows of six. (Model arranging the beans into 2 rows of six.)

T: How many rows did you make?

S: 2 rows.

- T: How many beans in each row?
 S: 6 beans.
 T: So, there are 2 equal groups of...?
 S: 6.

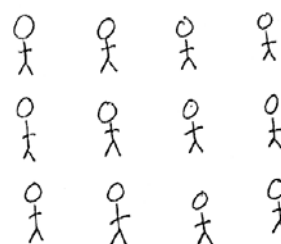
If students need more practice arranging objects into arrays, continue with the above sequence with the following numbers: 8, 9, and 20. Otherwise, allow students to begin the Application Problem.

Application Problem (14 minutes)

Mrs. White is in line at the bank. There are 4 teller windows, and 3 people are standing in line at each window.

- Draw an array to show the people in line at the bank.
- Write the total number of people.

Note: This Application Problem comes after the Concept Development to reinforce the concepts learned. The allotted time period includes 4 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.



12 people

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Compose arrays from rows and columns, and count to find the total using objects.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Roberto Date _____

1. Circle groups of four. Then draw the triangles into 2 equal rows.

2. Circle groups of two. Redraw the groups of two as rows and then as columns.

3. Circle groups of three. Redraw the groups of three as rows and then as columns.

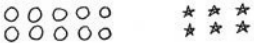
Any combination of the questions below may be used to lead the discussion.

- For Problem 1, how did circling equal groups prepare you for arranging the triangles into an **array**?
- For Problem 2, why did you make 3 **rows** or 3 **columns**? Could you have made an array with only 2 rows or columns? How?
- For Problem 3, what does the number of rows or columns represent? What does the number in each row or column represent? How does arranging the hearts into an array help you find the total more efficiently or easily?
- For Problem 5, show your partner how you redrew the arrays in Problem 4. What is the same between the new arrays and the old ones in Problem 4?
- For Problem 6, compare your arrays with a partner. How could you describe your arrays in terms of equal groups? How do rows and columns help us to organize groups?

4. Count the objects in the arrays from left to right by rows and by columns. As you count, circle the rows and then the columns.

a.  b. 

5. Redraw the circles and stars in Problem 4 as columns of two.



6. Draw an array with 15 triangles.



7. Show a different array with 15 triangles.



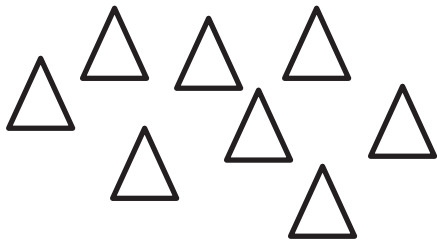
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

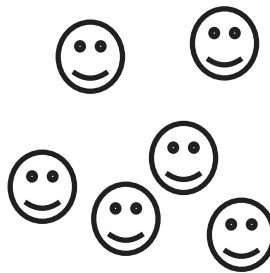
Name _____

Date _____

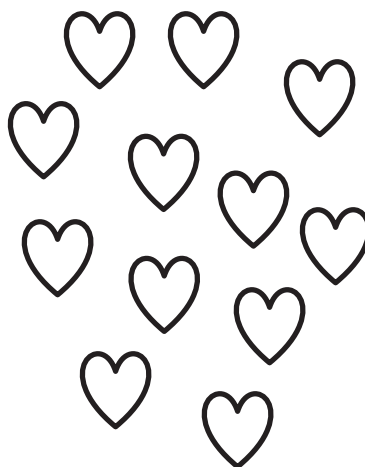
1. Circle groups of four. Then, draw the triangles into 2 equal rows.



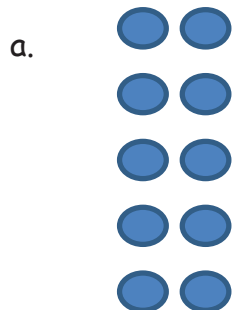
2. Circle groups of two. Redraw the groups of two as rows and then as columns.



3. Circle groups of three. Redraw the groups of three as rows and then as columns.



4. Count the objects in the arrays from left to right by rows and by columns. As you count, circle the rows and then the columns.



5. Redraw the circles and stars in Problem 4 as columns of two.

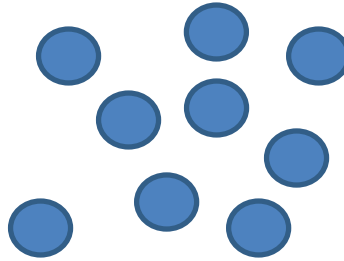
6. Draw an array with 15 triangles.

7. Show a different array with 15 triangles.

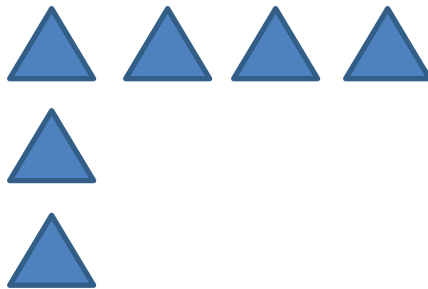
Name _____

Date _____

1. Circle groups of three. Redraw the groups of three as rows and then as columns.



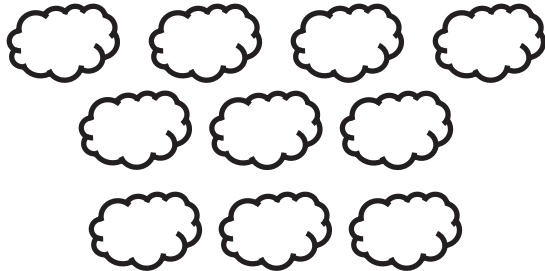
2. Complete the array by drawing more triangles. The array should have 12 triangles in all.



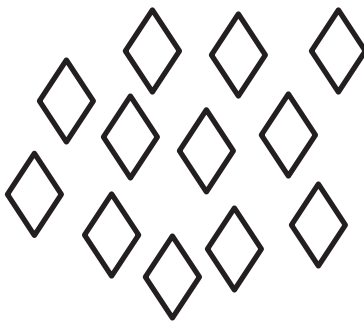
Name _____

Date _____

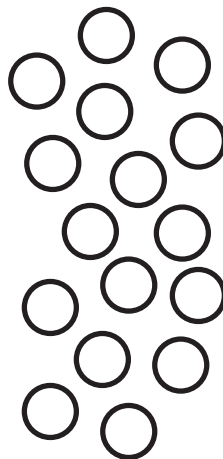
1. Circle groups of five. Then, draw the clouds into two equal rows.



2. Circle groups of four. Redraw the groups of four as rows and then as columns.



3. Circle groups of four. Redraw the groups of four as rows and then as columns.

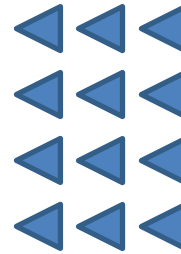


4. Count the objects in the arrays from left to right by rows and by columns. As you count, circle the rows and then the columns.

a.



b.



5. Redraw the smiley faces and triangles in Problem 4 as columns of three.

6. Draw an array with 20 triangles.

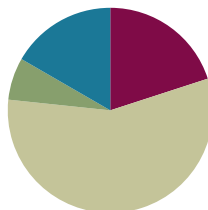
7. Show a different array with 20 triangles.

Lesson 6

Objective: Decompose arrays into rows and columns, and relate to repeated addition.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(4 minutes)
■ Concept Development	(34 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Making the Next Hundred Drill **2.NBT.5, 2.NBT.7** (4 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)
- Happy Counting by Tens: Crossing 100 **2.NBT.2** (3 minutes)

Making the Next Hundred Drill (4 minutes)

Note: This fluency activity reviews foundations that lead into today's lesson.

- T: (Write 170 on the board.) Let's find the missing part to make the next hundred. What is the next hundred?
- S: 200.
- T: If I say "170," you say the missing number needed to make 200. Ready? 170.
- S: 30.
- T: Tell me the addition sentence.
- S: $170 + 30 = 200$.

Continue with the following possible sequence: 190, 160, 260, 270, 370, 380, 580, 620, 720, 740, 840, 844, 846, 916, 914, and 924.

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 1 Core Fluency Practice Sets)

Note: During Topic B and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 1.

Happy Counting by Tens: Crossing 100 (3 minutes)

Note: Students skip-count by tens as a foundation for counting rows and columns in today's lesson.

T: Let's count by tens, starting at 90. Ready?

S: 90, 100, 110, 120, 130, 140 (switch) 130, 120, 110, 100, 90 (switch) 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220 (switch) 210, 200, 190, 180.

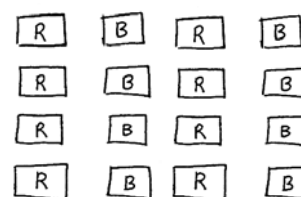
T: Excellent! Try it for 30 seconds with your partner, starting at 180. Partner B, you are the teacher today.

Application Problem (4 minutes)

Sam is organizing her greeting cards. She has 8 red cards and 8 blue cards. She puts the red cards in 2 columns and the blue ones in 2 columns to make an array.

- Draw a picture of Sam's greeting cards in the array.
- Write a statement about Sam's array.

Note: This Application Problem includes drawing a simple array in preparation for the Concept Development.



$4 + 4 + 4 + 4 = 16$
Sam has 16 cards.

Concept Development (34 minutes)

Materials: (T) 24 lima beans (or other counters), 21 1-inch tiles, ruler (S) Per pair: 21 1-inch tiles, 24 lima beans (or other counters), personal white board, 1 ruler

Distribute materials to students, and instruct them to create the arrays directly on their personal white boards. This way, they can count each row of beans and write the total at the end of the row. Then, they can write the repeated addition equation directly underneath the array.

T: (Show an array of 4 rows of 5 beans with a small space between each bean.)

T: How many rows do you see?

S: 4 rows.

T: Are my rows equal?

S: Yes!

T: For right now, let's call a row a group. How many equal groups are there?

S: 4 equal groups.

T: How many beans are in each group?

S: 5 beans.



T: I am going to pull this array apart so we can clearly see our 4 rows. (Using the ruler, separate the rows so there is space between each row as pictured.)

T: There are 5 beans in the first row. With your marker, write 5 to the right of the row. (Write 5 to the right of the row.)

T: There are 5 in the second row. (Write 5 to the right of the row as students do the same.) $5 + 5$ is...?

S: 10.

T: Add 5 more for the third row. (Write another 5 as students do the same.) $10 + 5$ is...?

S: 15.

MP.4 T: Add 5 more for the last row. (Write another 5 as students do the same.) $15 + 5$ is...?

S: 20.

T: Look at all these fives! (Point to the 4 fives along the right of the bean array.) What repeated addition equation can we write underneath to show the total number of beans?

S: $5 + 5 + 5 + 5 = 20$.

T: Yes! And how many addends do you see?

S: 4 addends!

T: So, there are 4 fives, and $5 + 5 + 5 + 5$ equals 20.

T: (Push the beans back together so there is no space between each row.)

T: Using your lima beans, work with your partner to make an array with 5 columns of 4 beans on your personal white board.

T: Watch now as I use my ruler to add space between each column. (Using the ruler, separate the columns so there are about 2 inches between each one, as students do the same.)

T: How many columns do you see?

S: 5 columns.

T: Are the columns equal?

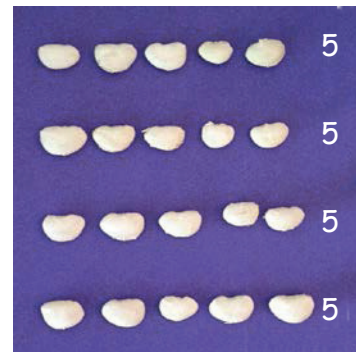
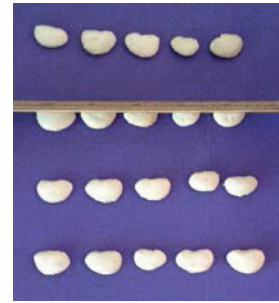
S: Yes!

T: Now, let's say a column is a group. How many equal groups are there?

S: 5 equal groups!

T: Let's count the number in each group.

S: (Count the beans.) 4 beans.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Set up games for students that involve arrays of different numbers of objects. For example, set up a memory game with 5 rows of 5 cards each. Then, have students try to keep track of where a card appeared by noting its position mentally (e.g., third row, second column).

- T: (Write 4 at the bottom of each column, just as with the rows.)
- T: If there are 4 beans in the first column, how many beans are in each column?
- S: 4 beans!
- T: Turn and talk: Can we make this into a repeated addition equation to find the total number of beans?
- S: Yes. Put a plus sign between each number. → Add the number in each group to find the total.
- T: Let's count to find the total.

T: $4 + 4$ is...?

S: 8.

T: Add 4 more.

S: 12.

T: Add 4 more.

S: 16.

T: And the last 4.

S: 20.

T: Do we have the same total number of beans as before?

S: Yes!

T: (Add a column of 4 beans to the right of the array as students do the same.)

T: How many columns do we have now?

S: 6 columns!

T: Turn and talk: How many beans do we have now? How do you know?

S: There are 24 beans. I know because $4 + 4 + 4 + 4 + 4 + 4 = 24$. → I know there are 24 beans now because there were 20, and I added on 4 more. → We added another group of 4, so there are 24.

T: Let's try another. With your partner, create an array with 3 rows of 6 tiles.

S: (Create an array with partners.)

T: How many equal groups of 6 do you see?

S: 3 equal groups!

T: Turn and talk: What repeated addition equation would you use to find the total?

S: $6 + 6 + 6 = 18$.

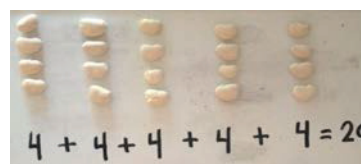
T: How many groups, or addends, are there?

S: 3 groups!

T: So, there are 3 sixes! And if you add them, $6 + 6 + 6$, you have 18.

T: (Push the tiles back together.) Turn and talk: How many columns do you see, and how many tiles are in each column?

S: There are 6 columns of 3 tiles.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students actively make the connection with multiplication at this point. Encourage them to also notice the connections between repeated addition of groups of objects and the multiplication facts.

T: Let's check that using our ruler. (Use the ruler to separate the tiles into 6 columns of 3 tiles.)

T: How many columns?

S: 6 columns!

T: How many tiles in each column?

S: 3 tiles!

T: Turn and talk: What repeated addition equation would you use to find the total?

S: $3 + 3 + 3 + 3 + 3 + 3 = 18$.

T: We made an array with 3 rows of 6 and 6 columns of 3, and the total was the same!

T: (Add another column of 3 tiles.)

T: Turn and talk: How many tiles are there now, and how do you know?

S: There are 21 tiles, because $3 + 3 + 3 + 3 + 3 + 3 = 18$. 18 plus 3 more is 21. → I know there were 18, so I added 3 more, and that makes 21.

Repeat the above process with an array of 2 by 7, and draw students' attention to the fact that when they remove a column or row, they remove a unit. Repeat as needed with arrays of 3 by 4 and 4 by 4. Afterward, allow students to move on to the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Decompose arrays into rows and columns, and relate to repeated addition.

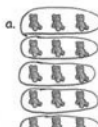
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Roberto Date _____

1. Complete each missing part describing each array.

a. Circle rows.



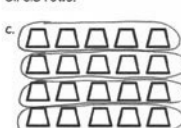
5 rows of 3 = 15
 $\underline{3} + \underline{3} + \underline{3} + \underline{3} + \underline{3} = \underline{15}$

b. Circle columns.



3 columns of 5 = 15
 $\underline{5} + \underline{5} + \underline{5} = \underline{15}$

c. Circle rows.



4 rows of 5 = 20
 $\underline{5} + \underline{5} + \underline{5} + \underline{5} = \underline{20}$

d. Circle columns.

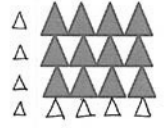


5 columns of 4 = 20
 $\underline{4} + \underline{4} + \underline{4} + \underline{4} + \underline{4} = \underline{20}$

Any combination of the questions below may be used to lead the discussion.

- For Problems 1(a) and (b), describe the arrays. Discuss how each repeated addition equation, or number sentence, matches the array.
- For Problems 1(c) and (d), describe the arrays. How many shapes are in each row or column? How does this match the repeated addition equation?
- For Problem 2, describe two different ways to break apart, or decompose, the 3 by 4 array using rows or columns. How is decomposing arrays similar to decomposing numbers?
- For Problem 2(d), how would adding one more row change the repeated addition equation?
- For Problem 3(a), did you write the repeated addition equation in terms of rows or columns? Why didn't it matter?
- For Problem 3(d), how did removing one row change the repeated addition equation?

2. Use the array of triangles to answer the questions below.



a. 3 rows of 4 = 12

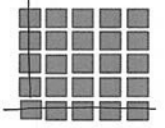
b. 4 columns of 3 = 12

c. 4 + 4 + 4 = 12

d. Add 1 more row. How many triangles are there now? 16

e. Add 1 more column to the new array you made in 2(d). How many triangles are there now? 20

3. Use the array of squares to answer the questions below.



a. 5 + 5 + 5 + 5 + 5 = 25

b. 5 rows of 5 = 25

c. 5 columns of 5 = 25

d. Remove 1 row. How many squares are there now? 20

e. Remove 1 column from the new array you made in 3(d). How many squares are there now? 16

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. Complete each missing part describing each array.

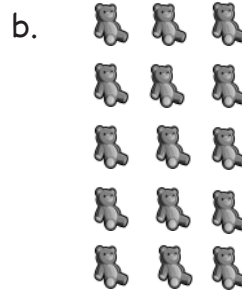
Circle rows.



5 rows of _____ = _____

_____ + _____ + _____ + _____ + _____ = _____

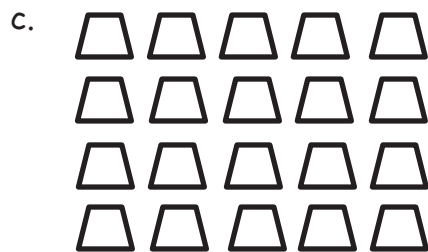
Circle columns.



3 columns of _____ = _____

_____ + _____ + _____ = _____

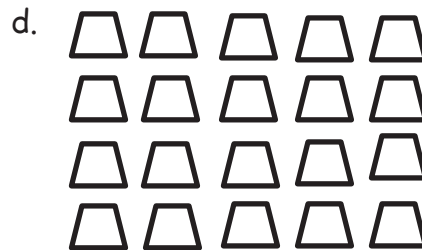
Circle rows.



4 rows of _____ = _____

_____ + _____ + _____ + _____ = _____

Circle columns.



5 columns of _____ = _____

_____ + _____ + _____ + _____ + _____ = _____

2. Use the array of triangles to answer the questions below.

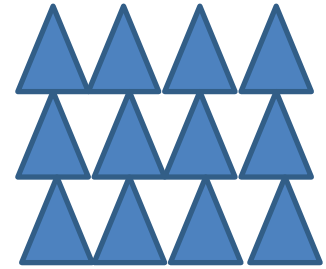
a. ____ rows of ____ = 12

b. ____ columns of ____ = 12

c. ____ + ____ + ____ = ____

d. Add 1 more row. How many triangles are there now? ____

e. Add 1 more column to the new array you made in 2(d). How many triangles are there now? ____



3. Use the array of squares to answer the questions below.

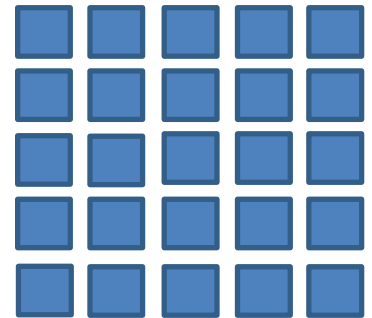
a. ____ + ____ + ____ + ____ + ____ = ____

b. ____ rows of ____ = ____

c. ____ columns of ____ = ____

d. Remove 1 row. How many squares are there now? ____

e. Remove 1 column from the new array you made in 3(d). How many squares are there now? ____



Name _____

Date _____

Use the array to answer the questions below.



a. _____ rows of _____ = _____

b. _____ columns of _____ = _____

c. _____ + _____ + _____ + _____ = _____

d. Add 1 more row. How many stars are there now? _____

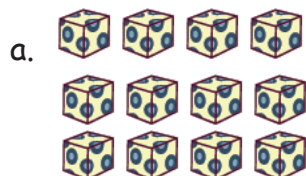
e. Add 1 more column to the new array you made in (d). How many stars are there now? _____

Name _____

Date _____

1. Complete each missing part describing each array.

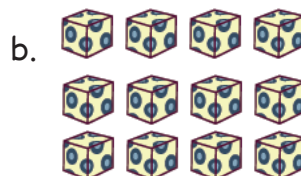
Circle rows.



3 rows of _____ = _____

_____ + _____ + _____ = _____

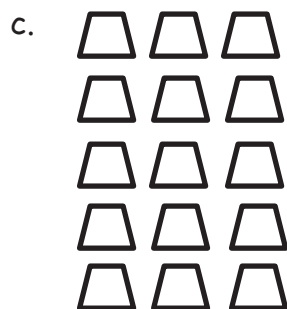
Circle columns.



4 columns of _____ = _____

_____ + _____ + _____ + _____ = _____

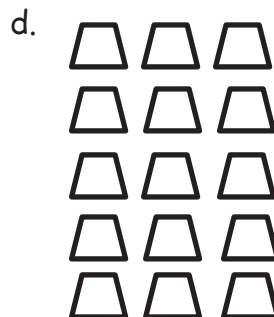
Circle rows.



5 rows of _____ = _____

_____ + _____ + _____ + _____ + _____ = _____

Circle columns.



3 columns of _____ = _____

_____ + _____ + _____ = _____

2. Use the array of smiley faces to answer the questions below.

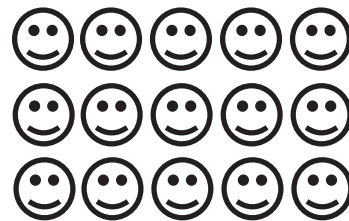
a. ____ rows of ____ = ____

b. ____ columns of ____ = ____

c. ____ + ____ + ____ = ____

d. Add 1 more row. How many smiley faces are there now? ____

e. Add 1 more column to the new array you made in 2(d). How many smiley faces are there now? ____



3. Use the array of squares to answer the questions below.

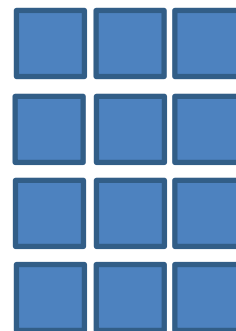
a. ____ + ____ + ____ + ____ = ____

b. ____ rows of ____ = ____

c. ____ columns of ____ = ____

d. Remove 1 row. How many squares are there now? ____

e. Remove 1 column from the new array you made in 3(d). How many squares are there now? ____

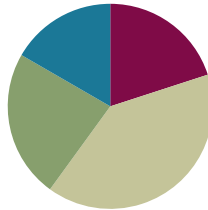


Lesson 7

Objective: Represent arrays and distinguish rows and columns using math drawings.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(24 minutes)
■ Application Problem	(14 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Coin Drop **2.OA.2** (3 minutes)
- Sprint: Sums to the Teens **2.NBT.5** (9 minutes)

Coin Drop (3 minutes)

Materials: (T) 10 dimes, 30 pennies, metal can or plastic container

Note: In this fluency activity, students prepare for Module 7 by adding and subtracting ones and tens using coins.

- T: (Hold up a penny.) Name my coin.
 S: A penny.
 T: How much is it worth?
 S: 1 cent.
 T: Listen carefully as I drop coins in my can. Count along in your minds.

Drop in some pennies, and ask how much money is in the can. Take out some pennies, and show students. Ask how much money is still in the can. Continue adding and subtracting pennies for a minute or so. Repeat the activity with dimes and then with dimes and pennies.

Sprint: Sums to the Teens (9 minutes)

Materials: (S) Sums to the Teens Sprint

Note: This Sprint gives practice in the grade level fluency of sums to 20.

Concept Development (24 minutes)

Materials: (S) Personal white board, 30 lima beans (per pair)

Note: Assign student partners with one Partner A and one Partner B.

- T: In our last lesson, we composed arrays with objects. Let's review together. With your partner and your lima beans, show me 3 groups of 5 on each of your personal white boards. Let's say that for right now, a group is a column.
- T: Remind your partner, should the column be vertical or horizontal?
- S: Vertical!
- T: Count with your partner to make each column.
- S: (Count as 5 beans are laid down in a column, then 5 more, and then 5 more.)
- T: Now, add a line between each column.
- S: (Draw lines.)
- T: These lines between each column will help us keep our array organized. They also help us see each column as a group, or unit, of 5.
- T: How many columns did you make?
- S: 3 columns!
- T: How many beans in each column?
- S: 5 beans!
- T: What repeated addition sentence can we use to find the total?
- S: $5 + 5 + 5 = 15$.
- T: Now, Partner A, change your array to show 5 rows of three. Don't forget to add lines between the rows.
- S: (Reorganize the beans.)
- T: Turn and talk with your partner: How do the arrays look similar, and how are they different?
- S: They look the same, but the lines show rows instead of columns. → They both have a total of 15 beans.
- T: Partner A, erase your lines between the rows of beans. Now, draw lines to show the columns.
- T: Partner B, erase your lines between the columns, and redraw them to show the rows.
- T: (Prompt students to compare their arrays again.)

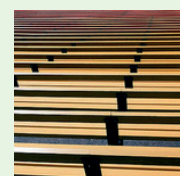


NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Post a picture of Greek or Roman columns with the word *column* beneath it and a photo of rows, such as rows of bleacher seats, to help students recall what the words mean.



Column



Row

T: If we want to show 3 rows of 5, what do we do first? Should we make a row first or organize the beans into equal groups of three?

S: Make a row of 5. → The beans are already in groups. Just move them to make rows.

T: Remind me, should the row be vertical or horizontal?

S: Horizontal!

T: With your partner, create an array using the same beans. Make 3 rows of 5. Then, draw lines between your rows to keep them organized.

S: (Make array.)

T: How many rows did you make?

S: 3 rows!

T: How many beans in each row?

S: 5 beans!

MP.4 T: So, each row is also a group, or unit, of...?

S: 5.

T: Say the repeated addition sentence.

S: $5 + 5 + 5 = 15$.

T: What was the total for 3 columns of 5 beans?

S: 15.

T: Yes! And what is the total for 3 rows of 5 beans?

S: 15.

T: So, how are 3 columns of 5 and 3 rows of 5 related?

S: They equal the same amount! → It's just a different way of grouping things to make 15.

T: This time, let's draw our arrays. (Direct students to put their beans away.)

T: If we want to make an array of 4 columns of 2, what should we do first?

S: Make a column of 2. → Start with the first column.

T: Yes. Follow me! (Draw a column with 2 X's, as students do the same on their personal white boards.)

T: (Model drawing the first column in the array as pictured.)

T: Turn and talk: How can we finish the array from here?

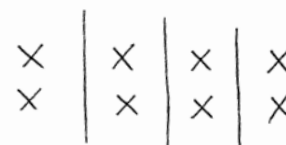
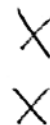
S: Draw more columns of 2. → Draw another group, and then another group, and then another group. → Draw 3 more groups with 2 X's in each one.

T: Let's do that together. What I draw, you draw. (Model drawing 3 more columns of two with vertical lines in between each group, as students do the same.)



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Some students may want to distribute one X each into the four columns and build the arrays that way. Have these students show and explain this alternate procedure to the class after the modeling portion of the Concept Development.



T: What do the vertical lines remind us of?

S: That each column is a group. → It separates the groups of two. → It's like each column is its own unit of two.

T: (Circulate to check for understanding as students complete the arrays.)

T: Turn and talk: What repeated addition sentence can we use to find the total?

S: $2 + 2 + 2 + 2 = 8$.

T: Now, let's switch it like we did before. Show me 4 rows of 2 X's. Write an equation to find the total. (Repeat the process above for the new array. Circulate as students draw 4 rows of 2, reminding them to draw horizontal lines between each row.)

$$\begin{array}{cc|c} \times & \times & 2 \\ \hline \times & \times & 2 \\ \hline \times & \times & 2 \\ \hline \times & \times & 2 \end{array} \quad 2+2+2+2=8$$

T: What was the total for 4 columns of 2 X's?

S: 8.

T: And what is the total for 4 rows of 2 X's?

S: 8.

T: This time, you will draw your array without my help.

T: Draw an array with 3 columns of 4 X's. Don't forget your vertical lines in between each column! Write a repeated addition equation below your array to find the total.

T: (Circulate as students draw the array and write the corresponding equation.)

T: How many columns did you draw?

S: 3 columns.

T: How many X's are in each column?

S: 4 X's.

T: So, each column is a group, or unit, of...?

S: 4.

T: What repeated addition equation can we use to find the total?

S: $4 + 4 + 4 = 12$.

T: Turn and talk: What would happen if you added one more column of 4?

S: The total would go up by 4. → There would be another group of 4. → The total would be 16.

T: Add another column of 4 to your array, and show me your new equation.

T: (Circulate as students add another column of 4 to their arrays and change their equations.)

T: Let's read our repeated addition equations together, including the total.

S: $4 + 4 + 4 + 4 = 16$.

T: Turn and talk: What would happen now if you erased two columns of 4? Go ahead and try that. Don't forget about your equation! (Circulate as students erase two columns and change their equations.)

$$\begin{array}{c|c|c} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{array} \quad 4 + 4 + 4 = 12$$

T: What happened?

S: The total went down by 8. → There are only 2 columns of 4. → Since we took away 8, I thought $16 - 8 = 8$.

T: Let's read our new repeated addition sentence together, including the total.

S: $4 + 4 = 8$.

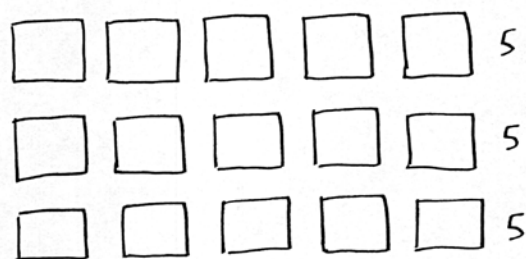
Complete the Concept Development by having students create an array with 4 rows of 3. Then, direct them to move on to the Application Problem.

Application Problem (14 minutes)

Bobby puts 3 rows of tile in his kitchen to make a design. He lays 5 tiles in each row.

- Draw a picture of Bobby's tiles.
- Write a repeated addition equation to solve for the total number of tiles Bobby used.

Note: This Application Problem comes after the Concept Development to give students the chance to practice new material. The allotted time period includes 4 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.



$$5 + 5 + 5 = 15$$

Bobby used 15 tiles total.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Represent arrays and distinguish rows and columns using math drawings.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problems 1(a) and (b), how do the vertical and horizontal lines help us?
- For Problems 2(a) and (b), compare your array drawings. How are they similar and different? Why did you write the same equation for both arrays?

- For Problem 3, share your array drawing with a partner. Which lines did you draw within the array: vertical or horizontal? Why? How many X's were in each group? Why is it important to know this?
- For Problem 4, share your new array with a partner. How did your drawing and equation change when you added 1 more row?
- For Problem 5, share your new array with a partner. How did your drawing and equation change when you removed 1 column? How many were in each group then? How did you group the X's to write a repeated addition equation (i.e., by groups of 2 or 5)?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

1.

a. One row of an array is drawn below. Complete the array with Xs to make 3 rows of 4. Draw horizontal lines to separate the rows.

```

X X X X
X X X X
X X X X

```

b. Draw an array with Xs that has 3 columns of 4. Draw vertical lines to separate the columns. Fill in the blanks.

```

X | X | X
X | X | X
X | X | X
X | X | X

```

$4 + 4 + 4 = 12$
3 rows of 4 = 12
3 columns of 4 = 12

2.

a. Draw an array of Xs with 5 columns of three.

```

X X X X X
X X X X X
X X X X X

```

b. Draw an array of Xs with 5 rows of three. Fill in the blanks below.

```

X X X
X X X
X X X
X X X
X X X

```

$3 + 3 + 3 + 3 + 3 = 15$
5 columns of three = 15
5 rows of three = 15

In the following problems, separate the rows or columns with horizontal or vertical lines.

3. Draw an array of Xs with 4 rows of 3.

```

X X X
X X X
X X X
X X X

```

$3 + 3 + 3 + 3 = 12$
4 rows of 3 = 12

4. Draw an array of Xs with 1 more row of 3 than the array in Problem 3. Write a repeated addition equation to find the total number of Xs.

```

X X X
X X X
X X X
X X X
X X X

```

$3 + 3 + 3 + 3 + 3 = 15$

5. Draw an array of Xs with 1 less column of 5 than the array in Problem 4. Write a repeated addition equation to find the total number of Xs.

```

X | X
X | X
X | X
X | X
X | X

```

$5 + 5 = 10$

A

Number Correct: _____

Sums to the Teens

1.	$9 + 2 =$	
2.	$9 + 3 =$	
3.	$9 + 4 =$	
4.	$9 + 7 =$	
5.	$7 + 9 =$	
6.	$10 + 1 =$	
7.	$10 + 2 =$	
8.	$10 + 3 =$	
9.	$10 + 8 =$	
10.	$8 + 10 =$	
11.	$8 + 3 =$	
12.	$8 + 4 =$	
13.	$8 + 5 =$	
14.	$8 + 9 =$	
15.	$9 + 8 =$	
16.	$7 + 4 =$	
17.	$10 + 5 =$	
18.	$6 + 5 =$	
19.	$7 + 5 =$	
20.	$9 + 5 =$	
21.	$5 + 9 =$	
22.	$10 + 6 =$	

23.	$4 + 7 =$	
24.	$4 + 8 =$	
25.	$5 + 6 =$	
26.	$5 + 7 =$	
27.	$3 + 8 =$	
28.	$3 + 9 =$	
29.	$2 + 9 =$	
30.	$5 + 10 =$	
31.	$5 + 8 =$	
32.	$9 + 6 =$	
33.	$6 + 9 =$	
34.	$7 + 6 =$	
35.	$6 + 7 =$	
36.	$8 + 6 =$	
37.	$6 + 8 =$	
38.	$8 + 7 =$	
39.	$7 + 8 =$	
40.	$6 + 6 =$	
41.	$7 + 7 =$	
42.	$8 + 8 =$	
43.	$9 + 9 =$	
44.	$4 + 9 =$	

B

Sums to the Teens

Number Correct: _____

Improvement: _____

1.	$10 + 1 =$	
2.	$10 + 2 =$	
3.	$10 + 3 =$	
4.	$10 + 9 =$	
5.	$9 + 10 =$	
6.	$9 + 2 =$	
7.	$9 + 3 =$	
8.	$9 + 4 =$	
9.	$9 + 8 =$	
10.	$8 + 9 =$	
11.	$8 + 3 =$	
12.	$8 + 4 =$	
13.	$8 + 5 =$	
14.	$8 + 7 =$	
15.	$7 + 8 =$	
16.	$7 + 4 =$	
17.	$10 + 4 =$	
18.	$6 + 5 =$	
19.	$7 + 5 =$	
20.	$9 + 5 =$	
21.	$5 + 9 =$	
22.	$10 + 8 =$	

23.	$5 + 6 =$	
24.	$5 + 7 =$	
25.	$4 + 7 =$	
26.	$4 + 8 =$	
27.	$4 + 10 =$	
28.	$3 + 8 =$	
29.	$3 + 9 =$	
30.	$2 + 9 =$	
31.	$5 + 8 =$	
32.	$7 + 6 =$	
33.	$6 + 7 =$	
34.	$8 + 6 =$	
35.	$6 + 8 =$	
36.	$9 + 6 =$	
37.	$6 + 9 =$	
38.	$9 + 7 =$	
39.	$7 + 9 =$	
40.	$6 + 6 =$	
41.	$7 + 7 =$	
42.	$8 + 8 =$	
43.	$9 + 9 =$	
44.	$4 + 9 =$	

Name _____

Date _____

1. a. One row of an array is drawn below. Complete the array with X's to make 3 rows of 4. Draw horizontal lines to separate the rows.

X X X X

- b. Draw an array with X's that has 3 columns of 4. Draw vertical lines to separate the columns. Fill in the blanks.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$3 \text{ rows of } 4 = \underline{\quad}$$

$$3 \text{ columns of } 4 = \underline{\quad}$$

2. a. Draw an array of X's with 5 columns of three.

- b. Draw an array of X's with 5 rows of three. Fill in the blanks below.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$5 \text{ columns of } three = \underline{\quad}$$

$$5 \text{ rows of } three = \underline{\quad}$$

In the following problems, separate the rows or columns with horizontal or vertical lines.

3. Draw an array of X's with 4 rows of 3.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$4 \text{ rows of } 3 = \underline{\hspace{1cm}}$$

4. Draw an array of X's with 1 more row of 3 than the array in Problem 3. Write a repeated addition equation to find the total number of X's.

5. Draw an array of X's with 1 less column of 5 than the array in Problem 4. Write a repeated addition equation to find the total number of X's.

Name _____ Date _____

Use horizontal or vertical lines to separate the rows or columns.

1. Draw an array of X's with 3 rows of 5.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$3 \text{ rows of } 5 = \underline{\quad}$$

2. Draw an array of X's with 1 more row than the above array. Write a repeated addition equation to find the total number of X's.

Name _____

Date _____

1. a. One row of an array is drawn below. Complete the array with X's to make 4 rows of 5. Draw horizontal lines to separate the rows.

 X X X X X

- b. Draw an array with X's that has 4 columns of 5. Draw vertical lines to separate the columns. Fill in the blanks.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$4 \text{ rows of } 5 = \underline{\hspace{1cm}}$$

$$6 \text{ columns of } 5 = \underline{\hspace{1cm}}$$

2. a. Draw an array of X's with 3 columns of 4.

- b. Draw an array of X's with 3 rows of 4. Fill in the blanks below.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$3 \text{ columns of } 4 = \underline{\hspace{1cm}}$$

$$3 \text{ rows of } 4 = \underline{\hspace{1cm}}$$

In the following problems, separate the rows or columns with horizontal or vertical lines.

3. Draw an array of X's with 3 rows of 3.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$3 \text{ rows of } 3 = \underline{\quad}$$

4. Draw an array of X's with 2 more rows of 3 than the array in Problem 3. Write a repeated addition equation to find the total number of X's.

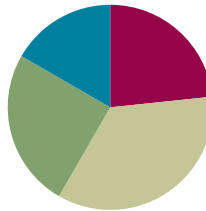
5. Draw an array of X's with 1 less column than the array in Problem 4. Write a repeated addition equation to find the total number of X's.

Lesson 8

Objective: Create arrays using square tiles with gaps.

Suggested Lesson Structure

Fluency Practice	(14 minutes)
Concept Development	(21 minutes)
Application Problem	(15 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (14 minutes)

- Using the Nearest Ten to Subtract **2.NBT.5** (5 minutes)
- Sprint: Subtraction from Teens **2.OA.2** (9 minutes)

Using the Nearest Ten to Subtract (5 minutes)

Note: Students use bonds of 10 when subtracting as a mental strategy to help subtract fluently with larger numbers.

T: (Post $16 - 9$ on the board.) Raise your hand when you know $16 - 9$.

S: 7.

T: (Write in the bond.) $10 - 9$ is...?

S: 1.

T: $6 + 1$ is...?

S: 7.

T: $16 - 9$ again is...?

S: 7.

T: (Post $26 - 9$ on the board.) Raise your hand when you know $26 - 9$.

S: 17.

T: (Write in the bond.) $10 - 9$ is...?

S: 1.

T: $16 + 1$ is...?

S: 17.

T: $26 - 9$ again is...?

S: 17.

$$\begin{array}{r} 16 - 9 = 7 \\ \swarrow \searrow \\ 6 \quad 10 \\ \quad \quad 1 \end{array}$$

$$\begin{array}{r} 26 - 9 = 17 \\ \swarrow \searrow \\ 16 \quad 10 \\ \quad \quad 1 \end{array}$$

Continue with the following possible sequence:

- a. $36 - 9$, $46 - 9$
- b. $11 - 9$, $21 - 9$, $41 - 9$
- c. $12 - 8$, $22 - 8$, $42 - 8$
- d. $11 - 8$, $41 - 8$, $61 - 8$
- e. $25 - 9$, $26 - 9$, $27 - 9$
- f. $35 - 19$, $45 - 19$, $55 - 9$

Sprint: Subtraction from Teens (9 minutes)

Materials: (S) Subtraction from Teens Sprint

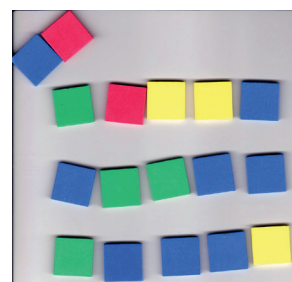
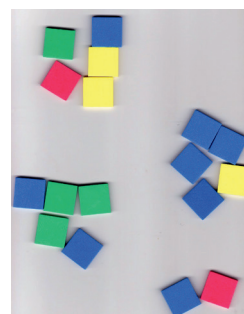
Note: This Sprint builds fluency with the grade level fluency goal of subtracting within 20 using mental strategies.

Concept Development (21 minutes)

Materials: (T/S) 25 square tiles, ruler

Distribute materials to each student before beginning the lesson.

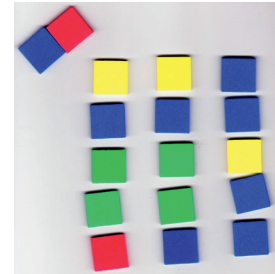
- T: Take out 17 tiles, and separate them into groups of 5 without putting them in rows or columns.
- S: (Create equal groups of 5 tiles.)
- T: How many groups of 5 did you make?
- S: 3 groups!
- T: Were there any tiles left?
- S: Yes!
- T: Why not put them into a group?
- S: That's not enough to make another group of 5. → They are extra.
- T: Put your remaining tiles, the ones not in a group, off to the side.
- T: Arrange your groups into 3 equal rows to make an array.
- T: How many rows, or groups, do you see?
- S: 3 rows!
- T: How many tiles are in each group?
- S: 5 tiles!
- T: What repeated addition sentence can we use to find the total?



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Deepen the meaning of the word *array* by showing real-life examples, such as arrays of solar panels in the desert, soldiers on parade, or even bumps on a Lego piece.

- S: $5 + 5 + 5 = 15$.
- T: Describe the array using the number of rows and the number of tiles in each row.
- S: Three rows of 5 tiles.
- T: This time, let's use the same groups, but make columns instead. (Pause as students work.)
- T: How many columns, or groups, do you see?
- S: 3 columns!
- T: How many tiles are in each group?
- S: 5 tiles!
- T: So, how many fives are there?
- S: 3 fives!
- T: Describe the array using the number of columns and the number of tiles in each column.
- S: Three columns of 5 tiles.
- T: What repeated addition equation can we use to find the total?
- S: $5 + 5 + 5 = 15$.
- T: Turn and talk: What is the same or different about the two arrays we made?
- S: They have the same total. → We made one with rows and one with columns. → They both had the same number of rows and columns. → They both had 3 groups of 5.



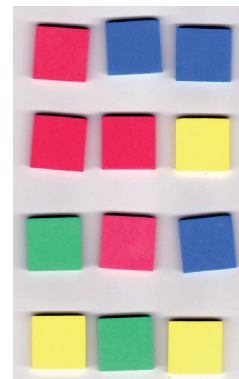
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students who struggle to line up manipulatives may benefit from the use of a Rekenrek, which makes even rows for them.

Repeat the above process with the following sequence: 15 tiles for a 2 by 7 array, 17 tiles for a 4 by 4 array, and 23 tiles for a 4 by 5 array.

MP.4

- T: Now, let's make some arrays one row at a time. Place 3 tiles in one row.
- S: (Make 1 row of 3.)
- T: Now, keep adding a row and then another row until you have a total of 12 tiles.
- S: (Add 3 more rows of 3.)
- T: How many rows of 3 did you make?
- S: 4 rows!
- T: How many equal groups of 3 did you make?
- S: 4 equal groups!
- T: Say the repeated addition equation to find the total.
- S: $3 + 3 + 3 + 3 = 12$.
- T: Describe the array using the number of rows and the number of tiles in each row.
- S: 4 rows of 3 tiles.



- T: This time, let's use our rulers to push our rows together to make one rectangle without spaces in between. (Model for students, if necessary.)
- S: (Push the tiles together using rulers.)
- T: Now, using your ruler, separate your array into columns. (Model for students, if necessary.)
- S: (Separate the columns using rulers.)
- T: How many columns did you make?
- S: 3 columns!
- T: How many tiles are in each column?
- S: 4 tiles!
- T: How many equal groups of 4 did you make?
- S: 3 equal groups!
- T: So, what repeated addition equation can we use to find the total?
- S: $4 + 4 + 4 = 12$.
- T: Describe the array using the number of columns and the number of tiles in each column.
- S: Three columns of 4 tiles.
- T: Did the total number of tiles change?
- S: No!
- T: Does an array made of 4 rows of 3 show the same total as an array made of 3 columns of 4?
- S: Yes!

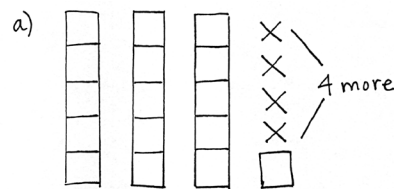
Repeat the above process with the following sequence: 2 by 4, 3 by 2, and 4 by 4.

Application Problem (15 minutes)

Charlie has 16 blocks in his room. He wants to build equal towers with 5 blocks each.

- Draw a picture of Charlie's towers.
- How many towers can Charlie make?
- How many more blocks does Charlie need to make equal towers of 5?

Note: This Application Problem comes after the Concept Development to give students an opportunity to apply the skills learned during the lesson. Students practice moving from the concrete to the pictorial stage of understanding in this problem, which prepares them for the Problem Set. The allotted time period includes 5 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.



- Charlie can make 3 towers. He has one block left over.
- Charlie needs 4 more blocks to make equal towers of 5.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Create arrays using square tiles with gaps.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1, how did you determine how many squares to put in each row? Describe the array: "There are ____ rows of ____."
- For Problem 2, how did you determine how many squares to put in each column? How is this array different from Problem 1, even though the total is the same?
- For Problem 4, compare your answers with a partner's. What repeated addition equation would describe 4 rows of 3? Could you redraw the squares to show an array with equal groups of 2? What would the repeated addition equation look like?
- For Problems 5 and 6, what steps did you take to draw the arrays? How many squares were in each group?

Name Roberto Date _____

1. Create an array with the squares.

2. Create an array with the squares from the set above.

3. Use the array of squares to answer the questions below.

a. There are 5 squares in each row.
 b. $\underline{5} + \underline{5} = \underline{10}$
 c. There are 2 squares in each column.
 d. $\underline{2} + \underline{2} + \underline{2} + \underline{2} + \underline{2} = \underline{10}$

4. Use the array of squares to answer the questions below.

a. There are 3 squares in one row.
 b. There are 4 squares in one column.
 c. $\underline{4} + \underline{4} + \underline{4} = \underline{12}$
 d. 3 columns of 4 = 4 rows of 3 = 12 total

5.

a. Draw an array with 8 squares that has 2 squares in each column.

b. Write a repeated addition equation to match the array.

$$2 + 2 + 2 + 2 = 8$$

6.

a. Draw an array with 20 squares that has 4 squares in each column.

b. Write a repeated addition equation to match the array.

$$4 + 4 + 4 + 4 + 4 = 20$$

c. Draw a tape diagram to match your repeated addition equation and array.

- For Problem 5(b), Soo Min wrote $4 + 4 = 8$. Tasha wrote $2 + 2 + 2 + 2 = 8$. Are they both correct? How do you know?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Subtraction from Teens

1.	$11 - 10 =$	
2.	$12 - 10 =$	
3.	$13 - 10 =$	
4.	$19 - 10 =$	
5.	$11 - 1 =$	
6.	$12 - 2 =$	
7.	$13 - 3 =$	
8.	$17 - 7 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 8 =$	
13.	$18 - 8 =$	
14.	$13 - 4 =$	
15.	$13 - 5 =$	
16.	$13 - 6 =$	
17.	$13 - 8 =$	
18.	$16 - 6 =$	
19.	$12 - 3 =$	
20.	$12 - 4 =$	
21.	$12 - 5 =$	
22.	$12 - 9 =$	

23.	$19 - 9 =$	
24.	$15 - 6 =$	
25.	$15 - 7 =$	
26.	$15 - 9 =$	
27.	$20 - 10 =$	
28.	$14 - 5 =$	
29.	$14 - 6 =$	
30.	$14 - 7 =$	
31.	$14 - 9 =$	
32.	$15 - 5 =$	
33.	$17 - 8 =$	
34.	$17 - 9 =$	
35.	$18 - 8 =$	
36.	$16 - 7 =$	
37.	$16 - 8 =$	
38.	$16 - 9 =$	
39.	$17 - 10 =$	
40.	$12 - 8 =$	
41.	$18 - 9 =$	
42.	$11 - 9 =$	
43.	$15 - 8 =$	
44.	$13 - 7 =$	

B

Subtraction from Teens

Number Correct: _____

Improvement: _____

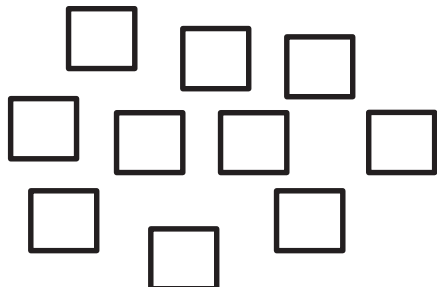
1.	$11 - 1 =$	
2.	$12 - 2 =$	
3.	$13 - 3 =$	
4.	$18 - 8 =$	
5.	$11 - 10 =$	
6.	$12 - 10 =$	
7.	$13 - 10 =$	
8.	$18 - 10 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 7 =$	
13.	$19 - 9 =$	
14.	$12 - 3 =$	
15.	$12 - 4 =$	
16.	$12 - 5 =$	
17.	$12 - 8 =$	
18.	$17 - 7 =$	
19.	$13 - 4 =$	
20.	$13 - 5 =$	
21.	$13 - 6 =$	
22.	$13 - 9 =$	

23.	$16 - 6 =$	
24.	$14 - 5 =$	
25.	$14 - 6 =$	
26.	$14 - 7 =$	
27.	$14 - 9 =$	
28.	$20 - 10 =$	
29.	$15 - 6 =$	
30.	$15 - 7 =$	
31.	$15 - 9 =$	
32.	$14 - 4 =$	
33.	$16 - 7 =$	
34.	$16 - 8 =$	
35.	$16 - 9 =$	
36.	$20 - 10 =$	
37.	$17 - 8 =$	
38.	$17 - 9 =$	
39.	$16 - 10 =$	
40.	$18 - 9 =$	
41.	$12 - 9 =$	
42.	$13 - 7 =$	
43.	$11 - 8 =$	
44.	$15 - 8 =$	

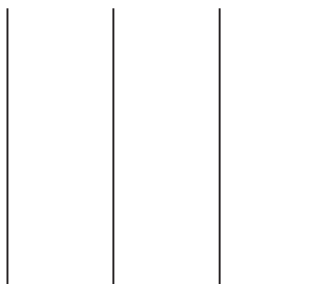
Name _____

Date _____

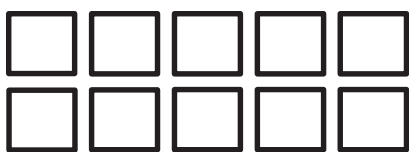
1. Create an array with the squares.



2. Create an array with the squares from the set above.



3. Use the array of squares to answer the questions below.



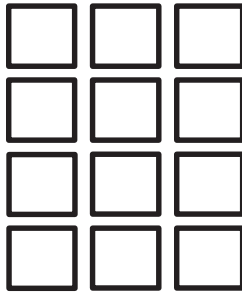
a. There are ____ squares in each row.

b. ____ + ____ = ____

c. There are ____ squares in each column.

d. ____ + ____ + ____ + ____ + ____ = ____

4. Use the array of squares to answer the questions below.



a. There are ____ squares in one row.

b. There are ____ squares in one column.

c. ____ + ____ + ____ = ____

d. 3 columns of ____ = ____ rows of ____ = ____ total

5. a. Draw an array with 8 squares that has 2 squares in each column.

b. Write a repeated addition equation to match the array.

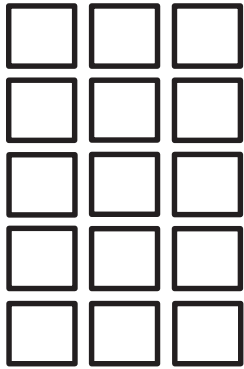
6. a. Draw an array with 20 squares that has 4 squares in each column.

b. Write a repeated addition equation to match the array.

c. Draw a tape diagram to match your repeated addition equation and array.

Name _____ Date _____

1. Use the array of squares to answer the questions below.



- a. There are ____ squares in one row.
- b. There are ____ squares in one column.
- c. ____ + ____ + ____ = ____
- d. 3 columns of ____ = ____ rows of ____ = ____ total

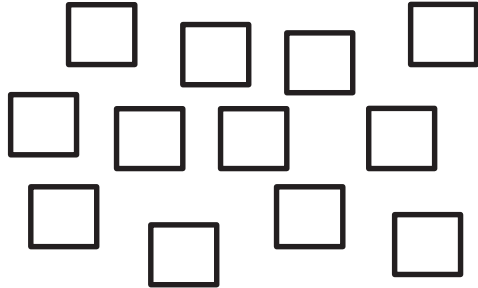
2. a. Draw an array with 10 squares that has 5 squares in each column.

- b. Write a repeated addition equation to match the array.

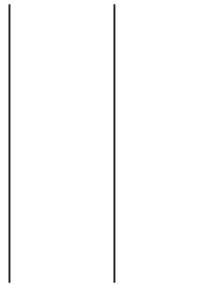
Name _____

Date _____

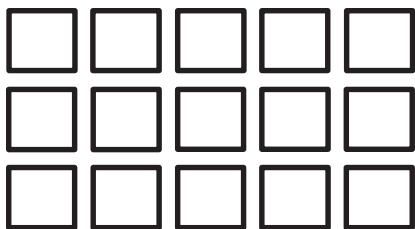
1. Create an array with the squares.



2. Create an array with the squares from the set above.



3. Use the array of squares to answer the questions below.



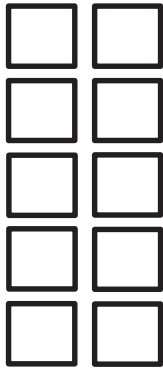
a. There are ____ squares in each row.

b. ____ + ____ + ____ = ____

c. There are ____ squares in each column.

d. ____ + ____ + ____ + ____ + ____ = ____

4. Use the array of squares to answer the questions below.



a. There are ____ squares in one row.

b. There are ____ squares in one column.

c. ____ + ____ = ____

d. 2 columns of ____ = ____ rows of ____ = ____ total

5. a. Draw an array with 15 squares that has 3 squares in each column.

b. Write a repeated addition equation to match the array.

6. a. Draw an array with 20 squares that has 5 squares in each column.

b. Write a repeated addition equation to match the array.

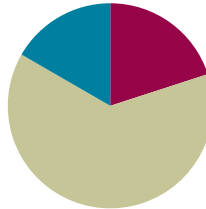
c. Draw a tape diagram to match your repeated addition equation and array.

Lesson 9

Objective: Solve word problems involving addition of equal groups in rows and columns.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Get the Ten Out and Subtract **2.NBT.5** (4 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)
- Happy Counting by Tens: Crossing 100 **2.NBT.2** (3 minutes)

Get the Ten Out and Subtract (4 minutes)

Note: Students practice taking out the ten when subtracting.

T: For every expression I give, subtract the ones from ten. When I say, “ $12 - 4$,” you say, “ $10 - 4 = 6$.” Ready?

T: $12 - 8$.

S: $10 - 8 = 2$.

T: $13 - 7$.

S: $10 - 7 = 3$.

Students practice taking the ten out of expressions fluently before adding the ones back.

T: Now, let’s add back the ones.

T: $12 - 8$. Take from ten.

S: $10 - 8 = 2$.

T: Now, add back the ones.

S: $2 + 2 = 4$.

T: $12 - 8$ is...?

S: 4.

T: True or false: $2 + 2 = 12 - 8$.

S: True.

Continue with the following possible sequence: $13 - 7$, $11 - 8$, $13 - 9$, $15 - 7$, and $14 - 8$.

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 1 Core Fluency Practice Sets)

Note: During Topic B and for the remainder of the year, each day's Fluency Practice includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 1.

Happy Counting by Tens: Crossing 100 (3 minutes)

Note: Students skip-count by tens as a foundation for counting equal groups in today's lesson.

- T: Let's count by tens, starting at 160. Ready? (Point up rhythmically until a change is desired. Close hand to indicate a stopping point. Point down to count in the opposite direction. Continue, periodically changing direction.)
- S: 160, 170, 180, 190, 200, 210, 220, 230, 240 (switch) 230, 220, 210, 200, 190 (switch) 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, 320 (switch) 310, 300, 290, 280.
- T: Excellent! Try it for 30 seconds with your partner, starting at 300. Partner B, you are the teacher today.

Concept Development (38 minutes)

Materials: (S) Personal white board

Problem 1: Anu wants to know how many eggs are in the carton. She sees 4 eggs in both rows. How many eggs are there?

- T: Read the problem aloud.
- T: Draw to show Anu's eggs.
- T: (Circulate as students draw.)
- T: What equation should we write to find the total?
- S: $4 + 4 = 8$.
- T: Write your equation, and label your answer.
- T: What did you draw?
- S: I drew 4 columns of 2 eggs. → I drew 2 rows of 4 eggs.
→ I drew mine by twos. It's just easier for me to match them up, and I knew there are 4 twos in 8.
- T: Read your statement to your partner.
- S: There are 8 eggs. → There are 8 eggs in the carton.

$$\begin{array}{cccc} 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 \end{array} \quad 4 + 4 = 8$$

Anu has 8 eggs.

MP.7

Problem 2: Miss Tam arranges desks into 4 rows of 5. How many desks are in her classroom?

Draw a picture to solve, and write a repeated addition equation. Then, write a statement of your answer.

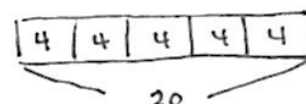
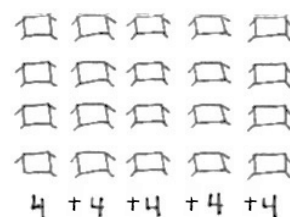
- T: Read the problem aloud.
- T: Draw to show Miss Tam's desks.
- T: (Circulate as students draw.)
- T: How many desks are in each column, or unit?
- S: 4 desks!
- T: Let's write that unit of 4 at the bottom of each column. (See the figure to the right.)
- T: So, what equation could we write to find the total?
- S: $4 + 4 + 4 + 4 + 4 = 20$.
- T: Yes! Turn and talk: How could we represent this problem using a tape diagram?
- S: I would make 5 parts, each with a four inside. → A bar that is separated into 5 fours. → Since we wrote units of 4 at the bottom of each column, we can make bars with fours inside.
- T: Great! Let's build our tape diagram together. (Draw one part at a time as students do the same.)
- T: Let's represent the first column of desks. How many units are inside?
- S: 4.
- T: Yes, and how many in the second part? (Continue for all five parts until the tape diagram is built.)
- T: Remind me, why are there 5 parts to our tape diagram?
- S: Because we drew 5 columns. → Well, there are 5 addends in the repeated addition.
- T: Yes. Our tape diagram is just another way to represent the problem. It's a great problem-solving strategy!
- T: Write an equation under your tape diagram and a statement of your answer. When you are writing your statement, be sure to check the original question. What is your statement?
- S: There are 20 desks in Miss Tam's room.
- T: You're on a roll! (Possibly repeat the sequence, this time making each row the unit of the tape diagram.)
- T: Let's represent this next problem using only a tape diagram. Here we go.



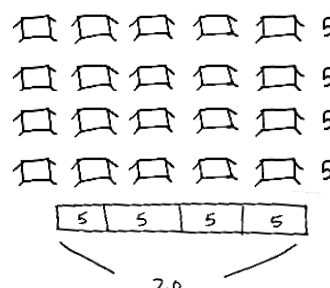
**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Challenge students with the following extension:

If there are 4 legs on each desk, how many legs are there altogether?



There are 20 desks
in Miss Tam's room.



Students may also solve the problem by adding the rows as pictured above.

Problem 3: Yehuda ate 4 cherries each in the morning, in the afternoon, and in the evening. How many cherries did Yehuda eat altogether?

- T: Read the problem aloud.
- T: Draw a tape diagram to show the cherries Yehuda eats.
- T: (Circulate as students draw.)
- T: How did you draw your tape diagram?
- S: I made 3 parts, each with a 4 inside. → I did a part that shows 4, then another that shows 4, and then another that shows 4. → I thought of 3 groups of 4. So, the tape diagram shows 3 fours.
- T: What repeated addition equation should we write to solve?
- S: $4 + 4 + 4 = 12$.
- T: Share your statement with your partner.
- S: Yehuda ate 12 cherries. → 4 in the morning plus 4 in the afternoon plus 4 in the evening is 12 altogether.

If students need more practice, create simple word problems using subjects that lend themselves to the drawing of arrays, e.g., windowpanes, muffin tins, or flowerbeds.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Solve word problems involving addition of equal groups in rows and columns.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide manipulatives to students who may still need to use them as a step before drawing the picture and then writing the repeated addition equation.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students may still need to draw dots instead of or before writing numbers to show the equal parts in the tape diagram.

4	4	4
cherries	cherries	cherries

$$4 + 4 + 4 = 12$$

Yehuda ate 12 cherries altogether.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1, share your array with a partner. How did Jason arrange his rocks? What addition equation matches your array?
- How did you determine how many chairs to put in each row in Problem 2? How did this match your equation?
- Share your array for Problem 3 with a partner. Did you draw rows or columns of 5? How did you solve?
- In Problem 4, how did you figure out how many windows face the street? Why are there 2 addends in the equation?
- How did you represent the situation in Problem 5 as a tape diagram? How did your tape diagram change for Problem 6? How is this like something we have done before (i.e., adding or removing rows and columns)?
- For Problem 6, how did you represent the situation as a tape diagram? Could you also have drawn an array for this problem? What would it look like?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

Draw an array for each word problem. Write a repeated addition equation to match each array.

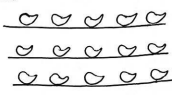
1. Jason collected some rocks. He put them in 5 rows with 3 stones in each row. How many stones did Jason have altogether?

 $3 + 3 + 3 + 3 + 3 = 15$
Jason had 15 stones altogether.

2. Abby made 3 rows of 4 chairs. How many chairs did Abby use?

 $4 + 4 + 4 = 12$
Abby used 12 chairs.

3. There are 3 wires and 5 birds sitting on each of them. How many birds in all are on the wires?

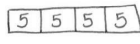
 $5 + 5 + 5 = 15$
There are 15 birds on the wires altogether.

4. Henry's house has 2 floors. There are 4 windows on each floor that face the street. How many windows face the street?

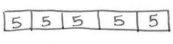
 $4 + 4 = 8$
There are 8 windows that face the street.

Draw a tape diagram for each word problem. Write a repeated addition equation to match each tape diagram.

5. Each of Maria's 4 friends has 5 markers. How many markers do Maria's friends have in all?

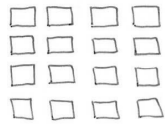
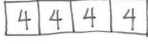
 Maria's friends have 20 markers in all.
 $5 + 5 + 5 + 5 = 20$

6. Maria also has 5 markers. How many markers do Maria and her friends have in all?

 Maria and her friends have 25 markers in all.
 $5 + 5 + 5 + 5 + 5 = 25$

Draw a tape diagram and an array. Then, write a repeated addition equation to match.

7. In a card game, 3 players get 4 cards each. One more player joins the game. How many total cards should be dealt now?

  $4 + 4 + 4 + 4 = 16$
16 cards should be dealt.

Name _____

Date _____

Draw an array for each word problem. Write a repeated addition equation to match each array.

1. Jason collected some rocks. He put them in 5 rows with 3 stones in each row. How many stones did Jason have altogether?
2. Abby made 3 rows of 4 chairs. How many chairs did Abby use?
3. There are 3 wires and 5 birds sitting on each of them. How many birds in all are on the wires?
4. Henry's house has 2 floors. There are 4 windows on each floor that face the street. How many windows face the street?

Draw a tape diagram for each word problem. Write a repeated addition equation to match each tape diagram.

5. Each of Maria's 4 friends has 5 markers. How many markers do Maria's friends have in all?

6. Maria also has 5 markers. How many markers do Maria and her friends have in all?

Draw a tape diagram and an array. Then, write a repeated addition equation to match.

7. In a card game, 3 players get 4 cards each. One more player joins the game. How many total cards should be dealt now?

Name _____

Date _____

Draw a tape diagram or an array for each word problem. Then, write a repeated addition equation to match.

1. Joshua cleans 3 cars every hour at work. He worked 4 hours on Saturday. How many cars did Joshua clean on Saturday?

2. Olivia put 5 stickers on each page in her sticker album. She filled 5 pages with stickers. How many stickers did Olivia use?

Name _____

Date _____

Draw an array for each word problem. Write a repeated addition equation to match each array.

1. Melody stacked her blocks in 3 columns of 4. How many blocks did Melody stack in all?
2. Marty arranged the desks in the classroom into 5 equal rows. There were 5 desks in each row. How many desks were arranged?
3. The baker made 5 trays of muffins. Each tray holds 4 muffins. How many muffins did the baker make?

4. The library books were on the shelf in 4 stacks of 4. How many books were on the shelf?

Draw a tape diagram for each word problem. Write a repeated addition equation to match each tape diagram.

5. Mary placed stickers in columns of 4. She made 5 columns. How many stickers did she use?

6. Jayden put his baseball cards into 5 columns of 3 in his book. How many cards did Jayden put in his book?

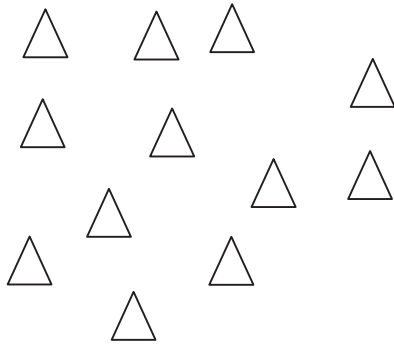
Draw a tape diagram and an array. Then, write a repeated addition equation to match.

7. The game William bought came with 3 bags of marbles. Each bag had 3 marbles inside. How many total marbles came with the game?

Name _____

Date _____

1. a. Redraw the objects below in an array.



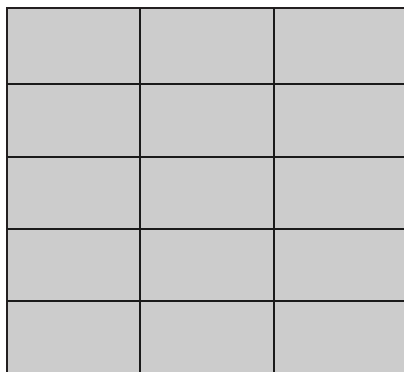
- b. Circle one column. Then, circle one row.



- c. Write a repeated addition number sentence to match the columns of hearts.

- d. Draw and label a tape diagram to match your addition sentence and array.

2. a. Circle all the expressions that describe the array.



$3 + 3 + 3 + 3$

$3 + 5$

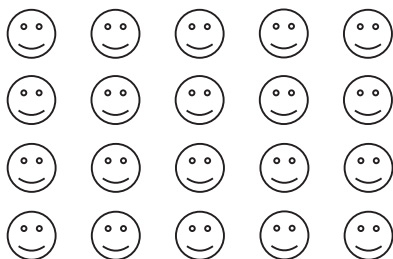
$5 + 5 + 5$

$5 + 5 + 5 + 5 + 5$

$3 + 3 + 3 + 3 + 3$

$10 + 3$

- b. Count the smiley faces one row at a time. Write a repeated addition number sentence to find the total.



- c. Draw an array to match $5 + 5 + 5 + 5$, where 5 is the number of objects in the column.

3. a. Draw an array with 15 squares where one row is made of 5 squares.

- b. Write a repeated addition sentence to match the array you drew in 3(a), showing the addition of the number in each row.

4. Sarah won a prize at school! Her teacher said that she would have two choices for the prize:

Choice 1: Get \$3 a day for the next 3 days.

Choice 2: Get \$2 a day for the next 5 days.

- a. Draw an array for each choice.

- b. Which way would Sarah get more money? Explain how you know.

Mid-Module Assessment Task Standard Addressed**Topics A–B****Work with equal groups of objects to gain foundations for multiplication.**

- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 2.OA.4	The student solves one out of four parts correctly.	The student solves two out of four parts correctly.	The student solves three out of four parts correctly.	The student correctly: a. Draws triangles in an array. Possible arrays include: 1 row of 12, 12 rows of 1, 2 rows of 6, 6 rows of 2, 3 rows of 4, or 4 rows of 3. b. Circles one row and one column. c. Answers $2 + 2 + 2 + 2 = 10$. d. Draws a tape diagram to match the addition sentence in Part (c).
2 2.OA.4	The student solves zero out of three parts correctly.	The student solves one out of three parts correctly.	The student solves two out of three parts correctly.	The student correctly: a. Circles both $5 + 5 + 5$ and $3 + 3 + 3 + 3 + 3$. b. Writes $5 + 5 + 5 + 5 = 20$ or $4 + 4 + 4 + 4 + 4 = 20$. c. Draws an array showing 4 columns of 5.
3 2.OA.4	The student solves zero out of two parts correctly.	The student solves one out of two parts correctly.	The student correctly shows an array and writes a matching equation for a sum other than 15.	The student correctly: a. Draws an array showing 3 rows of 5. b. Answers $5 + 5 + 5 = 15$.



A Progression Toward Mastery

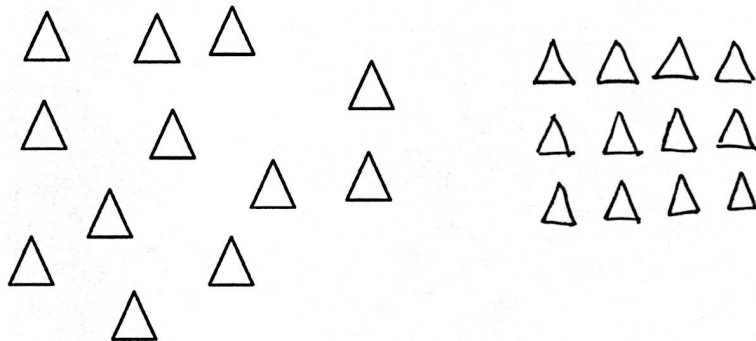
4 2.OA.4	The student solves zero out of two parts correctly.	The student only answers Part (a) or Part (b) correctly.	The student answers Parts (a) and (b) correctly but fails to provide a clear explanation.	The student correctly: a. Draws an array to show 3 rows of 3 and draws an array to show either 2 rows of 5 or 5 rows of 2. b. Clearly explains that Sarah would make more money with Choice 2.

Name Roberto

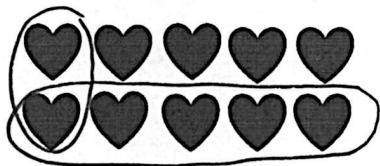
Date _____

1.

- a. Redraw the objects below in an array.



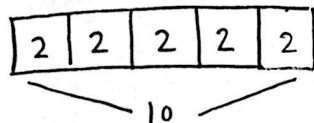
- b. Circle one column. Then, circle one row.



- c. Write a repeated addition number sentence to match the columns of hearts.

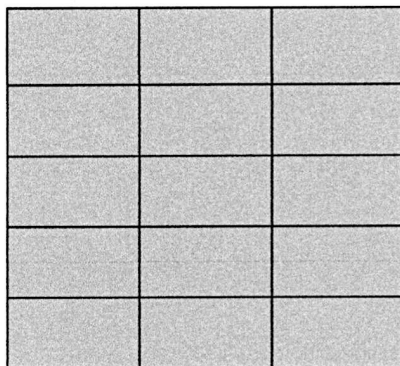
$$2 + 2 + 2 + 2 + 2 = 10$$

- d. Draw and label a tape diagram to match your addition sentence and array.



2.

- a. Circle all the expressions that describe the array.



$3 + 3 + 3 + 3$

$3 + 5$

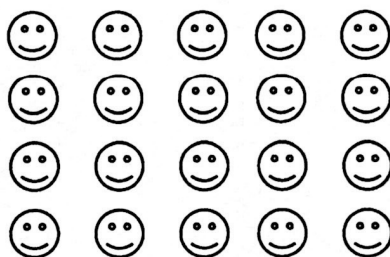
$5 + 5 + 5$

$5 + 5 + 5 + 5 + 5$

$3 + 3 + 3 + 3 + 3$

$10 + 3$

- b. Count the smiley faces one row at a time. Write a repeated addition number sentence to find the total.



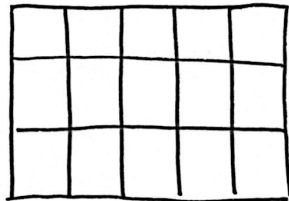
$$5 + 5 + 5 + 5 = 20$$

- c. Draw an array to match $5 + 5 + 5 + 5$, where 5 is the number of objects in the column.



3.

- a. Draw an array with 15 squares where one row is made of 5 squares.



- b. Write a repeated addition sentence to match the array you drew in 3(a), showing the addition of the number in each row.

$$5 + 5 + 5 = 15$$

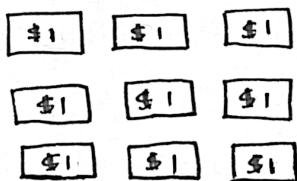
4. Sarah won a prize at school! Her teacher said that she would have two choices for the prize:

Choice 1: Get \$3 a day for the next 3 days.

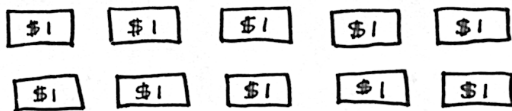
Choice 2: Get \$2 a day for the next 5 days.

- a. Draw an array for each choice.

CHOICE 1:



CHOICE 2:



- b. Which way would Sarah get more money? Explain how you know.

Sarah would get more money with choice 2 because
that would be \$10, and choice 1 would only
be \$9. Choice 1 comes out to \$9 because $3+3+3$
is 9. Choice 2 comes out to \$10 because
 $2+2+2+2+2$ is 10.



Topic C

Rectangular Arrays as a Foundation for Multiplication and Division

2.OA.4, 2.G.2

Focus Standards:	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
Instructional Days:	7	
Coherence	-Links from: G1–M2	Introduction to Place Value Through Addition and Subtraction Within 20
	-Links to: G2–M8	Time, Shapes, and Fractions as Equal Parts of Shapes
	G3–M4	Multiplication and Area

Topic C naturally follows Topic B, where students composed and manipulated the rows and columns of an array. Topic C is designed to deepen students' understanding of spatial structuring as they build and partition rectangles with rows and columns of same-size squares.

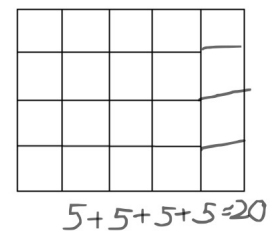
In Lessons 10 and 11, students compose a rectangle by making tile arrays with no gaps or overlaps. They use their prior knowledge of making equal groups and the spatial relationship between rows and columns to construct rectangular arrays. In Lesson 10, given a number of tiles (up to 25), students are asked to create rectangular arrays that show equal rows or columns (up to 5 by 5). In Lesson 11, students build upon this understanding, manipulating a set of 12 square tiles to compose various rectangles (e.g., 1 column of 12, 2 rows of 6, or 3 rows of 4). As students share their rectangles, they are encouraged to ask themselves, "How can I construct this differently?" They use repeated addition to find the total number of squares, alternating flexibly between the number in each row and the number in each column as the unit.

Lesson 12 introduces the added complexity of composing a rectangle by using math drawings. Once students have arranged square tiles into a specified rectangular array without gaps or overlaps, they trace to construct the same rectangle by iterating the square unit, much as they iterated a length unit in Module 2 to create a centimeter ruler. Next, students use the spatial reasoning developed up to this point in the module to draw the same rectangle without tracing, using their understanding of equal columns and equal rows.

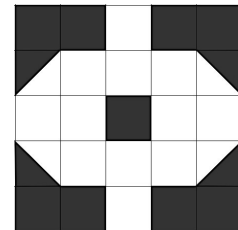
After students compose rectangles, they decompose, or partition, them using tiles in Lesson 13. For example, when working with an array of 5 rows of 3 (and a total of 15), they see that if they remove a row of 3, they have 4 rows of 3 (and a total of 12). Alternately, they see that instead of 3 columns of 5, they have 3 columns of 4.

In Lesson 14, students are encouraged to think flexibly as they use paper models to further develop their ability to visualize arrays. Given three 2 by 4 rectangles, students cut the first rectangle into 2 rows of 4 squares and the second rectangle into 4 columns of 2 squares. They use these models to answer questions and to analyze similarities and differences. Next, they cut each row or column into individual square units. As a result, they see that just as a rectangle is composed of equal rows or columns, each row or column is composed of squares, or iterated units. Students now have 16 same-size squares and can create different rectangular arrays with them (e.g., 1 by 16, 2 by 8, and 4 by 4). Finally, students cut out the squares from the third rectangle and create rectangular arrays using 24 square units.

Lesson 15 moves toward more abstract reasoning as students use math drawings to partition rectangles. With colored pencils and grid paper, students shade in rows or columns and relate them to the repeated addition equation (e.g., 5 rows of 3 squares is 15 squares, which is the same as $3 + 3 + 3 + 3 + 3 = 15$, or 5 threes). Then, given a rectangle with one row or one column missing, students draw in the remaining squares to complete the array (shown to the right) and find the total by relating their completed arrays to repeated addition.



In Lesson 16, students practice spatial structuring skills by working with grids and diagrams. They copy designs using same-size squares and triangles (half of the squares) as manipulatives. Students create their copies on paper with grid squares of the same size as the manipulative square (shown to the right). To successfully create these, students must pay careful attention to which grid square to color and how many spaces to leave. Finally, students use grid paper to design a tessellation using a core square composed of a 3 by 3 array of same-size squares. They create designs by coloring the 9 squares and then iterating that core unit. This provides students with the opportunity to sharpen their spatial structuring skills because they must count rows and columns to successfully create a quilt of their designs.



A Teaching Sequence Toward Mastery of Rectangular Arrays as a Foundation for Multiplication and Division

Objective 1: Use square tiles to compose a rectangle, and relate to the array model.
(Lessons 10–11)

Objective 2: Use math drawings to compose a rectangle with square tiles.
(Lesson 12)

Objective 3: Use square tiles to decompose a rectangle.
(Lesson 13)

Objective 4: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares.
(Lesson 14)

Objective 5: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition.
(Lesson 15)

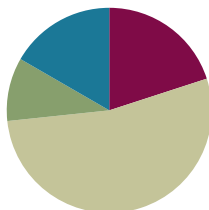
Objective 6: Use grid paper to create designs to develop spatial structuring.
(Lesson 16)

Lesson 10

Objective: Use square tiles to compose a rectangle, and relate to the array model.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Happy Counting by Tens: Crossing 100 **2.NBT.2** (3 minutes)
- Sprint: Sums to the Teens **2.OA.2** (9 minutes)

Happy Counting by Tens: Crossing 100 (3 minutes)

Note: Students skip-count by tens as a review of counting equal groups.

- T: Let's count by tens, starting at 270. Ready? (Point up rhythmically until a change is desired. Close hand to indicate a stopping point. Point down to count in the opposite direction. Continue, periodically changing direction.)
- S: 270, 280, 290, 300, 310, 320, 330, 340 (switch) 330, 320, 310, 300, 290 (switch) 300, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420 (switch) 410, 400, 390, 380 (switch) 390, 400, 410.
- T: Excellent! Try it for 30 seconds with your partner, starting at 300. Partner A, you are the teacher today.

Sprint: Sums to the Teens (9 minutes)

Materials: (S) Sums to the Teens Sprint

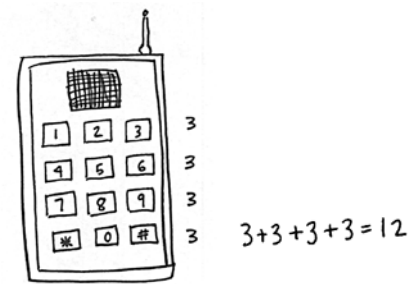
Note: This Sprint gives practice in the grade level fluency goal of sums to 20.

Application Problem (6 minutes)

Sandy's toy telephone has buttons arranged in 3 columns and 4 rows.

- Draw a picture of Sandy's telephone.
- Write a repeated addition equation to show the total number of buttons on Sandy's telephone.

Note: Students have a chance to apply their understanding of arrays to a real-world context.



Sandy's telephone has 12 buttons total.

Concept Development (32 minutes)

Materials: (S) 25 square tiles

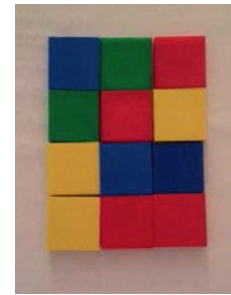
Part 1: Compose a rectangle with square tiles that has no gaps or overlaps.

- T: Place 10 tiles into 2 equal groups.
- T: Organize your tiles into 2 equal rows like you did yesterday, but this time, leave no spaces between the rows.
- S: (Count out 10 tiles, and create arrays.)
- T: How many rows did you make?
- S: 2 rows!
- T: How many tiles are in each row?
- S: 5 tiles!
- T: What repeated addition equation can we use to find the total for 2 rows of five?
- S: $5 + 5 = 10$.
- T: What do you notice about the shape of this array?
- S: It has corners like an L. → The top and bottom sides are the same length. → It's a rectangle!
- T: Using the same tiles, make 2 columns of 5, and again, leave no spaces between the columns to make a rectangle.
- S: (Construct array.)
- T: Turn and talk: What repeated addition equation can we use to find the total for 2 columns of 5?
- S: $5 + 5 = 10$.
- T: Are the equations and totals equal for both arrays?
- S: Yes!



- T: How is that so? Talk to your partner.
- S: It is the same rectangle, just turned on its side. → It uses the same number of tiles. → It doesn't matter whether you have 2 columns of 5 or 2 rows of 5, because you have 2 groups of 5. → It's a rectangle, too.
- T: Is this shape also a rectangle?
- S: Yes!

Repeat the above process with 15 tiles (5 by 3) and 12 tiles (3 by 4).



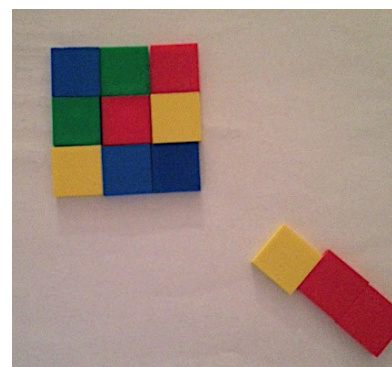
Part 2: Compose a square from rows and columns.

- T: Let's look at the array we just made (3 by 4). How can we change this rectangle from 3 columns of four to have the same number of rows and columns? Talk to your partner.
- S: Change it to 3 groups of 3. → Add another column to make it 4 groups of 4. → Take away one of the rows.
- T: (Model taking away a row to make equal rows and columns.)
- T: What do you notice about the shape of this array?
- S: It's a square. → The columns and rows are equal. → There are 3 rows and 3 columns.
- T: Let's see if we can make another square array. Talk to your partner about your thinking as you use all 25 tiles from your bag to create an array with equal rows and columns. (Circulate and offer hints to encourage all students to find the array.)
- S: I am going to start by making a row of 5 because I know I can count to 25 by fives. → I will separate the tiles into groups until they are all equal.
- T: What does your array look like?
- S: 5 rows of five! → 5 columns and 5 rows! → A square!
- T: Now, keep 16 tiles on your desk, and put the rest in your bag.
- T: Create an array with equal rows and columns.
- S: (Create equal rows and columns.)
- T: What strategies did you use to figure out how many rows and how many columns?
- S: I started by creating groups of 2. Then, I realized that if I made groups of 4, I would have 4 groups. → I know that $4 + 4 + 4 + 4 = 16$, so I made 4 rows of 4. → I made two rows of eight and then saw it was a double of 2 rows of 4, so I just moved half the tiles down.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

- Large foam floor tiles can be a tool to engage the class in a whole-group activity prior to giving students small tiles to handle themselves.
- In the absence of tiles, square sticky notes can be a good substitute for this activity.



MP.4

- T: So, what do you know about making an array with equal rows and columns?
- S: I know that if you have 4 rows, then there has to be 4 in each row. → I know that you need the same number of groups and the same number in each group. → You need the same number of tiles in the rows as in the columns. → It's a special rectangle: a square!
- T: Turn and talk: Could we make a square array with 10 tiles?
- S: No, because you can't make equal rows and columns. → Ten tiles can only be 2 rows of 5 or 1 row of 10. → Only certain numbers can make equal rows and columns.

Direct students to move on to the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use square tiles to compose a rectangle, and relate to the array model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1 (a) and (b), share your rectangles with a partner, and describe them using the words *rows* and *columns*. How do your rectangles match the repeated addition equations?
- For Problem 2 (a) and (b), share your rectangles with a partner. How are rectangles composed of equal groups? How does your rectangle match your equation?
- For Problem 3, explain to your partner how you arranged the tiles into a rectangle. How did making equal rows and columns help you to construct the rectangle?

Name Sarah Date _____

Use your square tiles to construct the following rectangles with no gaps or overlaps. Write a repeated addition equation to match each construction.

1.

a. Construct a rectangle with 2 rows of 3 tiles.

$3 + 3 = 6$

b. Construct a rectangle with 2 columns of 3 tiles.

$3 + 3 = 6$

2.

a. Construct a rectangle with 5 rows of 2 tiles.

$2 + 2 + 2 + 2 + 2 = 10$

b. Construct a rectangle with 5 columns of 2 tiles.

$2 + 2 + 2 + 2 + 2 = 10$

- Squares are special rectangles that have the same number of rows and columns. In Problem 4, you changed a rectangle into a square by removing a column. Is there a different way to make a square from the array? What repeated addition equation would we use to describe a square array with rows of 2? 3? 4? 5? What do you notice?
- Why don't we relate triangles to an array model?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Note: To assess student understanding using today's Exit Ticket, walk around, and directly observe students as they work with the tiles. Take note of how students are building their arrays. Look for understanding of rows and columns, as well as the importance of building with no gaps or spaces between the tiles.

3.

a. Construct a rectangle of 9 tiles that has equal rows and columns.

$3 + 3 + 3 = 9$

b. Construct a rectangle of 16 tiles that has equal rows and columns.

$4 + 4 + 4 + 4 = 16$

4.

a. What shape is the array pictured below? rectangle



b. Redraw the above shape with one column removed in the space below.



c. What shape is the array now? square

A

Number Correct: _____

Sums to the Teens

1.	$9 + 1 =$	
2.	$9 + 2 =$	
3.	$9 + 3 =$	
4.	$9 + 9 =$	
5.	$8 + 2 =$	
6.	$8 + 3 =$	
7.	$8 + 4 =$	
8.	$8 + 9 =$	
9.	$9 + 1 =$	
10.	$9 + 4 =$	
11.	$9 + 5 =$	
12.	$9 + 8 =$	
13.	$8 + 2 =$	
14.	$8 + 5 =$	
15.	$8 + 6 =$	
16.	$8 + 8 =$	
17.	$9 + 1 =$	
18.	$9 + 7 =$	
19.	$8 + 2 =$	
20.	$8 + 7 =$	
21.	$9 + 1 =$	
22.	$9 + 6 =$	

23.	$7 + 3 =$	
24.	$7 + 4 =$	
25.	$7 + 5 =$	
26.	$7 + 9 =$	
27.	$6 + 4 =$	
28.	$6 + 5 =$	
29.	$6 + 6 =$	
30.	$6 + 9 =$	
31.	$5 + 5 =$	
32.	$5 + 6 =$	
33.	$5 + 7 =$	
34.	$5 + 9 =$	
35.	$4 + 6 =$	
36.	$4 + 7 =$	
37.	$4 + 9 =$	
38.	$3 + 7 =$	
39.	$3 + 9 =$	
40.	$5 + 8 =$	
41.	$2 + 8 =$	
42.	$4 + 8 =$	
43.	$1 + 9 =$	
44.	$2 + 9 =$	

B

Sums to the Teens

Number Correct: _____

Improvement: _____

1.	$8 + 2 =$	
2.	$8 + 3 =$	
3.	$8 + 4 =$	
4.	$8 + 8 =$	
5.	$9 + 1 =$	
6.	$9 + 2 =$	
7.	$9 + 3 =$	
8.	$9 + 8 =$	
9.	$8 + 2 =$	
10.	$8 + 5 =$	
11.	$8 + 6 =$	
12.	$8 + 9 =$	
13.	$9 + 1 =$	
14.	$9 + 4 =$	
15.	$9 + 5 =$	
16.	$9 + 9 =$	
17.	$9 + 1 =$	
18.	$9 + 7 =$	
19.	$8 + 2 =$	
20.	$8 + 7 =$	
21.	$9 + 1 =$	
22.	$9 + 6 =$	

23.	$7 + 3 =$	
24.	$7 + 4 =$	
25.	$7 + 5 =$	
26.	$7 + 8 =$	
27.	$6 + 4 =$	
28.	$6 + 5 =$	
29.	$6 + 6 =$	
30.	$6 + 8 =$	
31.	$5 + 5 =$	
32.	$5 + 6 =$	
33.	$5 + 7 =$	
34.	$5 + 8 =$	
35.	$4 + 6 =$	
36.	$4 + 7 =$	
37.	$4 + 8 =$	
38.	$3 + 7 =$	
39.	$3 + 9 =$	
40.	$5 + 9 =$	
41.	$2 + 8 =$	
42.	$4 + 9 =$	
43.	$1 + 9 =$	
44.	$2 + 9 =$	

Name _____

Date _____

Use your square tiles to construct the following rectangles with no gaps or overlaps.
Write a repeated addition equation to match each construction.

1. a. Construct a rectangle with 2 rows of 3 tiles.

- b. Construct a rectangle with 2 columns of 3 tiles.

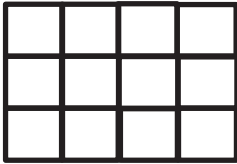
2. a. Construct a rectangle with 5 rows of 2 tiles.

- b. Construct a rectangle with 5 columns of 2 tiles.

3. a. Construct a rectangle of 9 tiles that has equal rows and columns.

- b. Construct a rectangle of 16 tiles that has equal rows and columns.

4. a. What shape is the array pictured below? _____



- b. Redraw the above shape with one column removed in the space below.

- c. What shape is the array now? _____

Name _____

Date _____

On this sheet, use your square tiles to construct the following arrays with no gaps or overlaps on this sheet. Write a repeated addition equation to match your construction.

1. a. Construct a rectangle with 2 rows of 5 tiles.

b. Write the repeated addition equation. _____

2. a. Construct a rectangle with 5 columns of 2 tiles.

b. Write the repeated addition equation. _____

Name _____

Date _____

Cut out the square tiles below, and construct the following arrays with no gaps or overlaps. On the line, write a repeated addition equation to match each construction on the line.

1. a. Construct a rectangle with
2 rows of 4 tiles.

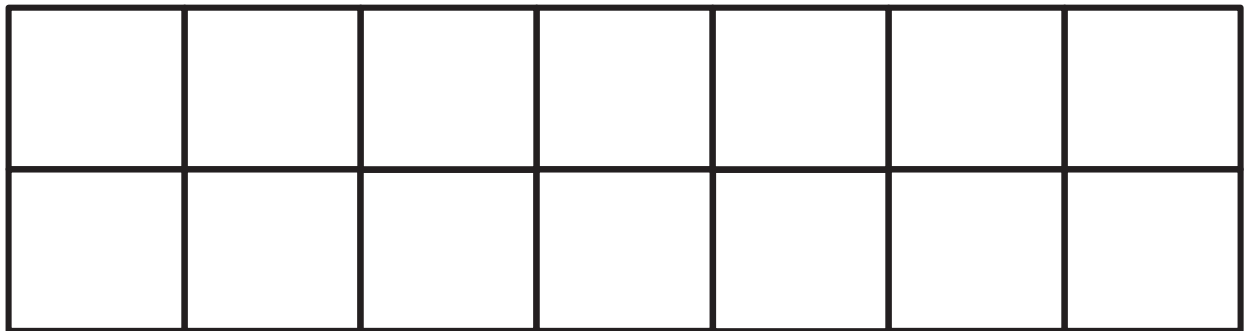
- b. Construct a rectangle with
2 columns of 4 tiles.

2. a. Construct a rectangle with
3 rows of 2 tiles.

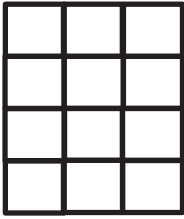
- b. Construct a rectangle with
3 columns of 2 tiles.

3. a. Construct a rectangle
using 10 tiles.

- b. Construct a rectangle
using 12 tiles.



4. a. What shape is the array pictured below? _____



- b. In the space below, redraw the above shape with one more column.

- c. What shape is the array now? _____

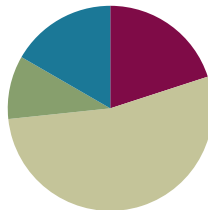
- d. Draw a different array of tiles that is the same shape as 4(c).

Lesson 11

Objective: Use square tiles to compose a rectangle, and relate to the array model.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Happy Counting by Tens: Crossing 100 **2.NBT.2** (3 minutes)
- Sprint: Subtraction Crossing Ten **2.OA.2** (9 minutes)

Happy Counting by Tens: Crossing 100 (3 minutes)

Note: Students skip-count by tens as a review of counting equal groups.

- T: Let's count by tens, starting at 360. Ready? (Point up rhythmically until a change is desired. Close hand to stop. Point down to count in the opposite direction.)
- S: 360, 370, 380, 390, 400, 410, 420, 430, 440 (switch) 430, 420, 410, 400, 390 (switch) 400, 410, 420, 430, 440, 450, 460, 470, 480, 490, 500, 510, 520 (switch) 510, 500, 490, 480.
- T: Excellent! Try it for 30 seconds with your partner, starting at 440. Partner B, you are the teacher today.

Sprint: Subtraction Crossing Ten (9 minutes)

Materials: (S) Subtraction Crossing Ten Sprint

Note: Students subtract a single-digit number from a teen number and continue the subtraction pattern when subtracting from multiples of ten and some ones.

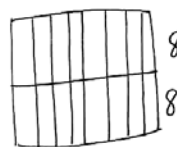
Application Problem (6 minutes)

MP.3

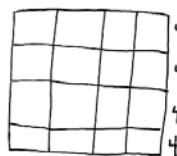
Ty bakes two pans of brownies. In the first pan, he cuts 2 rows of 8. In the second pan, he cuts 4 rows of 4.

- Draw a picture of Ty's brownie pans.
- Write a repeated addition equation to show the total number of brownies in each pan.
- How many brownies did Ty bake altogether? Write an equation and a statement to show your answer.

Note: This Application Problem gives students another real world context for seeing arrays. While sharing student work samples, encourage them to see the rectangular shape of the rows and columns.



$$8 + 8 = 16$$



$$4 + 4 + 4 + 4 = 16$$

$$\begin{array}{r} 16 + 16 \\ \quad \wedge \\ \quad 4 \quad 12 \\ 20 + 12 = 32 \end{array}$$

Ty made 32 brownies altogether.

Concept Development (32 minutes)

Materials: (T) 5 red 1-inch tiles, 5 green 1-inch tiles (S) 25 1-inch tiles, personal white board

Assign Partners A and B, and call students to sit in a circle at the front of the room.

Part 1: Compose rectangles from one row of tiles, and write addition sentences to match.

T: (Show a row of 10 tiles made with two colors as pictured.)

T: How many rows do you see?

S: 1 row.

T: How many tiles are in the row?

S: 10 tiles.

T: Did I make a rectangle with my tiles?

S: Yes. It's long and skinny!

T: It is also an array. Tell me about its rows and columns.

S: It has 1 row of 10. → Yes, you could say 10 columns of 1.

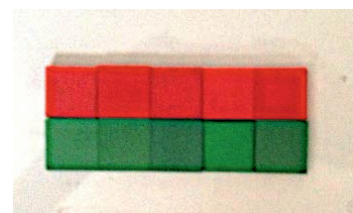
T: Turn and talk: How can we arrange these 10 tiles in a different way to form another rectangle?

S: Move the green tiles so they are under the red tiles. → I know I can count by 2s to 10, so you can make columns of 2 instead of columns of 1. → You can break the row apart, so there are 2 rows of 5.

T: Let's try that. (Move the green tiles to show 2 rows of 5 as pictured.)

T: Now how many rows do you see?

S: 2 rows!



- T: How many tiles are in each row?
- S: 5 tiles!
- T: How many columns do you see?
- S: 5 columns!
- T: How many tiles are in each column?
- S: 2 tiles!
- T: Turn and talk: What repeated addition equations can we make to represent either the columns or rows of this rectangle?
- S: $5 + 5 = 10$. $\rightarrow 2 + 2 + 2 + 2 + 2 = 10$.
- T: Now, make a row of 14 tiles on your personal white board.
- S: (Create a row.)
- T: Now, rearrange the tiles to make another array.
- S: (Make arrays.)
- T: How did you arrange the tiles to make the arrays?
- S: I broke the row into 2 rows and slid one under the other. $\rightarrow$ I made 2 rows of 7 since $7 + 7$ is 14. $\rightarrow$ It's like the last rectangle; I used the doubles fact to make 2 rows of 7, which makes 14.
- T: Add an equation underneath your rectangle.
- T: What equation did you write?
- S: I made columns of two. $2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$. $\rightarrow$ I broke the rectangle in half like we did with 10. $\rightarrow$ I moved half the array to the bottom row, so I have $7 + 7 = 14$.



$$7 + 7 = 14$$

Repeat the above process with 16 tiles.

Part 2: Compose varied rectangles from a given number of tiles.

- T: Turn and talk: Is there another rectangle we can make using the same 16 tiles?
- S: We can break it in half again and make 4 rows of 4. $\rightarrow$ We can make a square like we did yesterday.
- T: Partner A, with the help of Partner B, make a new array.
- T: Turn and talk: How are your arrays similar, and how are they different?
- S: They both have 16 tiles. $\rightarrow$ One has the same number of rows and columns, and the other doesn't. $\rightarrow$ You can turn an array with 4 rows of 4 on its side, and it looks exactly the same. You can't do that with 2 rows of 8.
- T: Now, each partner count out 12 tiles, and arrange them in a row.
- T: With your partner, construct two different rectangles using 12 tiles each. Write repeated addition equations below each rectangle. As you work, talk and compare your rectangles.



**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Encourage students to play games that involve arrays, such as Memory and Tic-Tac-Toe. This can also be a good home connection for families to connect with what students are learning in school.

- T: (Circulate as students create their rectangles and write their equations.)
- S: We can make one array that has 2 rows of 6 because we can break the row in half like we did before.
 → Let's try making a rectangle that has more than 2 rows. Hey, we can make 3 groups of 4 or 4 groups of 3. → We can make an array that has 3 rows of 4 because $4 + 4 + 4 = 12$.
- T: How are your rectangles similar, and how are they different? Discuss with your partner.
- S: They both have 12 tiles total. → My array has columns of 2, and yours has columns of 3. → Mine is 2 rows of 6, and yours is 3 rows of 4. → They are all rectangles.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use square tiles to compose a rectangle, and relate to the array model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Can we call the arrangement in Problem 1(a) an array? How can you describe it in terms of both rows and columns?
- For Problem 1, how is knowing how to make equal groups helpful in constructing a rectangle with 8 tiles? Explain how your equation matches your array.
- What strategy did you use in Problem 2 to construct a rectangle with 12 tiles? How are your two rectangles different? How are they the same? How did your rows and columns change when you rearranged your tiles to create a new rectangle for Problem 3?

Name: Roberto Date: _____

Use your square tiles to construct the following arrays with no gaps or overlaps. Write a repeated addition equation to match each construction.

1.

- a. Place 8 square tiles in a row.

- b. Construct an array with the 8 square tiles.

- c. Write a repeated addition equation to match the new array.
 $2 + 2 + 2 + 2 = 8$

2.

- a. Construct an array with 12 squares.

- b. Write a repeated addition equation to match the array.
 $4 + 4 + 4 = 12$
- c. Rearrange the 12 squares into a different array.

- d. Write a repeated addition equation to match the new array.
 $6 + 6 = 12$

- For Problem 4, explain how you know that $3 + 3 = 2 + 2 + 2$.
- You constructed two rectangles with 10 tiles for Problem 5. Is it possible to do the same with 11 tiles? Why not?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

- 3.
- Construct an array with 20 squares.
 - Write a repeated addition equation to match the array.
 $10 + 10 = 20$
 - Rearrange the 20 squares into a different array.
 - Write a repeated addition equation to match the new array.
 $5 + 5 + 5 + 5 = 20$
4. Construct 2 arrays with 6 squares.
- 2 rows of 3 = 6
 - 3 rows of 2 = 2 rows of 3
5. Construct 2 arrays with 10 squares.
- 2 rows of 5 = 10
 - 5 rows of 2 = 2 rows of 5

A

Number Correct: _____

Subtraction Crossing Ten

1.	$10 - 5 =$	
2.	$20 - 5 =$	
3.	$30 - 5 =$	
4.	$10 - 2 =$	
5.	$20 - 2 =$	
6.	$30 - 2 =$	
7.	$11 - 2 =$	
8.	$21 - 2 =$	
9.	$31 - 2 =$	
10.	$10 - 8 =$	
11.	$11 - 8 =$	
12.	$21 - 8 =$	
13.	$31 - 8 =$	
14.	$14 - 5 =$	
15.	$24 - 5 =$	
16.	$34 - 5 =$	
17.	$15 - 6 =$	
18.	$25 - 6 =$	
19.	$35 - 6 =$	
20.	$10 - 7 =$	
21.	$20 - 8 =$	
22.	$30 - 9 =$	

23.	$14 - 6 =$	
24.	$24 - 6 =$	
25.	$34 - 6 =$	
26.	$15 - 7 =$	
27.	$25 - 7 =$	
28.	$35 - 7 =$	
29.	$11 - 4 =$	
30.	$21 - 4 =$	
31.	$31 - 4 =$	
32.	$12 - 6 =$	
33.	$22 - 6 =$	
34.	$32 - 6 =$	
35.	$21 - 6 =$	
36.	$31 - 6 =$	
37.	$12 - 8 =$	
38.	$32 - 8 =$	
39.	$21 - 8 =$	
40.	$31 - 8 =$	
41.	$28 - 9 =$	
42.	$27 - 8 =$	
43.	$38 - 9 =$	
44.	$37 - 8 =$	

B

Subtraction Crossing Ten

Number Correct: _____

Improvement: _____

1.	$10 - 1 =$	
2.	$20 - 1 =$	
3.	$30 - 1 =$	
4.	$10 - 3 =$	
5.	$20 - 3 =$	
6.	$30 - 3 =$	
7.	$12 - 3 =$	
8.	$22 - 3 =$	
9.	$32 - 3 =$	
10.	$10 - 9 =$	
11.	$11 - 9 =$	
12.	$21 - 9 =$	
13.	$31 - 9 =$	
14.	$13 - 4 =$	
15.	$23 - 4 =$	
16.	$33 - 4 =$	
17.	$16 - 7 =$	
18.	$26 - 7 =$	
19.	$36 - 7 =$	
20.	$10 - 6 =$	
21.	$20 - 7 =$	
22.	$30 - 8 =$	

23.	$13 - 5 =$	
24.	$23 - 5 =$	
25.	$33 - 5 =$	
26.	$16 - 8 =$	
27.	$26 - 8 =$	
28.	$36 - 8 =$	
29.	$12 - 5 =$	
30.	$22 - 5 =$	
31.	$32 - 5 =$	
32.	$11 - 5 =$	
33.	$21 - 5 =$	
34.	$31 - 5 =$	
35.	$12 - 7 =$	
36.	$22 - 7 =$	
37.	$11 - 7 =$	
38.	$31 - 7 =$	
39.	$22 - 9 =$	
40.	$32 - 9 =$	
41.	$38 - 9 =$	
42.	$37 - 8 =$	
43.	$28 - 9 =$	
44.	$27 - 8 =$	

Name _____

Date _____

Use your square tiles to construct the following arrays with no gaps or overlaps. Write a repeated addition equation to match each construction.

1. a. Place 8 square tiles in a row.

b. Construct an array with the 8 square tiles.

c. Write a repeated addition equation to match the new array.

2. a. Construct an array with 12 squares.

a. Write a repeated addition equation to match the array.

c. Rearrange the 12 squares into a different array.

d. Write a repeated addition equation to match the new array.

3. a. Construct an array with 20 squares.

b. Write a repeated addition equation to match the array.

c. Rearrange the 20 squares into a different array.

d. Write a repeated addition equation to match the new array.

4. Construct 2 arrays with 6 squares.

a. 2 rows of _____ = _____

b. 3 rows of _____ = 2 rows of _____

5. Construct 2 arrays with 10 squares.

a. 2 rows of _____ = _____

b. 5 rows of _____ = 2 rows of _____

Name _____

Date _____

a. Construct an array with 12 square tiles.

b. Write a repeated addition equation to match the array.

Name _____

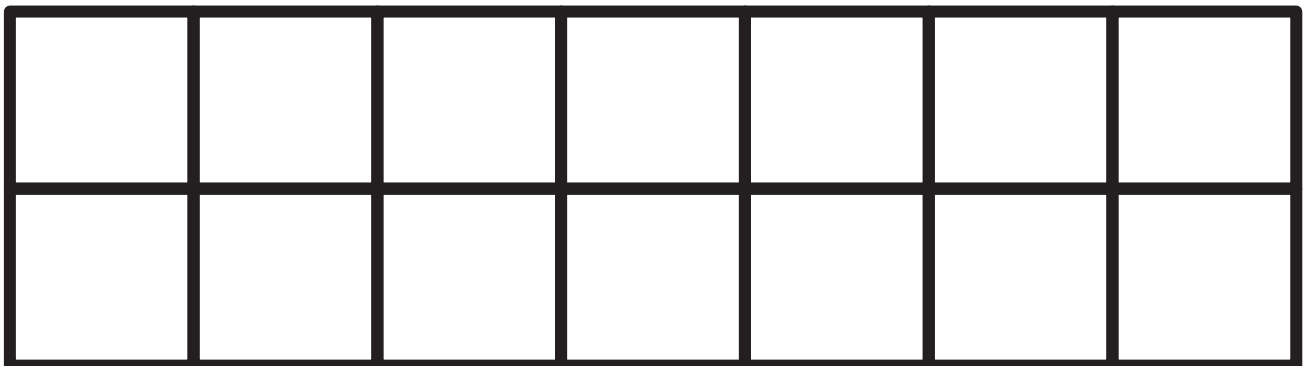
Date _____

1. a. Construct an array with 9 square tiles.
b. Write a repeated addition equation to match the array.

2. a. Construct an array with 10 square tiles.
b. Write a repeated addition equation to match the array.

- c. Rearrange the 10 square tiles into a different array.
- d. Write a repeated addition equation to match the new array.

Cut out each square tile. Use the tiles to construct the arrays in Problems 1-4.



3. a. Construct an array with 12 square tiles.
b. Write a repeated addition equation to match the array.

- c. Rearrange the 12 square tiles into a different array.
d. Write a repeated addition equation to match the new array.

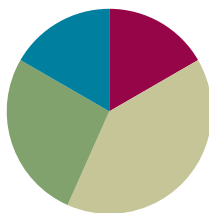
4. Construct 2 arrays with 14 square tiles.
a. 2 rows of _____ = _____
b. 2 rows of _____ = 7 rows of _____

Lesson 12

Objective: Use math drawings to compose a rectangle with square tiles.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Concept Development	(24 minutes)
Application Problem	(16 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Compensation **2.NBT.5** (5 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)

Compensation (5 minutes)

Note: This activity reviews the mental math strategy of compensation. By making a multiple of 10, students solve a much simpler addition problem. Draw a number bond for the first problem on the board to help students visualize the decomposition.

T: (Write $42 + 19 = \underline{\quad}$.) Let's use a mental math strategy to add.
How much more does 19 need to make the next ten?

S: 1 more.

T: Where can 19 get 1 more from?

S: From the 42.

T: Take 1 from 42, and give it to 19. Say the new simplified number sentence with the answer.

S: $41 + 20 = 61$.

T: So, $42 + 19$ is...?

S: 61.

T: $37 + 19$.

S: $36 + 20 = 56$.

$$\begin{array}{r}
 42 + 19 \\
 \swarrow \searrow \\
 41 \quad 1 \\
 41 + 20 = 61
 \end{array}$$

Continue with the following possible sequence: $29 + 23$, $38 + 19$, $52 + 19$, $24 + 18$, and $34 + 28$.

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets

Note: During Topic C and for the remainder of the year, each day's fluency activities include an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Five options are provided in this lesson for the Core Fluency Practice Set, with Set A being the most simple and Set E being the most complex. Start all students on Set A.

Students complete as many problems as they can in 120 seconds. One hundred percent accuracy and completion is recommended before moving to the next level. Collect any Practice Sets that have been completed within the 120 seconds, and check the answers. The next time Core Fluency Practice Sets are used, students who have successfully completed their set can be provided with the next level. Keep a record of student progress.

Consider assigning early finishers a counting pattern and start number (e.g., count by fives from 195). Celebrate improvement as well as advancement. Students should be encouraged to compete with themselves rather than their peers. Discuss with students possible strategies to solve. Notify caring adults of each student's progress.

Important note: These Practice Sets are different from the Practice Sets used in Topic A.

**NOTES ON
MULTIPLE MEANS
OF REPRESENTATION:**

Offer grid paper to students who need extra support to draw the grids. If the paper that is available does not match the size of the tiles, measure the tiles, and use a computer or pencil and ruler to create a custom grid that can be duplicated.

Students who have trouble drawing around small objects such as the tiles may work better with patty paper.

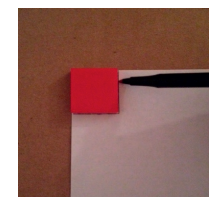
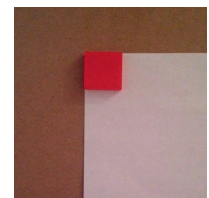
Concept Development (24 minutes)

Materials: (T) one 1-inch tile, plain white paper (S) bag of six 1-inch tiles, plain white paper

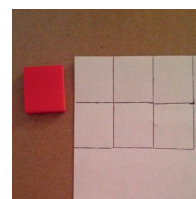
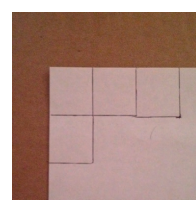
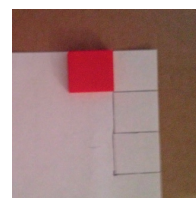
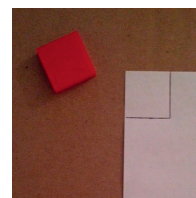
Note: For this lesson, students may convene in a circle on the floor with clipboards (or another solid work surface), or they may remain at their seats if there is a document camera or projector available.

Part 1: Trace a unit square to draw an array.

- T: Make an array with 2 rows of 3 on your paper using the tiles in your bag.
- S: (Create a 2 by 3 array.)
- T: To draw the same array, we can trace the tile 3 times to make a row and then trace to make another row underneath.
- T: Remove the tiles from your paper. Using one tile, make a square in the very top left corner of your paper like I do.
- T: We can use the edge of the paper as one of our lines to help keep the array straight.

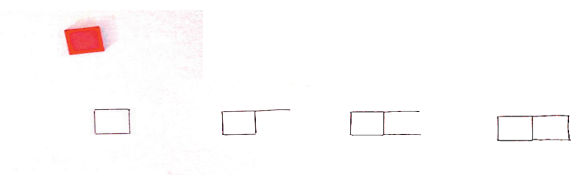


- S: (Draw the first square as shown.)
- T: Turn and talk: What should we do next to create 2 rows of 3?
- S: Make another square in the top row. → Make a square underneath the first one to start the second row.
- T: We could do it either way. For now, let's finish the row of 3, just as we did when we were building arrays row by row. Add two more squares by tracing your tile as I do.
- T: (Complete the first row as students do the same.)
- T: Turn and talk: What is our next step?
- S: Add another row. → Trace a tile on the bottom to start the second row.
- T: Let's try that. Add another square just below the first one. Do we need to trace the entire square?
- S: No!
- T: Why not?
- S: The bottom line from the first square will be the top line of this square!
- T: Draw what I draw. (Add the first square in the second row as pictured.)
- T: How many more squares do we need to complete this array?
- S: 2.
- T: Add the last two squares as I do. (Complete the array as pictured.)
- S: (Complete the arrays.)
- T: This process reminds me of creating our rulers. How is it the same? Talk to your partner.
- S: We are repeating a unit. → We are using a tool to help us make the spaces the same. → We are measuring using the mark and advance technique but going down, too!
- T: I hear good observations. This time, we are making a unit that is a square. Before, we were making a length unit.



Have students repeat the above process independently to create an array that is 2 units wide and 4 units long, below the first array. This time, the side of the paper guides them. Then, distribute another piece of paper.

Part 2: Draw an array without the use of a tile.



- T: This time, we will draw most of the array without the tile. To start, let's make one square in the middle of the page. (Model the tracing to make one square in the middle of the page as pictured.)



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Model this process outdoors, drawing with sidewalk chalk on pavement around foam squares or a square piece of construction paper.

T: Let's start with the top side of the next square. Tell me when to stop drawing. (Make the line as pictured on the previous page.)

S: Stop!

T: Is the length of the line about the same length as the first tile?

S: Yes!

T: Your turn. Draw that line on your paper.

T: Let's draw the bottom of the square. Again, say when to stop. (Add the line as pictured.)

S: Stop!

T: Is the bottom line the same length as the top line?

S: Yes!

T: Add that line to your drawing.

T: Let's close the square by making a third line. (Add the last line of the square as pictured.)

T: Does the square I drew look pretty much the same as the square I traced?

S: Yes!

T: Now it's your turn. Complete your square on your paper.

S: (Draw the line to close the square.)



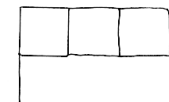
T: How many more squares do we need in this row to make 1 row of 3?

S: 1.

T: Draw one more square the way we made the last one. Then, hold your paper up with two hands for me to see.

T: (Check student work.)

T: Let's start the second line together.



T: I will draw the line, and you say when to stop. (Add another line to start the first square in the second row.)

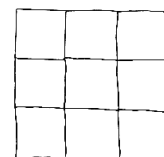
S: Stop!

T: Is this line the same length as the side of the first square?

S: Yes!

T: Add that line to your drawing. (Circulate.)

S: (Draw.)



T: Work to draw 3 rows of 3 squares.

T: What shape did you end up making?

S: A square!

T: What shape had you made after making 2 rows of 3?

S: A rectangle!

Check student work before moving on. When students are ready, have them finish the second row of the array independently. If more practice is needed, have them complete an array with 2 rows of 5, offering support when needed. Otherwise, move students directly to the Application Problem and then to the Problem Set.

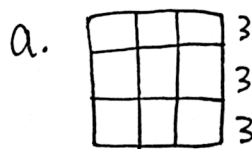
Application Problem (16 minutes)

Lulu made a pan of brownies. She cut them into 3 rows and 3 columns.

- Draw a picture of Lulu's brownies in the pan.
- Write a number sentence to show how many brownies Lulu has.
- Write a statement about Lulu's brownies.

MP.3

Extension: How should Lulu cut her brownies if she wants to equally serve 12 people? 16 people? 20 people?



b. $3 + 3 + 3 = 9$

c. Lulu has 9 brownies.

Note: This Application Problem serves as a practice of the previous day's lesson. The extension, however, invites students to apply understandings from today's lesson as well. For this reason, it follows the Concept Development. The allotted time period includes 6 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use math drawings to compose a rectangle with square tiles.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Roberto Date _____

1. Draw without using a square tile to make an array with 2 rows of 5.

2 rows of 5 = 10
 $\underline{5} + \underline{5} = \underline{10}$

2. Draw without using a square tile to make an array with 4 columns of 3.

4 columns of 3 = 12
 $\underline{3} + \underline{3} + \underline{3} + \underline{3} = \underline{12}$

Any combination of the questions below may be used to lead the discussion.

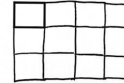
- For Problem 3 (a) and (b), what was your first step in drawing a rectangle?
- Explain to your partner how to draw a rectangle with one square tile. Why was precision important today? How is this different from drawing an array with Xs?
- For Problems 1 and 2, discuss with your partner how the repeated addition equation relates to the number of units in each rectangle.
- What was challenging about drawing a rectangle without tracing the square tile in Problem 3? What did you need to be sure to do?
- How does drawing a rectangle support the idea of composing a larger unit from smaller units? Use the terms *square*, *rows*, and *columns* in your response.

Exit Ticket (3 minutes)

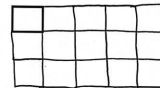
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

3. Complete the following arrays without gaps or overlaps. The first tile has been drawn for you.

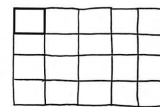
a. 3 rows of 4



b. 5 columns of 3



c. 5 columns of 4



Name _____

Date _____

1.	$10 + 2 =$	21.	$7 + 9 =$
2.	$10 + 7 =$	22.	$5 + 8 =$
3.	$10 + 5 =$	23.	$3 + 9 =$
4.	$4 + 10 =$	24.	$8 + 6 =$
5.	$6 + 11 =$	25.	$7 + 4 =$
6.	$12 + 2 =$	26.	$9 + 5 =$
7.	$14 + 3 =$	27.	$6 + 6 =$
8.	$13 + 5 =$	28.	$8 + 3 =$
9.	$17 + 2 =$	29.	$7 + 6 =$
10.	$12 + 6 =$	30.	$6 + 9 =$
11.	$11 + 9 =$	31.	$8 + 7 =$
12.	$2 + 16 =$	32.	$9 + 9 =$
13.	$15 + 4 =$	33.	$5 + 7 =$
14.	$5 + 9 =$	34.	$8 + 4 =$
15.	$9 + 2 =$	35.	$6 + 5 =$
16.	$4 + 9 =$	36.	$9 + 7 =$
17.	$9 + 6 =$	37.	$6 + 8 =$
18.	$8 + 9 =$	38.	$2 + 9 =$
19.	$7 + 8 =$	39.	$9 + 8 =$
20.	$8 + 8 =$	40.	$7 + 7 =$

Name _____

Date _____

1.	$10 + 6 =$	21.	$3 + 8 =$
2.	$10 + 9 =$	22.	$9 + 4 =$
3.	$7 + 10 =$	23.	$\underline{\hspace{1cm}} + 6 = 11$
4.	$3 + 10 =$	24.	$\underline{\hspace{1cm}} + 9 = 13$
5.	$5 + 11 =$	25.	$8 + \underline{\hspace{1cm}} = 14$
6.	$12 + 8 =$	26.	$7 + \underline{\hspace{1cm}} = 15$
7.	$14 + 3 =$	27.	$\underline{\hspace{1cm}} = 4 + 8$
8.	$13 + \underline{\hspace{1cm}} = 19$	28.	$\underline{\hspace{1cm}} = 8 + 9$
9.	$15 + \underline{\hspace{1cm}} = 18$	29.	$\underline{\hspace{1cm}} = 6 + 4$
10.	$12 + 5 =$	30.	$3 + 9 =$
11.	$\underline{\hspace{1cm}} = 2 + 17$	31.	$5 + 7 =$
12.	$\underline{\hspace{1cm}} = 3 + 13$	32.	$8 + \underline{\hspace{1cm}} = 14$
13.	$\underline{\hspace{1cm}} = 16 + 2$	33.	$\underline{\hspace{1cm}} = 5 + 9$
14.	$9 + 3 =$	34.	$8 + 8 =$
15.	$6 + 9 =$	35.	$\underline{\hspace{1cm}} = 7 + 9$
16.	$\underline{\hspace{1cm}} + 5 = 14$	36.	$\underline{\hspace{1cm}} = 8 + 4$
17.	$\underline{\hspace{1cm}} + 7 = 13$	37.	$17 = 8 + \underline{\hspace{1cm}}$
18.	$\underline{\hspace{1cm}} + 8 = 12$	38.	$19 = \underline{\hspace{1cm}} + 9$
19.	$8 + 7 =$	39.	$12 = \underline{\hspace{1cm}} + 7$
20.	$7 + 6 =$	40.	$15 = 8 + \underline{\hspace{1cm}}$

Name _____

Date _____

1.	$13 - 3 =$	21.	$16 - 8 =$
2.	$19 - 9 =$	22.	$14 - 5 =$
3.	$15 - 10 =$	23.	$16 - 7 =$
4.	$18 - 10 =$	24.	$15 - 7 =$
5.	$12 - 2 =$	25.	$17 - 8 =$
6.	$11 - 10 =$	26.	$18 - 9 =$
7.	$17 - 13 =$	27.	$15 - 6 =$
8.	$20 - 10 =$	28.	$13 - 8 =$
9.	$14 - 11 =$	29.	$14 - 6 =$
10.	$16 - 12 =$	30.	$12 - 5 =$
11.	$11 - 3 =$	31.	$11 - 7 =$
12.	$13 - 2 =$	32.	$13 - 8 =$
13.	$14 - 2 =$	33.	$16 - 9 =$
14.	$13 - 4 =$	34.	$12 - 8 =$
15.	$12 - 3 =$	35.	$16 - 12 =$
16.	$11 - 4 =$	36.	$18 - 15 =$
17.	$12 - 5 =$	37.	$15 - 14 =$
18.	$14 - 5 =$	38.	$17 - 11 =$
19.	$11 - 2 =$	39.	$19 - 13 =$
20.	$12 - 4 =$	40.	$20 - 12 =$

Name _____

Date _____

1.	$17 - 7 =$	21.	$16 - 7 =$
2.	$14 - 10 =$	22.	$17 - 8 =$
3.	$19 - 11 =$	23.	$18 - 7 =$
4.	$16 - 10 =$	24.	$14 - 6 =$
5.	$17 - 12 =$	25.	$17 - 8 =$
6.	$15 - 13 =$	26.	$12 - 8 =$
7.	$12 - 3 =$	27.	$14 - 7 =$
8.	$20 - 11 =$	28.	$15 - 8 =$
9.	$18 - 11 =$	29.	$13 - 5 =$
10.	$13 - 5 =$	30.	$16 - 8 =$
11.	$\underline{\hspace{1cm}} = 11 - 2$	31.	$14 - 9 =$
12.	$\underline{\hspace{1cm}} = 12 - 4$	32.	$15 - 6 =$
13.	$\underline{\hspace{1cm}} = 13 - 5$	33.	$13 - 6 =$
14.	$\underline{\hspace{1cm}} = 12 - 3$	34.	$\underline{\hspace{1cm}} = 13 - 8$
15.	$\underline{\hspace{1cm}} = 11 - 4$	35.	$\underline{\hspace{1cm}} = 15 - 7$
16.	$\underline{\hspace{1cm}} = 13 - 2$	36.	$\underline{\hspace{1cm}} = 18 - 9$
17.	$\underline{\hspace{1cm}} = 11 - 3$	37.	$\underline{\hspace{1cm}} = 20 - 14$
18.	$17 - 8 =$	38.	$\underline{\hspace{1cm}} = 20 - 7$
19.	$14 - 6 =$	39.	$\underline{\hspace{1cm}} = 20 - 11$
20.	$16 - 9 =$	40.	$\underline{\hspace{1cm}} = 20 - 8$

Name _____

Date _____

1.	$11 + 9 =$	21.	$13 - 7 =$
2.	$13 + 5 =$	22.	$11 - 8 =$
3.	$14 + 3 =$	23.	$15 - 6 =$
4.	$12 + 7 =$	24.	$12 + 7 =$
5.	$5 + 9 =$	25.	$14 + 3 =$
6.	$8 + 8 =$	26.	$8 + 12 =$
7.	$14 - 7 =$	27.	$5 + 7 =$
8.	$13 - 5 =$	28.	$8 + 9 =$
9.	$16 - 7 =$	29.	$7 + 5 =$
10.	$17 - 9 =$	30.	$13 - 6 =$
11.	$14 - 6 =$	31.	$14 - 8 =$
12.	$18 - 5 =$	32.	$12 - 9 =$
13.	$9 + 9 =$	33.	$11 - 3 =$
14.	$7 + 6 =$	34.	$14 - 5 =$
15.	$3 + 9 =$	35.	$13 - 8 =$
16.	$6 + 7 =$	36.	$8 + 5 =$
17.	$8 + 5 =$	37.	$4 + 7 =$
18.	$13 - 8 =$	38.	$7 + 8 =$
19.	$16 - 9 =$	39.	$4 + 9 =$
20.	$14 - 8 =$	40.	$20 - 8 =$

Name _____ Date _____

1. Draw without using a square tile to make an array with 2 rows of 5.

2 rows of 5 = _____

_____ + _____ = _____

2. Draw without using a square tile to make an array with 4 columns of 3.

4 columns of 3 = _____

_____ + _____ + _____ + _____ = _____

3. Complete the following arrays without gaps or overlaps. The first tile has been drawn for you.

a. 3 rows of 4



b. 5 columns of 3



c. 5 columns of 4



Name _____

Date _____

Draw an array of 3 columns of 3 starting with the square below without gaps or overlaps.



Name _____ Date _____

1. Cut out and trace the square tile to draw an array with 2 rows of 4.

Cut out
and trace.

2 rows of 4 = _____

_____ + _____ = _____

2. Trace the square tile to make an array with 3 columns of 5.

3 columns of 5 = _____

_____ + _____ + _____ = _____

3. Complete the following arrays without gaps or overlaps. The first tile has been drawn for you.

a. 4 rows of 5



b. 5 columns of 2



c. 4 columns of 3

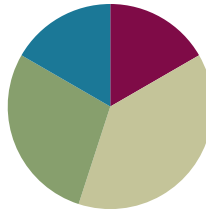


Lesson 13

Objective: Use square tiles to decompose a rectangle.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Concept Development	(23 minutes)
■ Application Problem	(17 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Making the Next Ten to Add **2.OA.2, 2.NBT.5** (5 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)

Making the Next Ten to Add (5 minutes)

Note: This fluency activity reviews making a ten to add.

T: When I say “9 + 4,” you say “10 + 3.” Ready? 9 + 4.

S: 10 + 3.

T: Answer.

S: 13.

Continue with the following possible sequences:

19 + 4, 29 + 4, 29 + 14, 59 + 14

9 + 6, 19 + 6, 19 + 16, 49 + 16

8 + 3, 18 + 3, 18 + 13

8 + 5, 18 + 5, 18 + 15, 38 + 15

7 + 6, 17 + 6, 17 + 16, 37 + 16

7 + 4, 17 + 4, 67 + 4

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 12 Core Fluency Practice Sets)

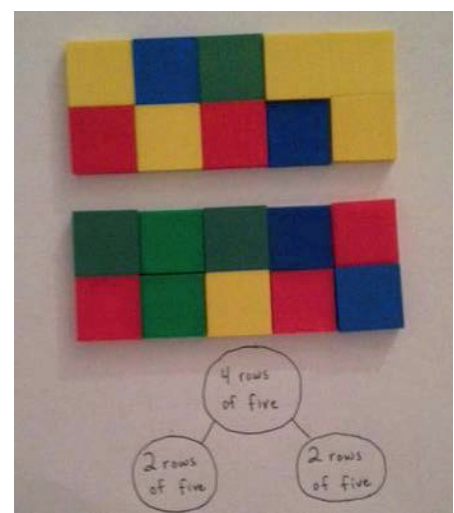
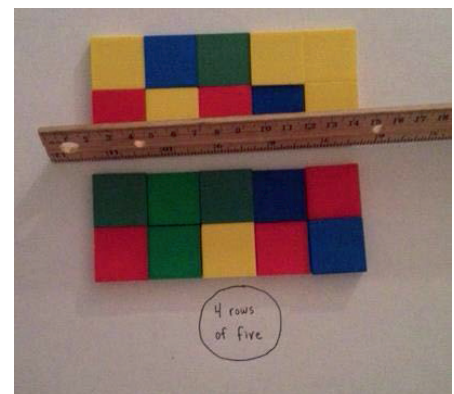
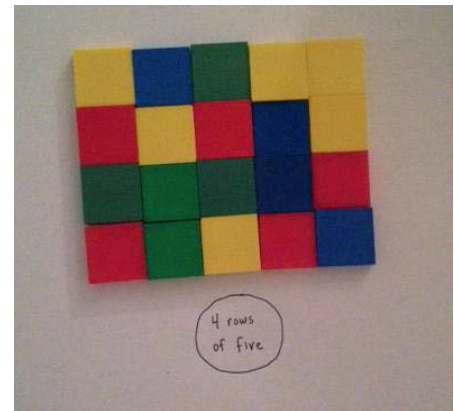
Note: During Topic C and for the remainder of the year, each day’s fluency activities include an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 12.

Concept Development (23 minutes)

Materials: (T/S) 25 square tiles or square tiles (Template), personal white board, ruler

For the following Concept Development, model the work for students using an overhead projector or document camera.

- T: With your partner, use the tiles to construct a rectangle with 4 rows of 5 on your personal white board. Tell your partner the total number of tiles in your rectangle and how you know.
- T: (Model a rectangle with 4 rows of 5 using the overhead projector.)
- S: There are 20 tiles because $5 + 5 + 5 + 5 = 20$. → There are 20 because $4 + 4 + 4 + 4 = 20$.
- T: Write the number of rows and the number in each row as the whole in your number bond as I do. (Model writing 4 rows of 5 under the rectangle.)
- S: (Write 4 rows of 5, as pictured.)
- T: Turn and talk: How can we decompose this rectangle into two equal parts?
- S: I know that $10 + 10$ makes 20, so we could put 10 on one side and 10 on the other. → If we split it across the middle like how we spread out the rows of lima beans with a ruler, we can make it half and half. → Two equal parts would be 2 rows of 5 on one side and 2 rows of 5 on the other.
- T: Use your ruler to break your rectangle into two equal parts as I do. (Model using the ruler to split the rectangle.)
- T: (Circulate as students decompose the rectangle as pictured.)
- T: How many rows do you have in each part?
- S: Two rows!
- T: How many tiles are in each row?
- S: 5 tiles!
- T: Write 2 rows of 5 for each part of your number bond.
- T: (Model writing 2 rows of 5 in each part of the number bond.)
- T: If $5 + 5 + 5 + 5$ represented the rectangle before we decomposed it, what number sentence can you use to describe each part?
- S: $5 + 5 = 10$.
- T: Write $5 + 5 = 10$ below the parts of the number bond. (Model writing the number sentences under each part.)
- S: (Write the number sentences.)



T: Tell your partner the two parts and the whole using a number sentence.

S: Two rows of 5 and 2 rows of 5 make 4 rows of 5.

Repeat the process with 6 columns of 2, decomposing by columns rather than by rows.

T: With your partner, count out 16 tiles. Then, construct a rectangle on your desk with 4 rows.

T: (Circulate as students work.)

T: How many rows did you make?

S: 4 rows!

T: How many tiles are in each row?

S: 4 tiles!

T: Say the repeated addition sentence.

S: $4 + 4 + 4 + 4 = 16$.

T: What do 4 rows of 4 equal?

S: 16.

T: Take away a row.

T: Turn and talk: What is the new total for the rectangle, and how do you know?

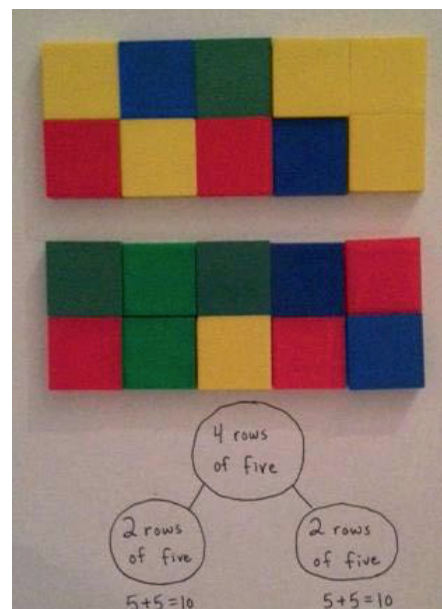
S: It is 12 because $4 + 4 + 4 = 12$. → It is 12 because $16 - 4 = 12$. → 3 fours is 12.

T: Remove one column.

T: Turn and talk: How many tiles do you have now, and how do you know?

S: We have 9 because there are 3 rows of 3, and $3 + 3 + 3 = 9$. → I see 3 threes, and that's 9. → There are 9 because $12 - 3 = 9$.

Repeat the above process with a rectangle of 25 tiles.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students are able to express their arrays in multiplication number sentences and are eager to show off their expertise. Encourage them to write both types of number sentences and share how they know with another student.

MP.7

Application Problem (17 minutes)

Ellie bakes a square pan of lemon bars, which she cut into nine equal pieces. Her brothers eat 1 row of her treats. Then, her mom eats 1 column.

- Draw a picture of Ellie's lemon bars before any are eaten. Write a number sentence to show how to find the total.
- Write an X on the bars that her brothers eat. Write a new number sentence to show how many are left.

- c. Draw a line through the bars that her mom eats. Write a new number sentence to show how many are left.
- d. How many bars are left? Write a statement.

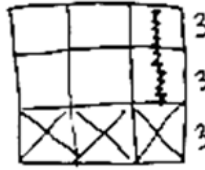
Note: This Application Problem provides an opportunity for students to apply understandings from today's lesson, so it follows the Concept Development. If necessary, provide manipulatives for students to use when solving the problem. The allotted time period includes 7 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

a) $3+3+3=9$

b) $9-3=6$
or
 $3+3=6$

c) $6-2=4$
or
 $2+2=4$

d) There are 4 lemon bars left.



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use square tiles to decompose a rectangle.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How does your number bond show how you decomposed, or broke apart, your rectangle in Problem 1?
- In Problem 2, what do you notice is the same in the whole and parts of your number bond? (A unit of two.) How does your repeated addition sentence change without one row?

Name Sam Date _____

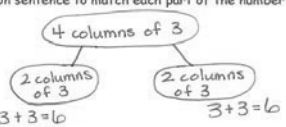
Use your square tiles to complete the steps for each problem.

Problem 1

Step 1: Construct a rectangle with 4 columns of 3.

Step 2: Separate 2 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Then, write a repeated addition sentence to match each part of the number bond.

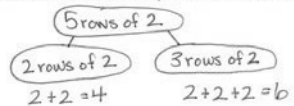


Problem 2

Step 1: Construct a rectangle with 5 rows of 2.

Step 2: Separate 2 rows of 2.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

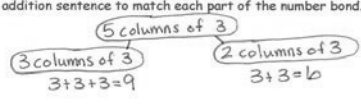


Problem 3

Step 1: Construct a rectangle with 5 columns of 3.

Step 2: Separate 3 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.



- In Problem 3, defend how you know that a rectangle can be decomposed into smaller rectangles. Describe the two smaller rectangles that you found in 5 columns of 3. Use the terms *rows*, *columns*, *units*, and *repeated addition*.
- What was your strategy for composing a rectangle with 12 squares for Problem 4? How many different possibilities are there?
- For Problem 5, how is removing a row from a rectangle with 2 rows of 10 different from removing a row from 5 rows of 4? Which one will leave you with more squares?
- For Problem 6, share with a partner all of the different ways that you could break apart a rectangle made up of 16 square tiles.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Note: Students need the square tile template to complete the homework.

4. Use 12 square tiles to construct a rectangle with 3 rows.
- 3 rows of 4 = 12
 - Remove 1 row. How many squares are there now? 8
 - Remove 1 column from the new rectangle you made in 4(b). How many squares are there now? 6
5. Use 20 square tiles to construct a rectangle.
- 5 rows of 4 = 20
 - Remove 1 row. How many squares are there now? 16
 - Remove 1 column from the new rectangle you made in 5(b). How many squares are there now? 12
6. Use 16 square tiles to construct a rectangle.
- 2 rows of 8 = 16
 - Remove 1 row. How many squares are there now? 8
 - Remove 1 column from the new rectangle you made in 6(b). How many squares are there now? 7

Name _____

Date _____

Use your square tiles to complete the steps for each problem.

Problem 1

Step 1: Construct a rectangle with 4 columns of 3.

Step 2: Separate 2 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Then, write a repeated addition sentence to match each part of the number bond.

Problem 2

Step 1: Construct a rectangle with 5 rows of 2.

Step 2: Separate 2 rows of 2.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

Problem 3

Step 1: Construct a rectangle with 5 columns of 3.

Step 2: Separate 3 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

4. Use 12 square tiles to construct a rectangle with 3 rows.

- a. _____ rows of _____ = 12
- b. Remove 1 row. How many squares are there now? _____
- c. Remove 1 column from the new rectangle you made in 4(b). How many squares are there now? _____

5. Use 20 square tiles to construct a rectangle.

- a. _____ rows of _____ = _____
- b. Remove 1 row. How many squares are there now? _____
- c. Remove 1 column from the new rectangle you made in 5(b). How many squares are there now? _____

6. Use 16 square tiles to construct a rectangle.

- a. _____ rows of _____ = _____
- b. Remove 1 row. How many squares are there now? _____
- c. Remove 1 column from the new rectangle you made in 6(b). How many squares are there now? _____

Name _____ Date _____

Use your square tiles to complete the steps for each problem.

Step 1: Construct a rectangle with 3 columns of 4.

Step 2: Separate 2 columns of 4.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of the number bond.

Name _____

Date _____

Cut out and use your square tiles to complete the steps for each problem.

Problem 1

Step 1: Construct a rectangle with 5 rows of 2.

Step 2: Separate 2 rows of 2.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of your number bond.

Problem 2

Step 1: Construct a rectangle with 4 columns of 3.

Step 2: Separate 2 columns of 3.

Step 3: Write a number bond to show the whole and two parts. Write a repeated addition sentence to match each part of your number bond.

3. Use 9 square tiles to construct a rectangle with 3 rows.

a. _____ rows of _____ = _____

b. Remove 1 row. How many squares are there now? _____

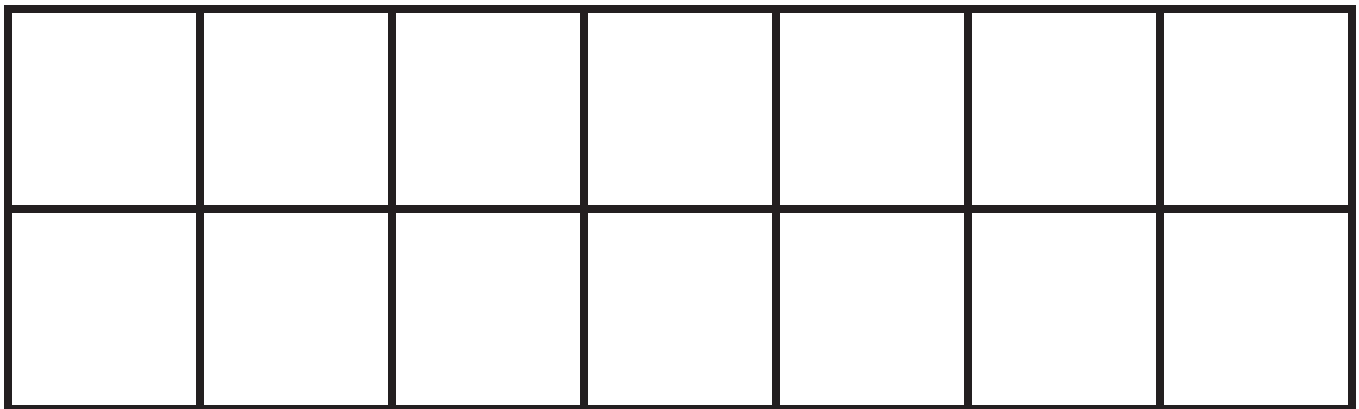
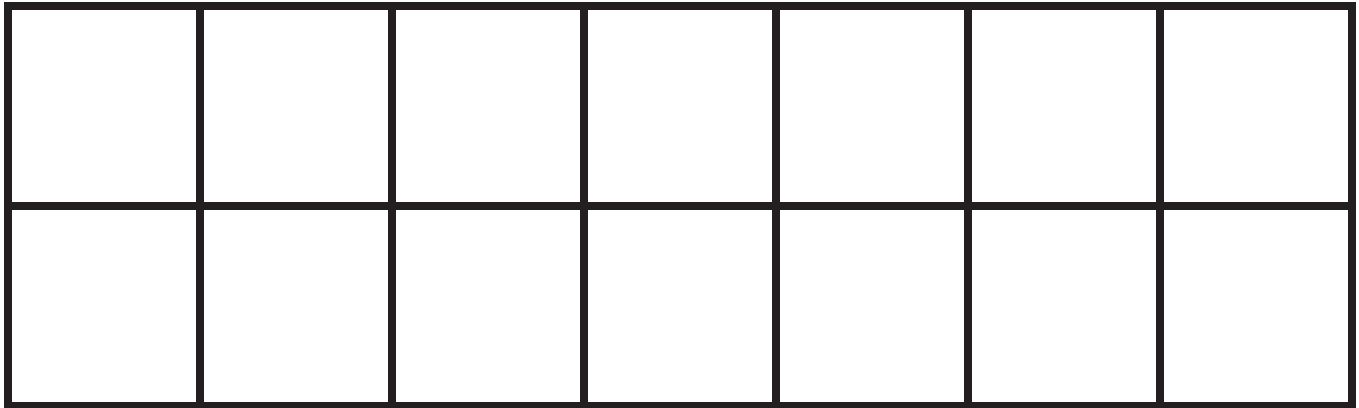
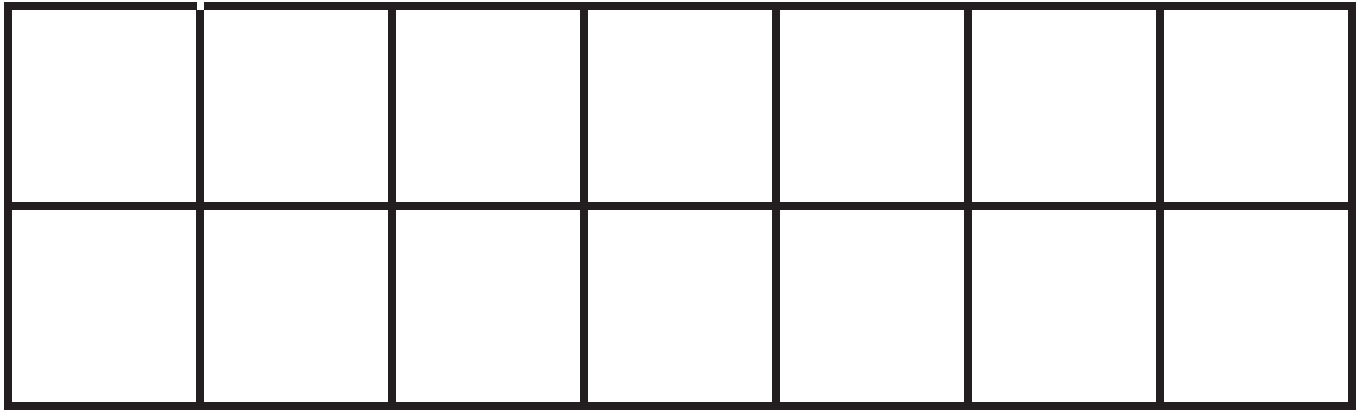
c. Remove 1 column from the new rectangle you made in 3(b). How many squares are there now? _____

4. Use 14 square tiles to construct a rectangle.

a. _____ rows of _____ = _____

b. Remove 1 row. How many squares are there now? _____

c. Remove 1 column from the new rectangle you made in 4(b). How many squares are there now? _____



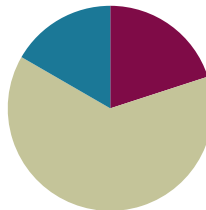
square tiles

Lesson 14

Objective: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Subtraction from Teens **2.OA.2** (8 minutes)
- Coin Drop **2.OA.2** (2 minutes)
- More and Less **2.NBT.5** (2 minutes)

Sprint: Subtraction from Teens (8 minutes)

Materials: (S) Subtraction from Teens Sprint

Note: Students practice subtraction from teens to gain mastery of the sums and differences within 20.

Coin Drop (2 minutes)

Materials: (T) 10 dimes, 10 pennies, metal or plastic can

Note: In this activity, students practice adding and subtracting ones and tens using coins in preparation for Module 7.

T: (Hold up a penny.) Name my coin.

S: A penny.

T: How much is it worth?

S: 1 cent.

T: Listen carefully as I drop coins in my can. Count along in your minds.

Drop in some pennies, and ask how much money is in the can. Take out some pennies, and show students. Ask how much money is still in the can. Continue adding and subtracting pennies for a minute or so. Repeat the activity with dimes and then with dimes and pennies.

More and Less (2 minutes)

Materials: (T) 10 dimes, 10 pennies

Note: In this activity, students practice adding and subtracting ones and tens using coins.

T: Let's count by tens. (Move dimes to the side while counting.)

S: 10, 20, 30, 40, 50, 60.

T: How many dimes are shown?

S: 6 dimes.

T: What is the value of 6 dimes?

S: 60 cents.

T: What is 5 cents more? (Move 5 pennies.)

S: 65 cents.

T: Give the number sentence.

S: $60 \text{ cents} + 5 \text{ cents} = 65 \text{ cents}$.

T: What is 10 cents less? (Move one dime.)

S: 55 cents.

T: Give the number sentence.

S: $65 \text{ cents} - 10 \text{ cents} = 55 \text{ cents}$.

Repeat this line of questioning by starting with 7 dimes, removing 3 dimes, and asking for the number sentence. Continue by adding 3 pennies and asking for the number sentence, adding 4 dimes and asking for the number sentence, and so forth.

Concept Development (38 minutes)

Materials: (T) Rectangles (Template) (S) Rectangles (Template), Problem Set, scissors

In this lesson, the Problem Set is used during the Concept Development.

T: Today, we're going to use the Problem Set for our lesson! We'll use the sentence frames to record our answers and to speak in complete sentences.

Pass out the template, Problem Set, and scissors. For each step of the instructions, model as students work. Circulate to be sure students are following the steps accurately.

T: Cut Rectangle A into rows, and complete Problem 1. Share your responses and thinking with your partner. (Allow students time to work and share.)

T: Cut Rectangle B into columns, and complete Problem 2. Share again.

**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

Assist students in using appropriate math vocabulary by providing a list of current words used in the module and adding to the list as new words are introduced. Post these words with definitions or illustrations prominently in the classroom so students can refer to them during the lessons.

MP.7

- T: Put both rectangles back together again so there are no gaps or overlaps. Move the rows of Rectangle A so they are sitting directly above the columns of Rectangle B. Talk to your partner about what you notice.
- S: They both show the same amount, and they're both the same size and shape. → I can see the same rectangle different ways: It's 2 rows of 4, 4 columns of 2, or 8 squares. → They're both made up of rows and columns with the same total.
- T: You've recognized that we can decompose the same rectangle into rows, columns, or individual square units.
- T: Take both your rows of 4 and cut them to show 4 twos instead of 2 fours.
- T: (Demonstrate as necessary.) Put together your twos to form one long rectangle that has 8 columns of 2.
- T: Imagine we were going to put 2 rows on top to make the exact same rectangle. Talk to your partner. Explain what those rectangles would be.
- S: I see it would be 2 rows of 8. → We need 2 eights. → They would be 2 of the long skinny rectangles.
- T: We can decompose this rectangle into 2 rows of 8 or 8 columns of 2.
- T: Cut out all your squares from Rectangles A and B. How many squares do you have now?
- S: 16.
- T: Use your 16 squares to answer Problem 3.
- T: To answer Problem 4, cut out your squares from Rectangles A, B, and C.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If there are students who miss lessons in the module and others who struggle to grasp the concept, ask those who struggle to explain the array concepts to the previously absent students. This helps students to clarify their thinking or discover what questions to ask to understand the concept better.

Circulate as students experiment with their squares to form their rectangles. Ask them questions about each rectangle to support their spatial structuring, such as “How many rows do you see in this rectangle?” or “How many columns?” This lesson’s intent is to give practice in seeing the same rectangle both as rows and columns. It is important for students to work with more tiles to increase the complexity of the work. Encourage them to view the array with 24 squares as a rectangle rather than as 24 individual squares. Students who struggle can work with fewer squares.

Student Debrief (10 minutes)

Lesson Objective: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. At the end of the Debrief today, review and clarify the directions for the Homework to check for student understanding.

Any combination of the questions below may be used to lead the discussion.

- What did we learn about our rectangles by first cutting them into rows and columns before cutting out each individual square?
- If you were to write a repeated addition sentence to describe the work we did in Problem 2, what would it look like? Why? How does this relate to the columns you cut out?
- For Problem 3, what was your strategy for composing a new rectangle? How did the rows and columns change?
- For Problem 4, what strategy did you use to compose a new rectangle with 24 squares?
- How many different possibilities can you think of for composing a rectangle with 24 squares? How many different repeated addition sentences? How do they match the rows and columns of your array?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

Cut out Rectangles A, B, and C. Then cut according to directions. Answer each of the following using Rectangles A, B, and C.¹

- Cut out each row of Rectangle A.
 - Rectangle A has 2 rows.
 - Each row has 4 squares.
 - 2 rows of 4 = 8
 - Rectangle A has 8 squares.
- Cut out each column of Rectangle B.
 - Rectangle B has 4 columns.
 - Each column has 2 squares.
 - 4 columns of 2 = 8
 - Rectangle B has 8 squares.

¹ Note: This Problem Set is used with a template of three identical 2 by 4 arrays. These arrays are labeled as Rectangles A, B, and C.

- Cut out each square from both Rectangles A and B.
 - Construct a new rectangle using all 16 squares.
 - My rectangle has 2 rows of 8.
 - My rectangle also has 8 columns of 2.
 - Write two repeated addition number sentences to match your rectangle.
 $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$ $8 + 8 = 16$
- Construct a new array using the 24 squares from Rectangles A, B, and C.
 - My rectangle has 4 rows of 6.
 - My rectangle also has 6 columns of 4.
 - Write two repeated addition number sentences to match your rectangle.
 $4 + 4 + 4 + 4 + 4 + 4 = 24$ $6 + 6 + 6 + 6 = 24$

Extension: Construct another array using the squares from Rectangles A, B, and C.

- My rectangle has 3 rows of 8.
- My rectangle also has 8 columns of 3.
- Write two repeated addition number sentences to match your rectangle.
 $8 + 8 + 8 = 24$ $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24$

A

Number Correct: _____

Subtraction from Teens

1.	$11 - 10 =$	
2.	$12 - 10 =$	
3.	$13 - 10 =$	
4.	$19 - 10 =$	
5.	$11 - 1 =$	
6.	$12 - 2 =$	
7.	$13 - 3 =$	
8.	$17 - 7 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 8 =$	
13.	$18 - 8 =$	
14.	$13 - 4 =$	
15.	$13 - 5 =$	
16.	$13 - 6 =$	
17.	$13 - 8 =$	
18.	$16 - 6 =$	
19.	$12 - 3 =$	
20.	$12 - 4 =$	
21.	$12 - 5 =$	
22.	$12 - 9 =$	

23.	$19 - 9 =$	
24.	$15 - 6 =$	
25.	$15 - 7 =$	
26.	$15 - 9 =$	
27.	$20 - 10 =$	
28.	$14 - 5 =$	
29.	$14 - 6 =$	
30.	$14 - 7 =$	
31.	$14 - 9 =$	
32.	$15 - 5 =$	
33.	$17 - 8 =$	
34.	$17 - 9 =$	
35.	$18 - 8 =$	
36.	$16 - 7 =$	
37.	$16 - 8 =$	
38.	$16 - 9 =$	
39.	$17 - 10 =$	
40.	$12 - 8 =$	
41.	$18 - 9 =$	
42.	$11 - 9 =$	
43.	$15 - 8 =$	
44.	$13 - 7 =$	

B

Number Correct: _____

Improvement: _____

Subtraction from Teens

1.	$11 - 1 =$	
2.	$12 - 2 =$	
3.	$13 - 3 =$	
4.	$18 - 8 =$	
5.	$11 - 10 =$	
6.	$12 - 10 =$	
7.	$13 - 10 =$	
8.	$18 - 10 =$	
9.	$11 - 2 =$	
10.	$11 - 3 =$	
11.	$11 - 4 =$	
12.	$11 - 7 =$	
13.	$19 - 9 =$	
14.	$12 - 3 =$	
15.	$12 - 4 =$	
16.	$12 - 5 =$	
17.	$12 - 8 =$	
18.	$17 - 7 =$	
19.	$13 - 4 =$	
20.	$13 - 5 =$	
21.	$13 - 6 =$	
22.	$13 - 9 =$	

23.	$16 - 6 =$	
24.	$14 - 5 =$	
25.	$14 - 6 =$	
26.	$14 - 7 =$	
27.	$14 - 9 =$	
28.	$20 - 10 =$	
29.	$15 - 6 =$	
30.	$15 - 7 =$	
31.	$15 - 9 =$	
32.	$14 - 4 =$	
33.	$16 - 7 =$	
34.	$16 - 8 =$	
35.	$16 - 9 =$	
36.	$20 - 10 =$	
37.	$17 - 8 =$	
38.	$17 - 9 =$	
39.	$16 - 10 =$	
40.	$18 - 9 =$	
41.	$12 - 9 =$	
42.	$13 - 7 =$	
43.	$11 - 8 =$	
44.	$15 - 8 =$	

Name _____

Date _____

Cut out Rectangles A, B, and C. Then, cut according to directions. Answer each of the following using Rectangles A, B, and C.¹

1. Cut out each row of Rectangle A.

a. Rectangle A has _____ rows.

b. Each row has _____ squares.

c. _____ rows of _____ = _____

d. Rectangle A has _____ squares.

2. Cut out each column of Rectangle B.

a. Rectangle B has _____ columns.

b. Each column has _____ squares.

c. _____ columns of _____ = _____

d. Rectangle B has _____ squares.

¹Note: This Problem Set is used with a template of three identical 2 by 4 arrays. These arrays are labeled as Rectangles A, B, and C.

3. Cut out each square from both Rectangles A and B.
 - a. Construct a new rectangle using all 16 squares.
 - b. My rectangle has _____ rows of _____.
 - c. My rectangle also has _____ columns of _____.
 - d. Write two repeated addition number sentences to match your rectangle.

4. Construct a new array using the 24 squares from Rectangles A, B, and C.
 - a. My rectangle has _____ rows of _____.
 - b. My rectangle also has _____ columns of _____.
 - c. Write two repeated addition number sentences to match your rectangle.

Extension: Construct another array using the squares from Rectangles A, B, and C.

- a. My rectangle has _____ rows of _____.
- b. My rectangle also has _____ columns of _____.
- c. Write two repeated addition number sentences to match your rectangle.

Name _____ Date _____

With your tiles, show 1 rectangle with 12 squares. Complete the sentences below.

I see _____ rows of _____.

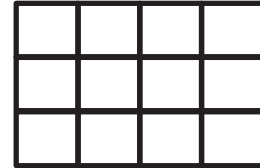
In the exact same rectangle, I see _____ columns of _____.

Name _____

Date _____

1. Imagine that you have just cut this rectangle into rows.

a. What do you see? Draw a picture.



How many squares are in each row? _____

b. Imagine that you have just cut this rectangle into columns. What do you see?
Draw a picture.

How many squares are in each column? _____

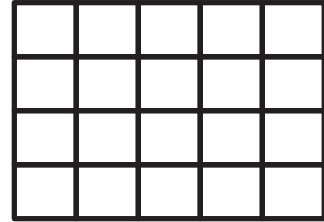
2. Create another rectangle using the same number of squares.

How many squares are in each row? _____

How many squares are in each column? _____

3. Imagine that you have just cut this rectangle into rows.

a. What do you see? Draw a picture.



How many squares are in each row? _____

b. Imagine that you have just cut this rectangle into columns. What do you see?
Draw a picture.

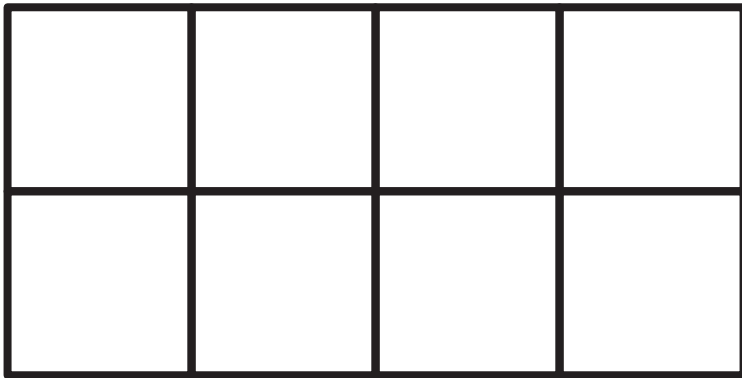
How many squares are in each column? _____

4. Create another rectangle using the same number of squares.

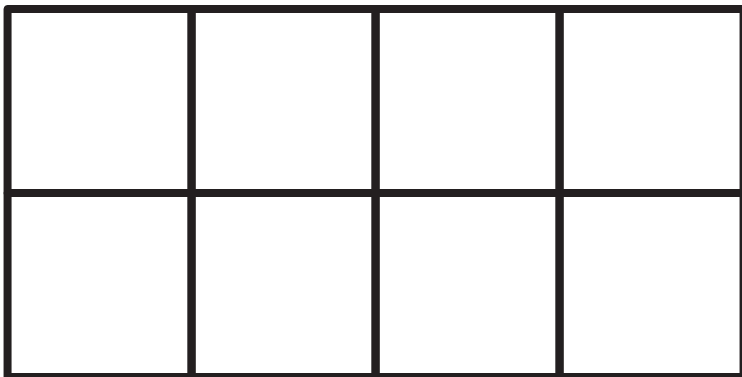
How many squares are in each row? _____

How many squares are in each column? _____

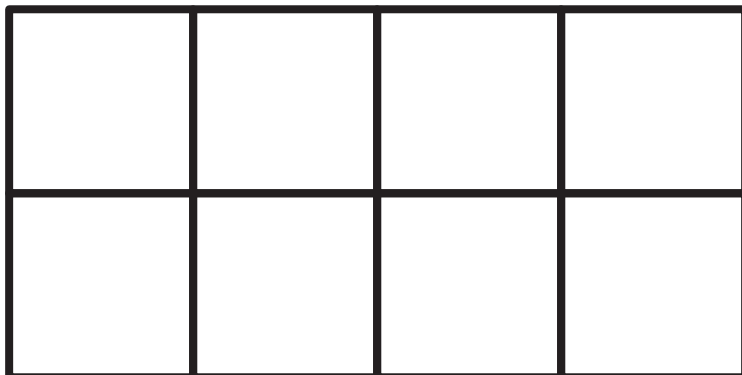
Rectangle A



Rectangle B



Rectangle C



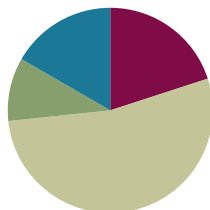
rectangles

Lesson 15

Objective: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Subtract Crossing the Ten **2.OA.2, 2.NBT.5** (8 minutes)
- Using the Nearest Ten to Subtract **2.NBT.5** (2 minutes)
- Subtract Common Units **2.NBT.5, 2.NBT.7** (2 minutes)

Sprint: Subtract Crossing the Ten (8 minutes)

Materials: (S) Subtract Crossing the Ten Sprint

Note: Students practice crossing the ten in preparation for the lesson, as well as to gain mastery of the sums and differences within 20.

Using the Nearest Ten to Subtract (2 minutes)

Note: Reviewing the Grade 1 skill of counting up and down to 10 to subtract gives students a mental strategy to subtract fluently with larger numbers.

- T: (Post $16 - 9$ on the board.) Raise your hand when you know the answer to $16 - 9$.
- S: 7.
- T: Break 16 apart into 10 and 6. (Write in the bond.) What is $10 - 9$?
- S: 1.
- T: $1 + 6$ is...?
- S: 7.

$ \begin{array}{r} 16 - 9 = \underline{\quad} \\ / \quad \backslash \\ 10 \quad 6 \end{array} $ $ \begin{array}{l} 10 - 9 = 1 \\ 1 + 6 = 7 \\ 16 - 9 = 7 \end{array} $

Continue with the following possible sequence: $14 - 9$, $15 - 8$, $16 - 7$, $13 - 7$, $12 - 9$, $12 - 7$, $22 - 7$, $25 - 7$, $25 - 9$, $26 - 9$, $27 - 9$, $27 - 19$, $37 - 9$, $37 - 19$, $35 - 19$, $45 - 19$, $47 - 18$, and $48 - 29$.

Subtract Common Units (2 minutes)

Materials: (S) Personal white board

Note: Reviewing this mental math Fluency Practice prepares students for understanding the importance of the subtraction algorithm and place value.

T: (Project 44.) Say the number in unit form.

S: 4 tens 4 ones.

T: (Write $44 - 22 = \underline{\quad}$.) Say the subtraction sentence and answer in unit form.

S: 4 tens 4 ones – 2 tens 2 ones = 2 tens 2 ones.

T: Write the subtraction sentence on your personal white board.

Continue with the following possible sequence: $77 - 33$, $88 - 55$, $99 - 33$, $199 - 33$, and $999 - 33$.

Application Problem (6 minutes)

Rick is filling his muffin pan with batter. He fills 2 columns of 4. One column of 4 is empty.

- Draw to show the muffins and the empty column.
- Write a repeated addition equation to tell how many muffins Rick makes.



$$4 + 4 = 8$$

Rick makes 8 muffins.

Note: This problem is intended for independent practice. Students apply learning from the previous day's lesson (distinguishing units within units) to a familiar situation. This leads into today's Concept Development wherein students shade in given arrays.

Concept Development (32 minutes)

Materials: (T) Extra 6 by 8 grids for independent practice (S) Problem Set, crayons or colored pencils

Note: In this lesson, the Problem Set comprises the Concept Development. Circulate as students complete arrays, monitoring comprehension.

T: Remind me what we discovered in the last lesson when we worked with rectangles.

S: We can cut up a rectangle into rows and columns. → There are lots of small squares inside the rectangle. → I was thinking how it's kind of like there are smaller numbers inside bigger numbers.

T: What an interesting connection! Yes, in all kinds of ways, smaller units combine to make larger units, and larger units can be decomposed into smaller units.

T: We're going to use the Problem Set again for today's lesson. Look at Problem 1. Tell your partner what you see.

S: A large rectangle. → A lot of small squares inside the large rectangle. → An array.

- T: That's all true. Problem 1 says shade, or color in, an array with 2 rows of 3. Are we going to color in the whole rectangle?
- S: No!
- T: Starting in the upper left corner, how many squares are we going to color in the first row?
- S: 3 squares!
- T: Let's color that first row green. (Model as students do the same.)
- T: What should we do next?
- S: Color in another row of 3 under the first row.
- T: Yes! Do that with me, this time using a different color. (Model as students do the same.)
- T: Now, tell your partner what you see.
- S: I see 2 rows of 3. → I see 6 colored squares, $3 + 3$. → I see 2 threes.
- T: Ah! I like the way you related repeated addition to the array. There are 2 threes, so $3 + 3$. Let's write that on the line next to the array. (Write $3 + 3 = 6$ as students do the same.)
- T: Let's do Problem 2. How many rows are we going to color?
- S: 4 rows!
- T: How many squares in each row do we need to color?
- MP.8** S: 3 squares!
- T: Color in an array that shows 4 rows of 3. Use a different color for each row. (Model as students do the same.)
- T: Now, write the repeated addition equation for the array, and share what you wrote with your partner.
- S: $3 + 3 + 3 + 3 = 12$.
- T: That's right! Let's read Problem 3: "Shade in an array with 5 columns of 4." How many columns are we going to color?
- S: 5 columns!
- T: How many squares should we color in each column?
- S: 4 squares!
- T: All right. Color in 5 columns of 4. Again, use a different color for each column. (Model as students do the same.)



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Offer students who may be struggling a chance to model the lesson using square tiles on grid paper prior to coloring in the arrays.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Some early finishers may notice that a Sprint contains arrays of squares and rectangles. Encourage them to color in the squares and figure out the arrays they make. They may even cut out the squares and arrange them into different arrays.

Before moving on to the next activity, provide support to struggling students. Allow students who demonstrate proficiency to work independently with the following sequence on the extra grids without changing colors for each row or column: 5 rows of 5, 3 columns of 4, and an array of their choosing, writing a repeated addition equation that represents each array.

- T: Let's try something different! Look at Problem 4. Draw one more column of 2 to make a new array.
- T: Are we adding another row or another column?
- S: Another column!
- T: Draw another column of 2. (Model as students do the same.)
- T: How many columns are there now?
- S: 5 columns!
- T: How many in each column?
- S: 2.
- T: Yes! Each column has a group, or unit, of two! How many twos are there altogether?
- S: 5 twos!
- T: What is a repeated addition equation for the new array?
- S: $2 + 2 + 2 + 2 + 2 = 10$.
- T: Excellent! Now, work with your partner to complete Problems 5 and 6. Be sure to read the directions carefully, and use your models to explain why your repeated addition equations match.

Provide support for students who may be struggling while the rest of the class works independently.

Student Debrief (10 minutes)

Lesson Objective: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

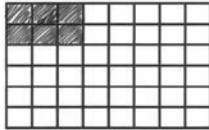
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In what way did your array change from Problem 1 to Problem 2? How did your equation change? How are the totals related?
- In Problem 3, each column is like a unit of how many? How does that relate to your equation?

Name Gloria Date _____

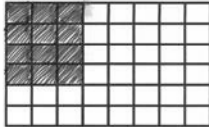
1. Shade in an array with 2 rows of 3.



Write a repeated addition equation for the array.

$3 + 3 = 6$

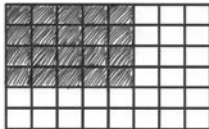
2. Shade in an array with 4 rows of 3.



Write a repeated addition equation for the array.

$3 + 3 + 3 + 3 = 12$

3. Shade in an array with 5 columns of 4.



Write a repeated addition equation for the array.

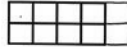
$4 + 4 + 4 + 4 + 4 = 20$

- For Problem 4, if you were to continue adding columns of 2, would your new array look more like a train or a tower? If you wanted to increase the total number of tiles quickly, would you suggest adding more rows or columns? Why?
- Why couldn't you draw another column of 2 in Problem 5? Given what you know about rectangles, what did you need to be sure to do? Explain how you know that your equation is correct by matching it to your drawing.
- How many squares are in your 2 new columns in Problem 6? Why? In what way does this array show that big units are made up of smaller units? (Use *rows*, *columns*, *square*, and *rectangle* in your response.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

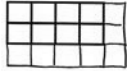
4. Draw one more column of 2 to make a new array.



Write a repeated addition equation for the new array.

$$5 + 5 = 10$$

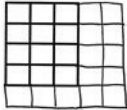
5. Draw one more row of 4 and then one more column to make a new array.



Write a repeated addition equation for the new array.

$$5 + 5 + 5 = 15$$

6. Draw one more row and then two more columns to make a new array.



Write a repeated addition equation for the new array.

$$5 + 5 + 5 + 5 = 25$$

A

Number Correct: _____

Subtract Crossing the Ten

1.	$10 - 1 =$	
2.	$10 - 2 =$	
3.	$20 - 2 =$	
4.	$40 - 2 =$	
5.	$10 - 2 =$	
6.	$11 - 2 =$	
7.	$21 - 2 =$	
8.	$51 - 2 =$	
9.	$10 - 3 =$	
10.	$11 - 3 =$	
11.	$21 - 3 =$	
12.	$61 - 3 =$	
13.	$10 - 4 =$	
14.	$11 - 4 =$	
15.	$21 - 4 =$	
16.	$71 - 4 =$	
17.	$10 - 5 =$	
18.	$11 - 5 =$	
19.	$21 - 5 =$	
20.	$81 - 5 =$	
21.	$10 - 6 =$	
22.	$11 - 6 =$	

23.	$21 - 6 =$	
24.	$91 - 6 =$	
25.	$10 - 7 =$	
26.	$11 - 7 =$	
27.	$31 - 7 =$	
28.	$10 - 8 =$	
29.	$11 - 8 =$	
30.	$41 - 8 =$	
31.	$10 - 9 =$	
32.	$11 - 9 =$	
33.	$51 - 9 =$	
34.	$12 - 3 =$	
35.	$82 - 3 =$	
36.	$13 - 5 =$	
37.	$73 - 5 =$	
38.	$14 - 6 =$	
39.	$84 - 6 =$	
40.	$15 - 8 =$	
41.	$95 - 8 =$	
42.	$16 - 7 =$	
43.	$46 - 7 =$	
44.	$68 - 9 =$	

B

Subtract Crossing the Ten

Number Correct: _____

Improvement: _____

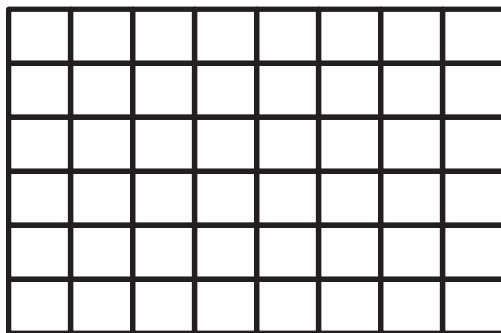
1.	$10 - 2 =$	
2.	$20 - 2 =$	
3.	$30 - 2 =$	
4.	$50 - 2 =$	
5.	$10 - 2 =$	
6.	$11 - 2 =$	
7.	$21 - 2 =$	
8.	$61 - 2 =$	
9.	$10 - 3 =$	
10.	$11 - 3 =$	
11.	$21 - 3 =$	
12.	$71 - 3 =$	
13.	$10 - 4 =$	
14.	$11 - 4 =$	
15.	$21 - 4 =$	
16.	$81 - 4 =$	
17.	$10 - 5 =$	
18.	$11 - 5 =$	
19.	$21 - 5 =$	
20.	$91 - 5 =$	
21.	$10 - 6 =$	
22.	$11 - 6 =$	

23.	$21 - 6 =$	
24.	$41 - 6 =$	
25.	$10 - 7 =$	
26.	$11 - 7 =$	
27.	$51 - 7 =$	
28.	$10 - 8 =$	
29.	$11 - 8 =$	
30.	$61 - 8 =$	
31.	$10 - 9 =$	
32.	$11 - 9 =$	
33.	$31 - 9 =$	
34.	$12 - 3 =$	
35.	$92 - 3 =$	
36.	$13 - 5 =$	
37.	$43 - 5 =$	
38.	$14 - 6 =$	
39.	$64 - 6 =$	
40.	$15 - 8 =$	
41.	$85 - 8 =$	
42.	$16 - 7 =$	
43.	$76 - 7 =$	
44.	$58 - 9 =$	

Name _____

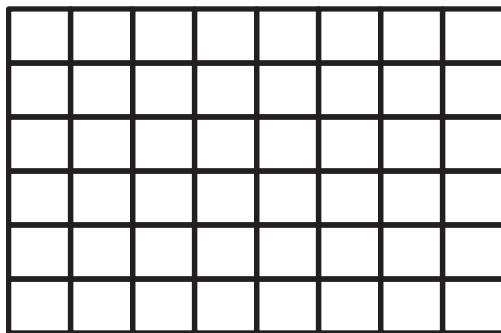
Date _____

1. Shade in an array with 2 rows of 3.



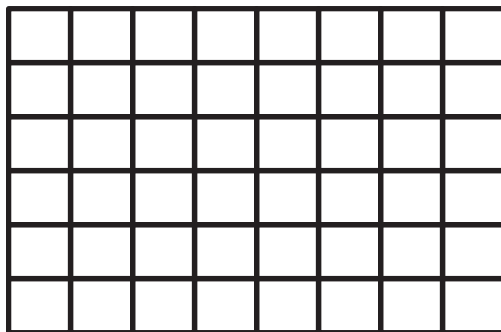
Write a repeated addition equation for the array.

2. Shade in an array with 4 rows of 3.



Write a repeated addition equation for the array.

3. Shade in an array with 5 columns of 4.



Write a repeated addition equation for the array.

4. Draw one more column of 2 to make a new array.



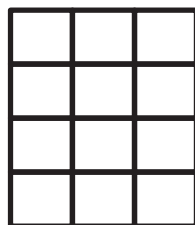
Write a repeated addition equation for the new array.

5. Draw one more row of 4 and then one more column to make a new array.



Write a repeated addition equation for the new array.

6. Draw one more row and then two more columns to make a new array.

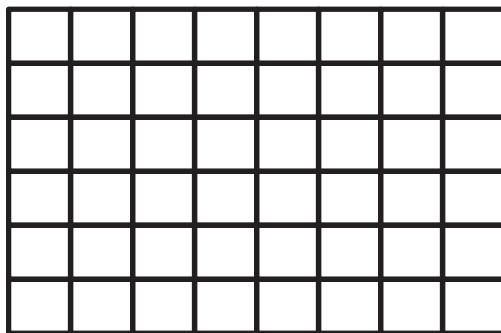


Write a repeated addition equation for the new array.

Name _____

Date _____

Shade in an array with 3 rows of 5.

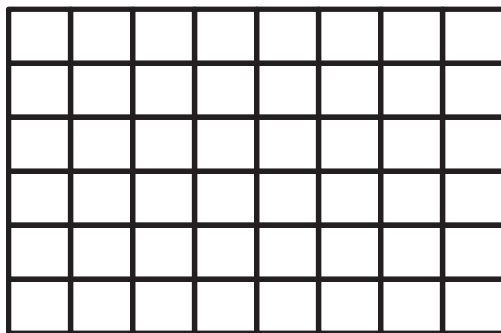


Write a repeated addition
equation for the array.

Name _____

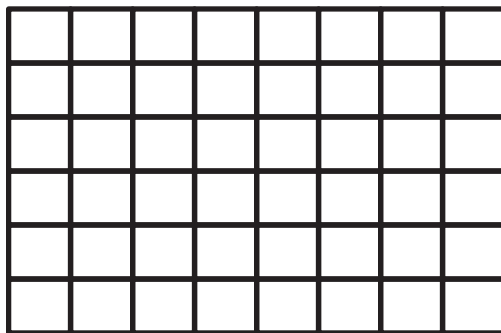
Date _____

1. Shade in an array with 3 rows of 2.



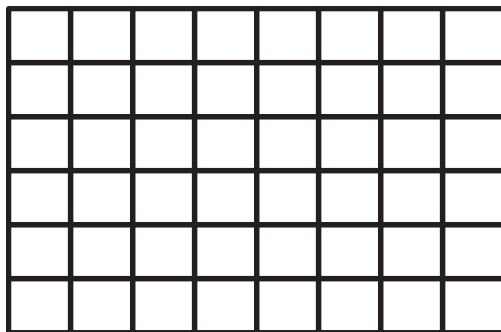
Write a repeated addition equation for the array.

2. Shade in an array with 2 rows of 4.



Write a repeated addition equation for the array.

3. Shade in an array with 4 columns of 5.



Write a repeated addition equation for the array.

4. Draw one more column of 2 to make a new array.



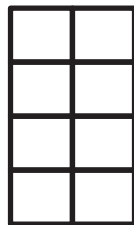
Write a repeated addition equation for the new array.

5. Draw one more row of 3 and then one more column to make a new array.



Write a repeated addition equation for the new array.

6. Draw one more row and then two more columns to make a new array.



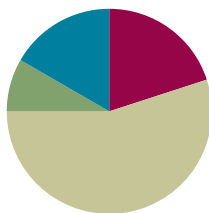
Write a repeated addition equation for the new array.

Lesson 16

Objective: Use grid paper to create designs to develop spatial structuring.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Get to 10, 20, or 30 **2.OA.2** (4 minutes)
- Count by Ten or One with Dimes and Pennies **2.OA.2** (3 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)

Get to 10, 20, or 30 (4 minutes)

Materials: (S) 3 dimes and 10 pennies

Note: This fluency activity uses dimes and pennies to help students become familiar with coins while simultaneously providing practice with missing addends to ten(s).

For the first two minutes:

- Step 1: Lay out 0–10 pennies in a 5-group formation, and ask students to identify the amount shown (e.g., 9 cents).
- Step 2: Ask for the addition sentence to get to a dime (e.g., 9 cents + 1 cent = 1 dime).

For the next two minutes:

- Repeat Steps 1 and 2. Then, add a dime, and ask students to identify the amount shown (e.g., 1 dime 9 cents + 1 cent = 2 dimes).

Count by Ten or One with Dimes and Pennies (3 minutes)

Materials: (T) 10 dimes and 10 pennies

Note: This activity uses dimes and pennies as abstract representations of tens and ones to help students become familiar with coins while simultaneously providing practice with counting forward and back by ten or one.

- First minute: Place and take away dimes in a 5-group formation as students count along by 10.
- Second minute: Begin with 2 pennies. Ask how many ones there are. Instruct students to start at 2 and add and subtract 10 as dimes are placed and taken away.
- Third minute: Begin with 2 dimes. Ask how many tens there are. Instruct students to begin at 20 and add and subtract 1 as pennies are placed and taken away.

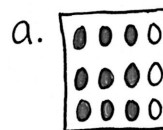
Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 12 Core Fluency Practice Sets)

Note: During Topic C and for the remainder of the year, each day's fluency activities include an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 12.

Application Problem (5 minutes)

Rick is baking muffins again. He filled 3 columns of 3 and left one column of 3 empty.



- MP.3**
- Draw a picture to show what the muffin pan looked like. Shade the columns that Rick filled.
 - Write a repeated addition equation to tell how many muffins Rick makes. Then, write a repeated addition equation to tell how many muffins would fit in the whole pan.

b. $3+3+3=9$
Rick makes 9 muffins.
 $3+3+3+3=12$
12 muffins would fit in the whole pan.

Note: This problem is intended for independent practice of the previous day's concept. Given an array with one column missing, students fill in the missing units to complete the array. Then, they find the total by relating the completed array to repeated addition.

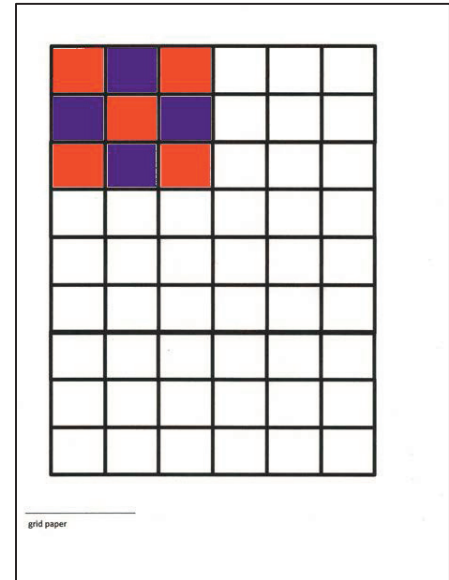
Concept Development (33 minutes)

Materials: (T) Grid paper (Template), 1-inch tiles (S) Problem Set, grid paper (Template), scissors, colored pencils or crayons, personal white board, 2 grid papers (Template) on 2 different colored papers (per group of four students)

Note: The Problem Set is completed during the course of the Concept Development today rather than at the end.

In this lesson, students extend their earlier work of composing and decomposing rectangles using tiles. Here, they create tessellations by using 1-inch tiles to design a core unit. They then iterate that core unit with no gaps or overlaps to make patterns that could, theoretically, extend indefinitely. This highly engaging activity serves the important purpose of further developing students' spatial structuring ability, preparing them to work with area in Grade 3, while generating work well suited for classroom display.

- T: We've used square tiles to put together and break apart rectangles, but did you know we can also use them to create designs?
- T: (Project the grid paper.) Watch how I place my tiles. (Place the tiles on top of the grid to create the image pictured.)
- T: Talk with your partner about the design. How can you describe it?
- S: It goes red, blue, red, blue, red, blue, red, blue, red. → It shows 5 red squares and 4 blue squares. → It looks like an array with 3 rows and 3 columns.
- T: Good attention to detail, and I like the way you used math language! I can create the same design again by shading in this same pattern. (Shade in the identical design on the grid paper.)



Be sure students are seated in groups of four for this activity. Pass out one white grid paper per student and two different colored grid papers for every four students. Instruct students to slide the white grid paper into their personal white boards and have their dry erase markers ready. Instruct each student to cut off one row of squares from each of the colored grid paper templates for themselves and then pass the papers to the next person in the group.

- T: Now, it's your turn to try! Cut out 5 single squares from each of the colored grid papers. (Pause.)
- T: Use the squares to copy my design on top of the grid paper that is in your personal white board.
- T: Then, carefully remove the squares and, with your marker, shade in the squares to create the design on your personal white board.

Circulate to check for understanding as students recreate the design.

- T: Now you get to create your own design! Listen and follow my directions carefully.

Part 1: Create a design using 10 tiles.

- T: Now, cut one square in half diagonally so that you have two triangles. (Pause.)
- T: Use each of your 9 square tiles and the 2 triangle tiles to create a design on top of your grid. Then, use your marker to shade in your design on your personal white board.
- T: Pay careful attention to which grid squares to color and how many spaces to leave.
- T: When you have finished, share your design with a partner.
- T: (Allow students time to work and share.) Now, hold up your design so we can admire each other's creative work!

Circulate to provide support as students work.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

- Search online for tessellations to show their connection to artwork and patterns, including real-life tessellations in quilts, historical mosaics, and tiling in buildings and homes, as well as many works by M.C. Escher.
- Offer students puzzles such as Perpetual Puzzles or other puzzles that use same-shaped pieces, and point out how they form tessellations.

Part 2: Create a design using 16 tiles.

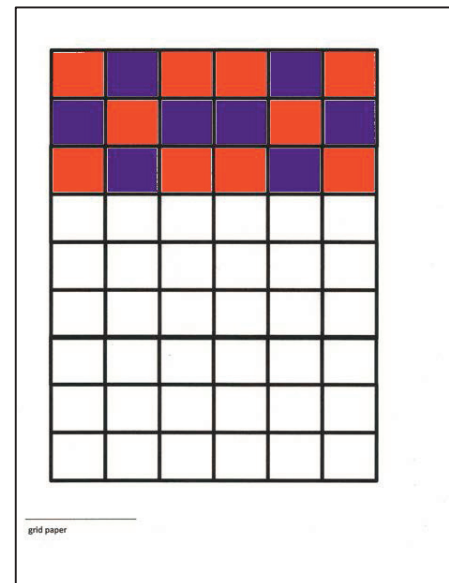
- T: Cut out 3 more squares from each colored grid. (Allow students time to cut.)
- T: Cut 2 of those square tiles in half diagonally. (Allow students time to cut.)
- T: Now, create a new design with your tiles, and then shade in your design on your personal white board.

Part 3: Share and check your partner's work.

- T: Share your second design with your partner. Check each other's copy to be sure it matches the tile design.
- T: Describe your partner's design: How would you describe it as an array? How many tiles do you see in the second row? Do any columns look the same? (Allow students time to discuss their partner's work.)
- T: Hold up those designs again, and look around! Oh, I see how they are becoming more intricate with more tiles and triangles!

Part 4: Create a tessellation.

- T: Let's look at my design again. (Project the original 9-tile core unit as shown.)
- T: You noticed that this is a 3 by 3 array, made up of 3 rows and 3 columns, which is a total of 9 squares. I can also call it my *core unit*.
- T: Watch how I can make this pattern go on and on by repeating the core unit. (Place the next core unit.)
- T: Notice how the tiles touch but don't overlap. I could keep going this way and cover the entire space of this page. Can you visualize how this paper would look if I repeated my design until the paper was full?
- T: I continue my pattern right up to the edge of the grid.
- T: Now you're going to work in pairs.
- T: One partner, take the grid paper out of your personal white board. You'll be using your colored pencils for this activity.
- T: You and your partner will now create your own 3 by 3 design, which will be your core unit.
- T: Work together, starting in one corner of the paper, to create a pattern and color a design that covers a 3 by 3 area.
- T: Next, fill your whole paper by copying that design over and over, repeating the core unit with no gaps or overlaps.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

As a class project, create a mosaic from the tessellations students create. If there is enough parental involvement, consider creating a class quilt from the pattern.

Circulate as students work. Once all have finished, consider having a gallery walk if time permits so that students can view each other's tessellations.

T: What you and your partner just created is called a **tessellation**! Isn't that a great word? Say it with me.

T/S: Tessellation!

T: How would you describe a tessellation?

S: It's like a bunch of tiles! → We made copies of a pattern over and over. → We made tiles with a pattern on them and arranged them in an array.

T: Good descriptions! The process of creating a tessellation is often described as tiling.

Student Debrief (10 minutes)

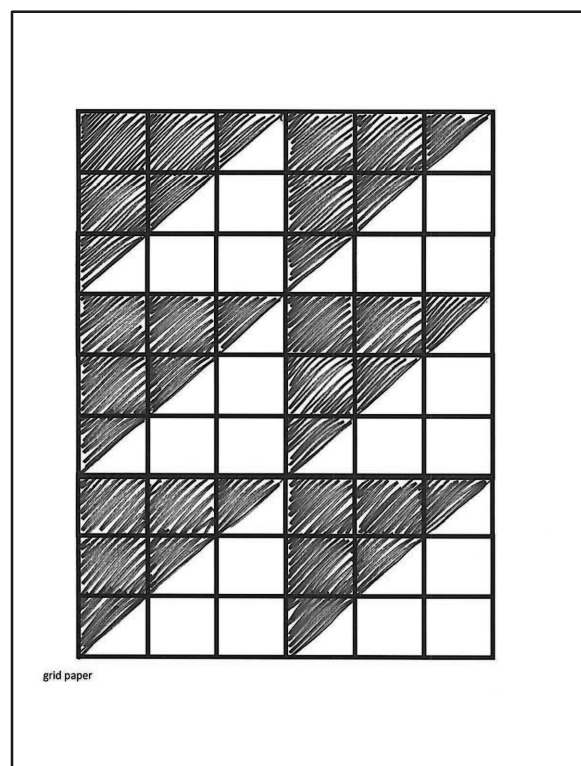
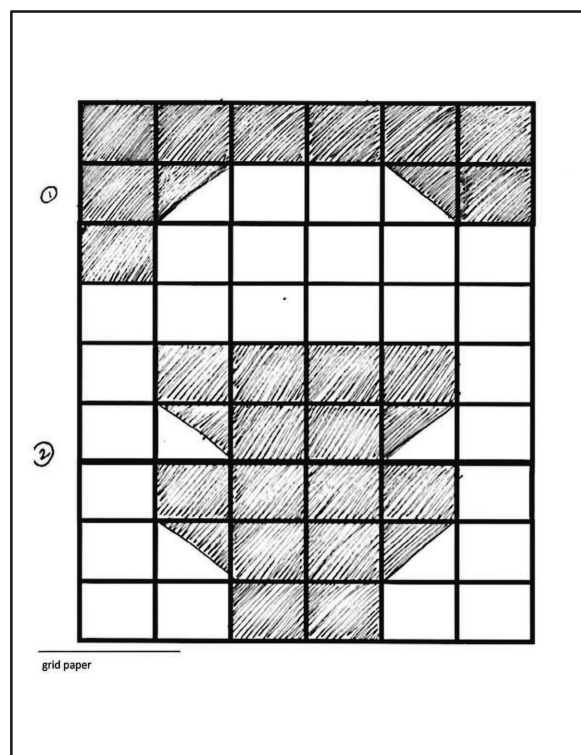
Lesson Objective: Use grid paper to create designs to develop spatial structuring.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problems 1–3, how is knowing how to build an array helpful in creating designs with the tiles?
- What was the most challenging part of today's Problem Set? Why?
- What exciting new math vocabulary did we learn today? How would you describe a **tessellation** to a first grader?
- How is making copies of a unit similar to something we have done before?
- Where do you see tessellations at school? At home? Outside?



- Why do you think we spent time creating designs and learning about tessellations today?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

Use your square tiles and grid paper to complete the following problems.

Problem 1

- a. Cut out 10 square tiles.
- b. Cut one of your square tiles in half diagonally.
- c. Create a design.
- d. Shade in your design on grid paper.

Problem 2

- a. Use 16 square tiles.
- b. Cut two of your square tiles in half diagonally.
- c. Create a design.
- d. Shade in your design on grid paper.
- e. Share your second design with your partner.
- f. Check each other's copy to be sure it matches the tile design.

Problem 3

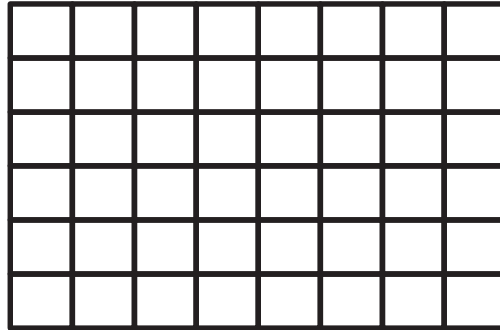
- a. Create a 3 by 3 design with your partner in the corner of a new piece of grid paper.
- b. With your partner, copy that design to fill the entire paper.

Name _____

Date _____

Use your square tiles and grid paper to complete the following.

- Create a design with the paper tiles you used in the lesson.
- Shade in your design on the grid paper.

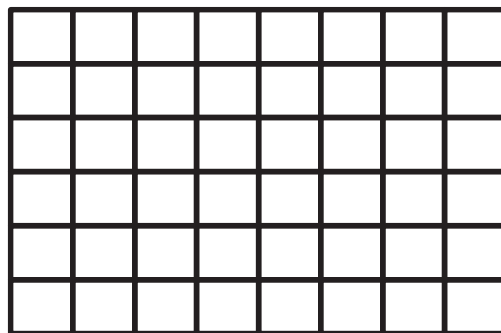
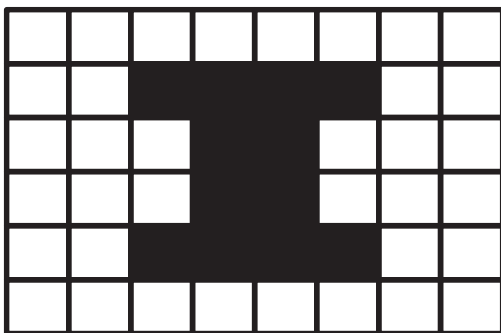


Name _____

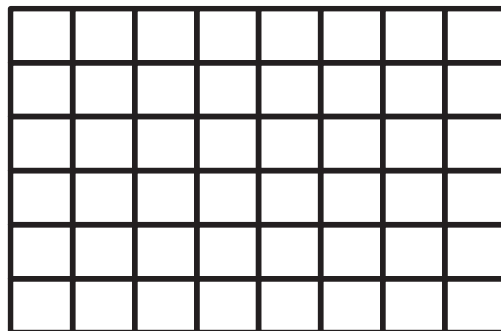
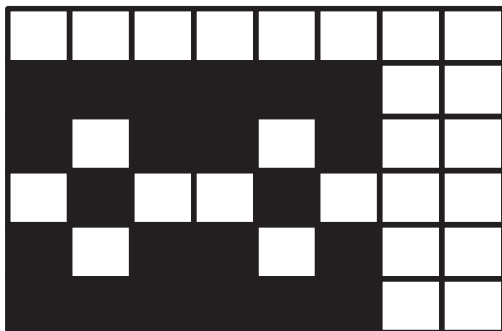
Date _____

1. Shade to create a copy of the design on the empty grid.

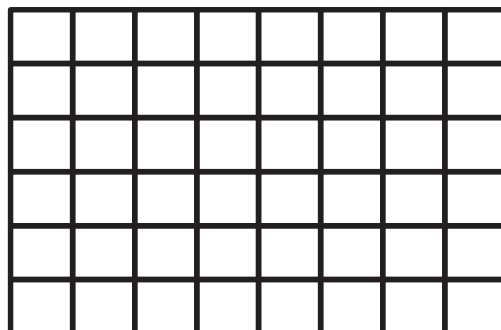
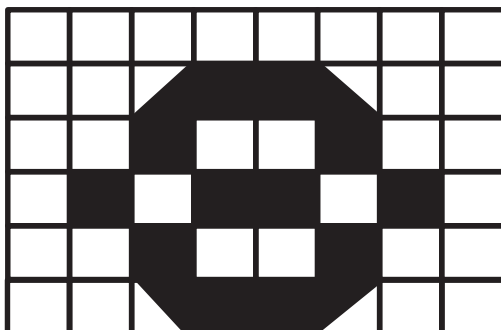
a.



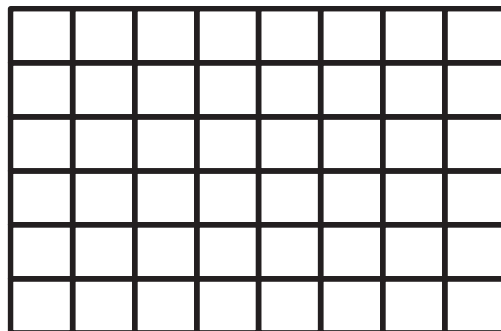
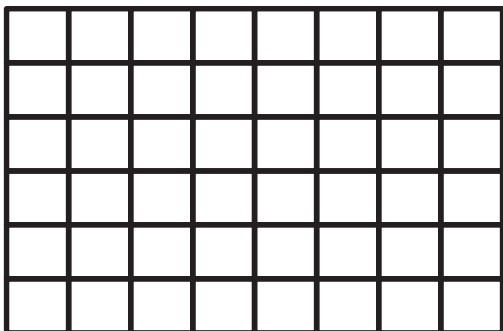
b.



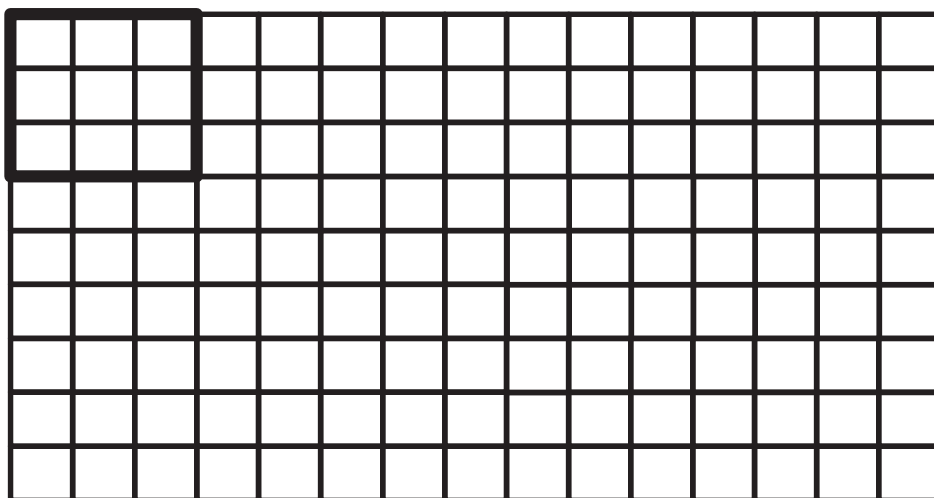
c.

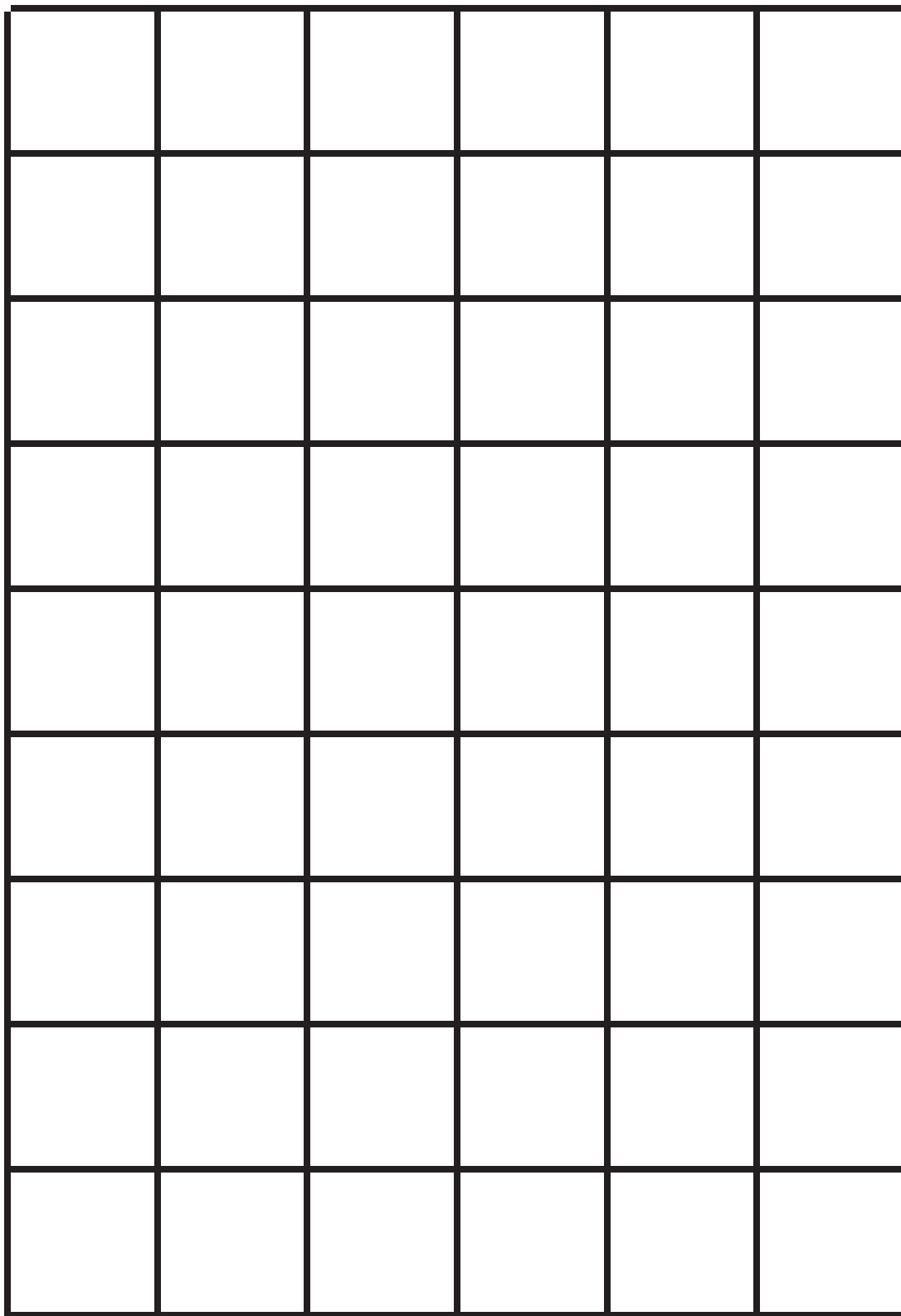


2. Create two different designs.



3. Use colored pencils to create a design in the bolded square section. Create a tessellation by repeating the design throughout.





grid paper



Topic D

The Meaning of Even and Odd Numbers

2.OA.3

Focus Standard:	2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
Instructional Days:	4	
Coherence	-Links from:	G2–M5 Addition and Subtraction Within 1,000 with Word Problems to 100
	-Links to:	G3–M4 Multiplication and Area

In Topic D, students explore the meaning of even and odd numbers, learning various interpretations and relating these interpretations to addition. Lesson 17 introduces even numbers via doubles. In other words, when any number from 1 to 10 is doubled, the resulting number is even, and any even number can be written as a doubles fact. Students arrange doubles into rectangular arrays (e.g., 2 rows of 7, or 2 sevens) and write an equation to show the total as a sum of two equal addends (e.g., $7 + 7 = 14$). They discover that doubles facts yield even numbers even when the number being doubled is not even.

In Lesson 18, students pair up to 20 objects and see that, when objects are paired with none remaining, the number is even (**2.OA.3**). They see that objects arranged in columns of two also create two equal groups. For example, a 2 by 7 array may be seen as 7 columns of 2 or 2 rows of 7. Students also see that even numbers occur when we count by twos (e.g., 1 two, 2 twos, ..., 7 twos, or 2, 4, 6, ..., 14). They count by twos up to 20 and then back down. When they reach zero, the question is posed: “Does this mean zero is even? Can I write 0 as a doubles fact?” As a result, students see that 0 is even. This practice lays the groundwork for the multiplication table of two in Grade 3.

By Lesson 19, students have a keen understanding of how to determine whether or not a number is even. Now, they learn a faster way to identify even numbers by looking for 0, 2, 4, 6, or 8 in the ones place. First, students use square tiles to build an array made up of columns of 2 and relate it to even numbers on a number path up to 20. As multiples of two are circled on the number path, students observe that the ones digits are 0, 2, 4, 6, and 8. Now, equipped with this interpretation, as well as the previously learned interpretations of even numbers, students are ready to name all other whole numbers as odd. They learn that odd numbers can be identified in contrast to even (i.e., if a number is not even, then it is odd).

Next, they learn that when one is added to or subtracted from any even number, the resulting number is odd. Finally, students apply all of the interpretations they have learned to discern whether or not numbers larger than 20 are even. For example, a student might reason that 41 is odd because it is an even number plus one or because it does not end in 0, 2, 4, 6, or 8.

Topic D culminates with students using arrays to investigate even and odd numbers. Students build arrays, as they did in Lesson 17, using even numbers, and they see concretely that when a number is even, it can be decomposed into two equal parts or groups of two. They then either remove or add on 1 square unit to make an odd number. This enables students to solidify the understanding that an odd number is either one more or one less than an even number and that it cannot be decomposed into two equal groups or groups of two, although it may be decomposed into more than two groups (e.g., a 3 by 3 array). Students add even numbers to other even numbers, odd numbers to other odd numbers, and even numbers to odd numbers to see what happens to the sum in each case. They discover that the sum of two even numbers is even, and the sum of two odd numbers is even. They also discover that the sum of an odd number and an even number is odd. Through these explorations, students build an intuitive understanding of prime, composite, and square numbers, which is foundational for later grade levels.

A Teaching Sequence Toward Mastery of The Meaning of Even and Odd Numbers

Objective 1: Relate doubles to even numbers, and write number sentences to express the sums.
(Lesson 17)

Objective 2: Pair objects and skip-count to relate to even numbers.
(Lesson 18)

Objective 3: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.
(Lesson 19)

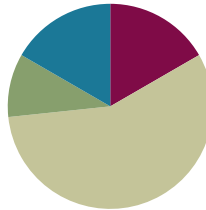
Objective 4: Use rectangular arrays to investigate odd and even numbers.
(Lesson 20)

Lesson 17

Objective: Relate doubles to even numbers, and write number sentences to express the sums.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(6 minutes)
Concept Development	(34 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Subtraction Patterns **2.NBT.5** (5 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)

Subtraction Patterns (5 minutes)

Materials: (S) Personal white board, math notebook or loose-leaf paper

Note: Students practice subtraction in order to gain mastery of the sums and differences within 20 and to see the relationship with larger numbers.

- T: After I say a basic fact for you to solve, make a pattern sequence by adding 10 to the whole and then subtracting. Continue until I say to stop. So, after solving $11 - 9$, you would solve $21 - 9$, and then...?
- S: $31 - 9$, $41 - 9$, $51 - 9$.
- T: Yes. Solve as many as you can on your personal white board before I give the signal to stop. Let's begin. $11 - 9$.

When every student has completed at least two problems, stop the class, and give the next expression.

Continue with the following possible sequence: $12 - 8$, $11 - 8$, and $13 - 9$.

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 12 Core Fluency Practice Sets)

Note: During Topic D and for the remainder of the year, each day's fluency activities includes an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 12.

Application Problem (6 minutes)

Seven students sit on one side of a lunch table. Seven more students sit across from them on the other side of the table.

- Draw an array to show the students.
- Write an addition equation that matches the array.

Three more students sit down on each side of the table.

- Draw an array to show how many students there are now.
- Write an addition equation that matches the new array.

a. 

b. $7 + 7 = 14$

c. 

d. $10 + 10 = 20$

Note: This problem is intended for independent practice. It leads directly into today's Concept Development, utilizing prior knowledge about arrays.

Concept Development (34 minutes)

Materials: (S) 20 counters (per pair), personal white board

- T: Put your elbows on your desk with your thumbs pointing up.
- T: What addition sentence, or equation, describes the number of fingers pointing up?
- S: $1 + 1 = 2$.
- T: (Record the addition equation on the board.) Pop up your index fingers. What is our new addition sentence?
- S: $2 + 2 = 4$.
- T: (Record the addition equation below $1 + 1 = 2$.) Pop up the next finger on each hand. What is our next addition equation?
- S: $3 + 3 = 6$.
- T: (Record the addition equation.) Pop up the next finger, and give me the addition equation.
- S: $4 + 4 = 8$.
- T: (Record the addition equation on the board.) Next finger up. What is our next addition equation?
- S: $5 + 5 = 10$.
- T: (Point to the list of addition sentences.) What do we call it when both addends are the same?

MP.8



**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Many students quickly recognize the doubles pattern, but do not allow them to call out. They are bursting to tell what they know by the time they reach $5 + 5$, which heightens their excitement and engagement. It also allows think time for students who do not recognize the pattern as quickly.



**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

Many students already have an understanding of *even* in the context of sharing fairly, as in, "That's not fair; it's not even!" Allow them to share these connections to deepen their understanding of the term.

MP.8

- S: Doubles!
- T: What patterns do you see in the totals? Discuss with your partner.
- S: It's like skip-counting, 2, 4, 6, 8. → Doubles is like counting by twos. → Each addend goes up by 1, so the answer goes up by 2.
- T: All these numbers are even. Tell your partner a way to think about **even numbers**.
- S: Even numbers mean it's the same number, like in a game of handball, if we both have 8 points, the score is even. → If you share with a friend fairly, you both have the same number of things. It's even. → When you add the same number two times, the answer is even. → When you skip-count by twos, the answer is even.
- T: Good thoughts. Let's do some more work with even numbers. (Pass out the counters.)
- T: Partner A, make one row of 6 counters. (Pause.)
- T: Partner B, double that row by creating a second row of 6. (Pause.)
- T: Look at your array. There are 6 counters in each group. What addition equation describes it?
- S: $6 + 6 = 12$.
- T: (Add this equation to the doubles facts on the board.) Partner B, make a row of 7 counters.
- T: How many counters should Partner A add to double 7?
- S: 7.
- T: Do that, Partner A, and let's add the doubles fact to our list. What should we write?
- S: $7 + 7 = 14$.
- T: Is 14 even?
- S: Yes!
- T: What is the next addition equation we will write?
- S: $8 + 8 = 16$.
- T: Of course! How did you know?
- S: The addends are going in order, $1 + 1$, $2 + 2$, ..., so $8 + 8$ comes after $7 + 7$. → The answers are going by twos, 2, 4, 6, 8, 10, 12, 14, so 16 is next.
- T: Brilliant! We can express this idea in another way. We can say "5 doubled is 10." Say that with me.
- T/S: 5 doubled is 10.
- T: What is 3 doubled? Give me the complete sentence.
- S: 3 doubled is 6.
- T: Excellent! Continue using your counters to make all the combinations of even numbers that you can. For each set, record the doubles equation on your personal white board.

As students form and record the sets of doubles, work with students who need support. As individuals demonstrate understanding, allow them to move on to the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Relate doubles to even numbers, and write number sentences to express the sums.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

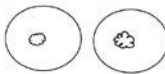
- In Problem 1, does doubling a number always result in an **even number**? Does it matter how many clouds are in each group?
- For Problem 1, if $4 + 4$ is even, is $5 + 4$ even? Why not? Fill in $5 + 4 + \underline{\quad}$ is an even number.
- Can you look at an array in Problem 2 and immediately determine if there is an even number of objects? How?
- What patterns do you notice in Problem 3? What connections do you see between even numbers and skip-counting?
- What new math word did we use today? How would you define an even number?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

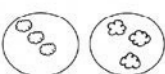
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

1. Draw to double the group you see. Complete the sentence, and write an addition equation.

a.  There is 1 cloud in each group.
1 + 1 = 2

b.  There are 2 clouds in each group.
2 + 2 = 4

c.  There are 3 clouds in each group.
3 + 3 = 6

d.  There are 4 clouds in each group.
4 + 4 = 8

e.  There are 5 clouds in each group.
5 + 5 = 10

2. Draw an array for each set. Complete the sentences. The first one has been drawn for you.

a. 2 rows of 6

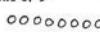
 2 rows of 6 = 12
6 + 6 = 12
 6 doubled is 12

b. 2 rows of 7

 2 rows of 7 = 14
7 + 7 = 14
 7 doubled is 14

c. 2 rows of 8

 2 rows of 8 = 16
8 + 8 = 16
 8 doubled is 16

d. 2 rows of 9

 2 rows of 9 = 18
9 + 9 = 18
 9 doubled is 18

e. 2 rows of 10

 2 rows of 10 = 20
10 + 10 = 20
 10 doubled is 20

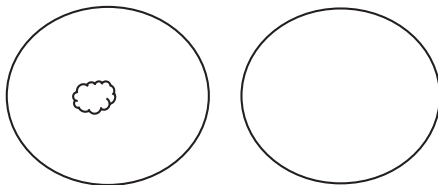
3. List the totals from Problem 1. 2 4 6 8 10
 List the totals from Problem 2. 12 14 16 18 20
 Are the numbers you have listed even or not even? even
 Explain in what ways the numbers are the same and different.
 Both sets of numbers have the same number of ones, but the numbers in Problem 2 have one more 10 than the numbers in Problem 1.

Name _____

Date _____

1. Draw to double the group you see. Complete the sentence, and write an addition equation.

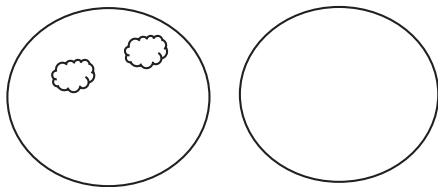
a.



There is _____ cloud in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

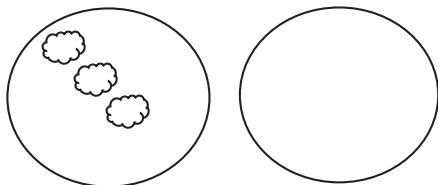
b.



There are _____ clouds in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

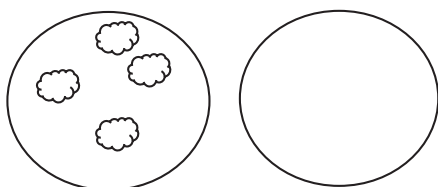
c.



There are _____ clouds in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

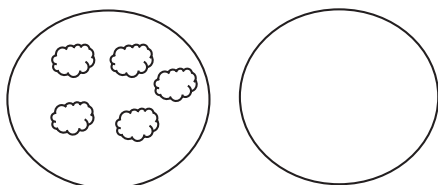
d.



There are _____ clouds in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

e.



There are _____ clouds in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

2. Draw an array for each set. Complete the sentences. The first one has been drawn for you.

a. **2 rows of 6**



2 rows of 6 = _____

_____ + _____ = _____

6 doubled is _____.

b. **2 rows of 7**

2 rows of 7 = _____

_____ + _____ = _____

7 doubled is _____.

c. **2 rows of 8**

2 rows of 8 = _____

_____ + _____ = _____

8 doubled is _____.

d. **2 rows of 9**

2 rows of 9 = _____

_____ + _____ = _____

9 doubled is _____.

e. **2 rows of 10**

2 rows of 10 = _____

_____ + _____ = _____

10 doubled is _____.

3. List the totals from Problem 1. _____

List the totals from Problem 2. _____

Are the numbers you have listed even or not even? _____

Explain in what ways the numbers are the same and different.

Name _____

Date _____

Draw an array for each set. Complete the sentences.

- a. 2 rows of 5

2 rows of 5 = _____

_____ + _____ = _____

Circle one: 5 doubled is even/not even.

- b. 2 rows of 3

2 rows of 3 = _____

_____ + _____ = _____

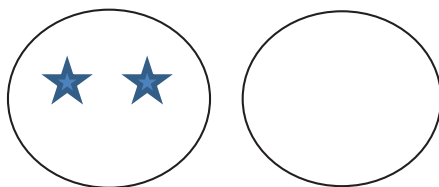
Circle one: 3 doubled is even/not even.

Name _____

Date _____

1. Draw to double the group you see. Complete the sentences, and write an addition equation.

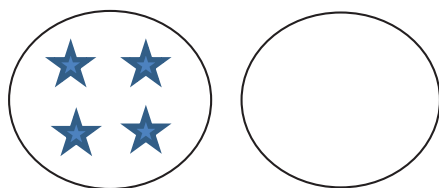
a.



There are _____ stars in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

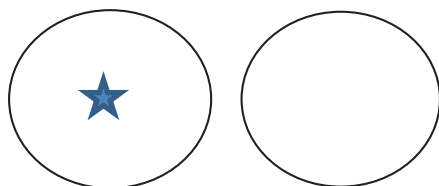
b.



There are _____ stars in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

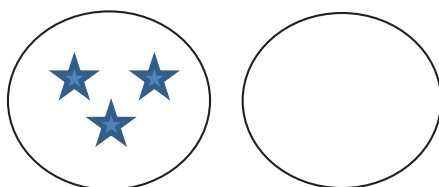
c.



There is _____ star in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

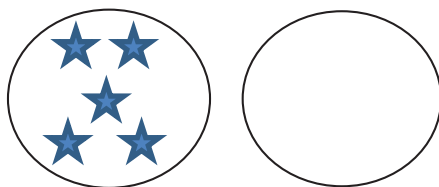
d.



There are _____ stars in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

e.



There are _____ stars in each group.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

2. Draw an array for each set. Complete the sentences. The first one has been drawn for you.

- a. **2 rows of 6**



2 rows of 6 = _____

_____ + _____ = _____

6 doubled is _____.

- b. **2 rows of 7**

2 rows of 7 = _____

_____ + _____ = _____

7 doubled is _____.

- c. **2 rows of 8**

_____ rows of _____ = _____

_____ + 8 = _____

8 doubled is _____.

- d. **2 rows of 9**

2 rows of 9 = _____

_____ + _____ = _____

9 doubled is _____.

- e. **2 rows of 10**

_____ rows of _____ = _____

10 + _____ = _____

10 doubled is _____.

3. List the totals from Problem 1. _____

List the totals from Problem 2. _____

Are the numbers you have listed even or not even? _____

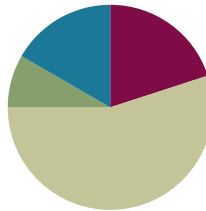
Explain in what ways the numbers are the same and different.

Lesson 18

Objective: Pair objects and skip-count to relate to even numbers.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Skip-Counting by Twos **2.OA.3** (4 minutes)
- Sprint: Subtraction from Teens **2.OA.2** (8 minutes)

Skip-Counting by Twos (4 minutes)

Note: This fluency activity is foundational to understanding the relationship between skip-counting and multiplication and division in Grade 3.

- T: On my signal, count by ones from 0 to 20 in a whisper. Ready? (Tap the desk while students are counting; knock on the twos. For example, tap, knock, tap, knock, etc.)
- T: Did anyone notice what I was doing while you were counting? I started out tapping by ones, but I knocked on every other number. Let's count again, and try knocking and tapping with me.
- S: 1 (tap), 2 (knock), 3 (tap), 4 (knock), 5 (tap), 6 (knock), ...
- T: Now, let's count only when we knock. Ready?
- S: (Tap), 2 (knock), (tap), 4 (knock), (tap), 6 (knock), (tap), 8 (knock), ...

Continue this routine up to 20 and back down again.

Sprint: Subtraction from Teens (8 minutes)

Materials: (S) Subtraction from Teens Sprint

Note: Students practice subtraction from teens in order to gain mastery of the sums and differences within 20.

Application Problem (5 minutes)

Eggs come in cartons of 12. Use pictures, numbers, or words to explain whether 12 is even or not even.

Note: This problem is intended for independent practice and bridges the concepts of Lessons 17 and 18. Egg cartons present a familiar real-life connection to solidify the idea of even. Allow students to share their reasoning.

12 is even because the eggs
look like this. 

2 rows of 6 is the same
as $6 + 6$ so 12 is even.

Concept Development (33 minutes)

Materials: (S) Personal white board, 20 counters (per pair)

- T: Let's keep exploring even numbers. (Call two students to the front of the class.)
- T: Line up side by side like you're going to lunch.
- S: (Pair up.)
- T: (Address the whole class.) Tell your partner whether 2 is even or not even.
- S: It's even because we can say 2, 4, 6, 8, like we did yesterday. → It's even because we can say $1 + 1$, so it's a double. → They each have a partner, so they're even.
- T: Interesting! You say they both have a partner. (Call another student up to continue the lineup.)
- T: So what do you think? Is 3 an even number?
- S: It's not even because Samuel doesn't have a partner. → It's not even because we don't say 3 when we count by twos. → It's not even because $1 + 2$ isn't a double.
- T: Excellent thinking! Let's experiment with counters to see what else we can discover about even numbers.
- T: Partner A, place 7 counters between you and your partner. Working together, pair up your counters to decide if 7 is even or not even.



**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Challenge students who grasp the concept of even numbers quickly to extend the pattern. Can they prove whether 52 is even? How about 73?

Circulate as students work. Some may arrange the counters in arrays, while others may have them scattered in groups of two. Either is fine, so long as they are pairing the counters.

- T: What did you decide?
- S: 7 is not even because there's a counter left over. → 7 is not even because we don't say 7 when counting by twos. → You can't make a doubles sentence for 7.
- T: Move your counters into array form. What addition sentence, or equation, matches what you see?
- S: $3 + 4 = 7$.
- T: That's not a doubles fact, is it?
- S: No!

T: Partner A, keep those 7 counters. Partner B, get 7 more counters. Pair your counters to decide if you'll get an even number. (Pause.)

T: What did you find?

S: It's 14, and that's even because every counter had a partner. → We made 2 rows of 7. → We can say $7 + 7 = 14$, and if it's doubles, it's even. → There are 7 columns of 2, so we can skip-count, 2, 4, 6, 8, 10, 12, 14.

T: So, even if one of the addends isn't even, when we double it, we get an even number. True?

S: True!

T: Now, let's see how many even numbers you can find! Working with your partner, continue to pair counters to see what numbers are even. Write the addition sentence for any even number on your personal white board.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

To make the content accessible for English language learners, model student and teacher explanations using counters, and encourage them to express their understanding by modeling. Also, pair them with students who have strong English skills, and encourage them to repeat what their partners say during pair-sharing.

Allow enough time for students to make all of the pairings up to 20. Circulate and support any students who need it.

T: Now that you've had a chance to work with the counters, make an array out of all 20. (Pause to allow students time to do so.)

T: Let's count by twos up and back. Ready?

T: But wait! Should we start at 2 or 0? What do you think? Is 0 even? Talk about that with your partner.

S: It can't be even because there's nothing there.

T: Thumbs up if you agree. (If any students disagree, allow them the chance to explain their thinking.)

S: We start at 0 when we skip-count during Fluency Practice, so it's part of counting by twos. → I see a pattern in the ones place. There are 2 ones in 2, and there are 2 ones in 12. There are 4 ones in 4 and 4 ones in 14, and it keeps going like that. And there are 0 ones in 10, and there are 0 ones in 20, so 0 is even.

T: You make very good arguments. And I have one more! What is $0 + 0$?

S: 0.

T: Yes! Just like $1 + 1 = 2$, 2 is a doubles fact. 0 is another doubles fact because you get 0 when you add $0 + 0$.

T: So, that means 0 is an even number! When you double **whole numbers**, like 0, 1, 2, 3, 4, the answer is an even number.

MP.8

Have students count by twos up to 20 and back down, starting at 0.

T: So, we only get a doubles fact when all of the objects have a partner. If any objects are left over without a partner, it can't be even. True?

S: True.

When they are able to demonstrate fluency, allow students to move on to the Problem Set. Continue working with students who need extra support.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Pair objects and skip-count to relate even numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1, what connections can you make between pairing objects and equal groups? If a number is even, can you make equal groups? If it is not even? If so, how many are in each equal group?
- Look at the pattern in Problem 2. How can you describe each picture in terms of rows or columns? Each time you add another pair, what happens to the rows and columns? (Use the frame, "There are ____ rows/columns of ____.")
- For Problem 3, do you think we should start with 0? Does 0 follow the pattern?
- For Problem 6, which array matches a drawing in Problem 2? Describe it in terms of rows or columns.

Name Roberto Date _____

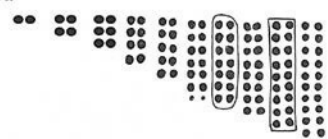
1. Pair the objects to decide if the number of objects is even.

 Even / Not Even

 Even / Not Even

 Even / Not Even

2. Draw to continue the pattern of the pairs in the space below until you have drawn 10 pairs.



3. Write the number of dots in each array in Problem 2 in order from least to greatest.

2 4 6 8 10 12 14 16 18 20

4. Circle the array in Problem 2 that has 2 columns of 7.

5. Box the array in Problem 2 that has 2 columns of 9.

6. Redraw the following sets of dots as columns of two or 2 equal rows.

a.  There are 10 dots.
Is 10 an even number? yes

b.  There are 15 dots.
Is 15 an even number? no

7. Circle groups of two. Count by twos to see if the number of objects is even.



a. There are 9 twos. There are 0 left over.
b. Count by twos to find the total.
2 4 6 8 10 12 14 16 18
c. This group has an even number of objects. True or False

- What is different about Problem 6(b)? Can you talk about your drawing in terms of rows and columns? Between which two even numbers does this number fall? Find those numbers in Problem 2. What is the difference between them?
- If you can circle groups of 2 with 0 left over, what do you know for sure?
- When you double a **whole number**, is the answer even or odd? How can you prove it?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Subtraction from Teens

1.	$10 - 3 =$	
2.	$11 - 3 =$	
3.	$12 - 3 =$	
4.	$10 - 2 =$	
5.	$11 - 2 =$	
6.	$10 - 5 =$	
7.	$11 - 5 =$	
8.	$12 - 5 =$	
9.	$14 - 5 =$	
10.	$10 - 4 =$	
11.	$11 - 4 =$	
12.	$12 - 4 =$	
13.	$13 - 4 =$	
14.	$10 - 7 =$	
15.	$11 - 7 =$	
16.	$12 - 7 =$	
17.	$15 - 7 =$	
18.	$10 - 6 =$	
19.	$11 - 6 =$	
20.	$12 - 6 =$	
21.	$14 - 6 =$	
22.	$10 - 9 =$	

23.	$11 - 9 =$	
24.	$12 - 9 =$	
25.	$17 - 9 =$	
26.	$10 - 8 =$	
27.	$11 - 8 =$	
28.	$12 - 8 =$	
29.	$16 - 8 =$	
30.	$10 - 6 =$	
31.	$13 - 6 =$	
32.	$15 - 6 =$	
33.	$10 - 7 =$	
34.	$13 - 7 =$	
35.	$14 - 7 =$	
36.	$16 - 7 =$	
37.	$10 - 8 =$	
38.	$13 - 8 =$	
39.	$14 - 8 =$	
40.	$17 - 8 =$	
41.	$10 - 9 =$	
42.	$13 - 9 =$	
43.	$14 - 9 =$	
44.	$18 - 9 =$	

B

Subtraction from Teens

Number Correct: _____

Improvement: _____

1.	$10 - 2 =$	
2.	$11 - 2 =$	
3.	$10 - 4 =$	
4.	$11 - 4 =$	
5.	$12 - 4 =$	
6.	$13 - 4 =$	
7.	$10 - 3 =$	
8.	$11 - 3 =$	
9.	$12 - 3 =$	
10.	$10 - 6 =$	
11.	$11 - 6 =$	
12.	$12 - 6 =$	
13.	$15 - 6 =$	
14.	$10 - 5 =$	
15.	$11 - 5 =$	
16.	$12 - 5 =$	
17.	$14 - 5 =$	
18.	$10 - 8 =$	
19.	$11 - 8 =$	
20.	$12 - 8 =$	
21.	$17 - 8 =$	
22.	$10 - 7 =$	

23.	$11 - 7 =$	
24.	$12 - 7 =$	
25.	$16 - 7 =$	
26.	$10 - 9 =$	
27.	$11 - 9 =$	
28.	$12 - 9 =$	
29.	$18 - 9 =$	
30.	$10 - 5 =$	
31.	$13 - 5 =$	
32.	$10 - 6 =$	
33.	$13 - 6 =$	
34.	$14 - 6 =$	
35.	$10 - 7 =$	
36.	$13 - 7 =$	
37.	$15 - 7 =$	
38.	$10 - 8 =$	
39.	$13 - 8 =$	
40.	$14 - 8 =$	
41.	$16 - 8 =$	
42.	$10 - 9 =$	
43.	$16 - 9 =$	
44.	$17 - 9 =$	

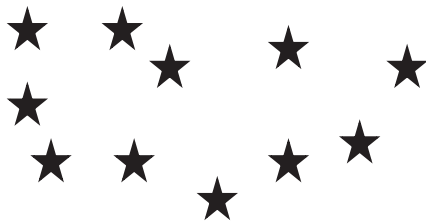
Name _____

Date _____

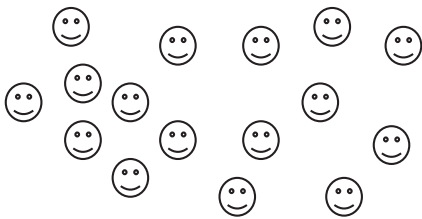
1. Pair the objects to decide if the number of objects is even.



Even/Not Even



Even/Not Even



Even/Not Even

2. Draw to continue the pattern of the pairs in the space below until you have drawn 10 pairs.



3. Write the number of dots in each array in Problem 2 in order from least to greatest.
4. Circle the array in Problem 2 that has 2 columns of 7.
5. Box the array in Problem 2 that has 2 columns of 9.
6. Redraw the following sets of dots as columns of two or 2 equal rows.

a.



b.



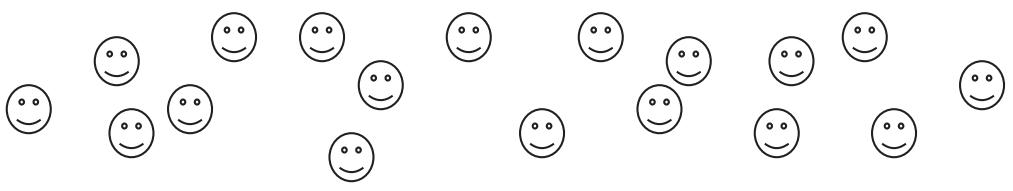
There are _____ dots.

There are _____ dots.

Is _____ an even number? _____

Is _____ an even number? _____

7. Circle groups of two. Count by twos to see if the number of objects is even.



a. There are _____ twos. There are _____ left over.

b. Count by twos to find the total.
 _____, _____, _____, _____, _____, _____, _____, _____

c. This group has an even number of objects: True or False

Name _____

Date _____

Redraw the following sets of dots as columns of two or 2 equal rows.

1.



2.



There are _____ dots.

Is _____ an even number? _____

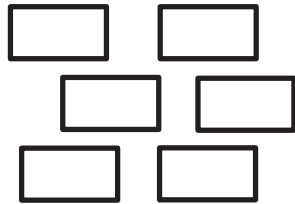
There are _____ dots.

Is _____ an even number? _____

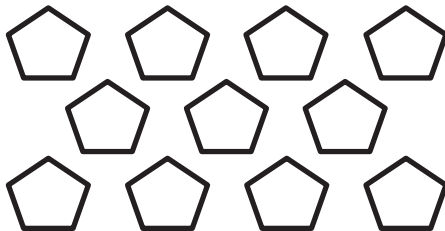
Name _____

Date _____

1. Pair the objects to decide if the number of objects is even.



Even/Not Even

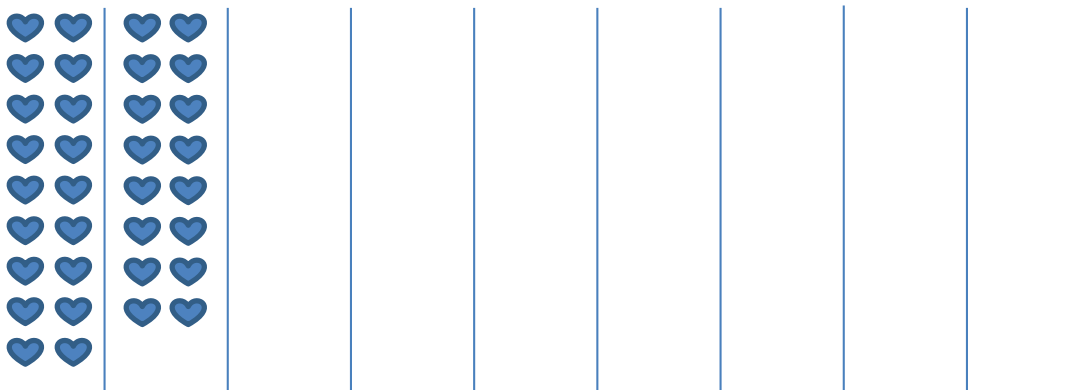


Even/Not Even



Even/Not Even

2. Draw to continue the pattern of the pairs in the spaces below until you have drawn zero pairs.



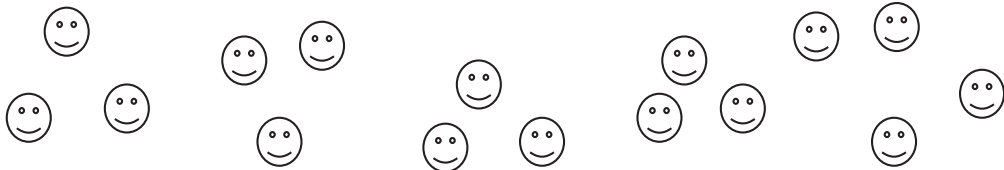
3. Write the number of hearts in each array in Problem 2 in order from greatest to least.
4. Circle the array in Problem 2 that has 2 columns of 6.
5. Box the array in Problem 2 that has 2 columns of 8.
6. Redraw the set of stars as columns of two or 2 equal rows.



There are _____ stars.

Is _____ an even number? _____

7. Circle groups of two. Count by twos to see if the number of objects is even.



a. There are _____ twos. There are _____ left over.

b. Count by twos to find the total.
 _____, _____, _____, _____, _____, _____, _____, _____

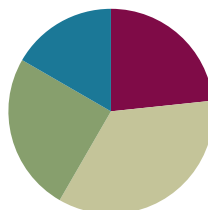
c. This group has an even number of objects: True or False.

Lesson 19

Objective: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.

Suggested Lesson Structure

Fluency Practice	(14 minutes)
Concept Development	(21 minutes)
Application Problem	(15 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (14 minutes)

- Making the Next Ten to Add **2.OA.2, 2.NBT.5** (5 minutes)
- Sprint: Sums to the Teens **2.OA.2** (9 minutes)

Making the Next Ten to Add (5 minutes)

Note: Students practice this mental strategy to gain mastery of sums within 20 and to relate the strategy to larger numbers.

T: When I say “9 + 4,” you say “10 + 3.” Ready? 9 + 4.

S: 10 + 3.

T: Answer.

S: 13.

Continue with the following possible sequences:

9 + 6, 9 + 16, 19 + 16

8 + 7, 8 + 17, 18 + 17

7 + 6, 7 + 16, 17 + 16

9 + 8, 9 + 18, 19 + 18

8 + 5, 8 + 15, 18 + 15

7 + 4, 17 + 4, 17 + 14

Sprint: Sums to the Teens (9 minutes)

Materials: (S) Sums to the Teens Sprint

Note: Students practice crossing the ten when adding to gain mastery of sums within 20.

Concept Development (21 minutes)

Materials: (S) 20 square tiles

- T: (Write numbers to 20 in a straight line across the board.) Make a column of 2 tiles.
- T: How many rows do you have?
- S: 2 rows!
- T: How many tiles do you have in each row?
- S: 1 tile.
- T: Say the doubles equation by adding the number in each row.
- S: $1 + 1 = 2$.
- T: (Draw the array on the board above a number path as shown to the right.)
- T: Is 2 an even number?
- S: Yes!
- T: As we make our arrays, let's keep track of all the even numbers we find by circling them. (Circle 2.)
- T: Add another column of 2. Now how many columns of 2 do you have?
- S: 2 columns!
- T: How many rows do you have?
- S: 2 rows!
- T: Say the doubles equation.
- S: $2 + 2 = 4$.
- T: Turn and talk: Is 4 an even number?
- S: Yes, because none are sticking out. → Four is even because I can count 2, 4.
- T: Let's circle 4 because it's an even number. (Circle 4.)
- T: Add another column of 2.
- S: (Add tiles to array, as teacher does the same on the board.)
- T: How many columns of 2 do you have now?
- S: 3 columns of 2.

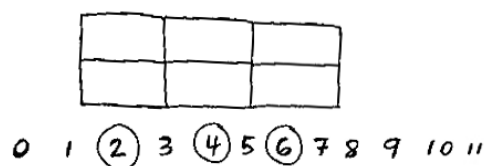
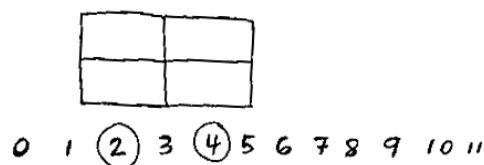
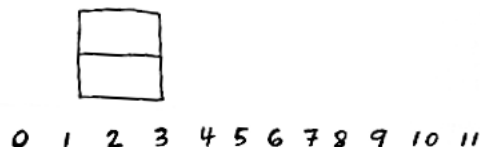
Briskly continue the above sequence until all even numbers up to 20 are circled.

- T: Turn and talk: What do you notice about the numbers we circled? Do you see a pattern?
- MP.8** S: They are all even. → It starts at 2 and keeps going 2, 4, 6, 8, 0 in the ones place. → It is every other number that is circled.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Make a body and science connection by inviting students to observe structures in the human body that appear in twos: two eyes, two ears, two hands, etc. Demonstrate that even parts that appear to be singular have two halves, like the two sides of the nose and mouth.



MP.8

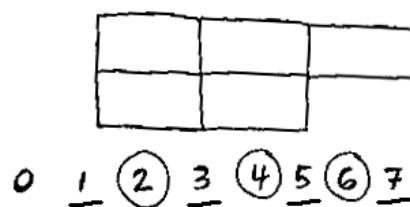
- T: It is true that all even numbers have 0, 2, 4, 6, or 8 in the ones place. This is one way we can identify even numbers. What do you notice about the numbers that are not circled?
- S: They are not even. → It is every other number that is not circled. → They are one more and one less than the even number.
- T: All the numbers that are not circled are called **odd numbers**.
- T: Take 1 tile away from your array of 20. How many tiles do you have left?
- S: 19 tiles!
- T: Is 19 an even number?
- S: No!
- T: Why not?
- S: You can't make pairs. → There is one sticking out. It has no partner. → Because it's not $10 + 10$ anymore since we took 1 away. → The number sentence would be $10 + 9 = 19$ since there are 10 on the bottom and 9 on the top. → It's not an array with 2 rows or columns of 2.
- T: That means that 19 is odd. Let's underline the odd numbers as we take away 1 from each even number. (Underline 19.)
- T: Take away another tile. How many tiles do you have now?
- S: 18.
- T: We know 18 is even. Take away a tile. How many tiles do you have now?
- S: 17.
- T: Turn and talk: Is 17 even or odd?
- S: One tile doesn't have a partner. → 17 is odd because there is no doubles sentence. → $8 + 9 = 17$, so that's odd. → I can't count by twos to 17, so it's not even.
- T: 17 is odd, so let's underline it. (Underline 17.)
- Briskly continue taking away 1 tile from each even number and underlining the odd numbers down to 1.
- T: What happened when we had an even number of tiles and we took 1 away?
- S: The number left over was an odd number. → An even number take away 1 is odd.
- T: Turn and talk: What will happen when we add 1 to an even number?
- S: It's going to make one extra stick out. → You won't have a double but a double plus 1 more. → Adding 1 to an even will be odd because $8 + 1 = 9$, and that is odd. → Adding 1 on is just like taking 1 away. It will make an odd. → I can see on the number path you can add 1 to an even to make an odd or take one away.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

For students who already understand even and odd numbers and are eager to move on, have them color a number path up to 100 with different colors for even and odd numbers.

For an even greater challenge, have them then color every 5 squares with a different color, and ask them to notice what the new blended color is when the multiple of 5 is odd and when it is even.



- T: Test what we just noticed. Take two minutes to use your tiles with your partner. Partner A, build an even number. Partner B, add one, and then take away one from the array of the even number. See if you get an odd number. Then, switch. Partner B, make an even number.
- S: (Work.)
- T: What happens when we add 1 to an even number?
- S: We make an odd!
- T: What happens when we take 1 away from an even number?
- S: We make an odd!
- T: Let's practice using what we know on some bigger numbers.
- T: (Write 40 on the board.)
- T: Turn and talk: Is 40 even or odd?
- S: 40 is even because it ends in 0. → 40 is even because I can count by twos up to 40. → I know that 40 is even because $20 + 20 = 40$.
- T: Is 41 even or odd?
- S: 41 must be odd, because it's an even plus 1. → 41 is odd because it doesn't end in 0, 2, 4, 6, or 8.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Provide an extension for students by encouraging them to build other arrays with odd numbers of tiles. Deepen understanding that an even number can be broken into pairs or groups of 2, but that does not mean that odd numbers cannot be broken into equal groups. For example, 9 can be constructed with 3 rows of 3.

Continue the above process starting with numbers within 50 that are easier for students to verify with concrete materials, doubles, or counting by twos, such as 26, 30, 44, and 50.

Application Problem (15 minutes)

Eggs come in cartons of 12. Joanna's mom used 1 egg. Use pictures, numbers, or words to explain whether the amount left is even or odd.

Note: This problem is intended for independent practice and bridges the concepts of Lessons 18 and 19. It also mirrors Lesson 18's Application Problem. Allow students to share their reasoning. The allotted time period includes 5 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

000000
000000

$$12 - 1 = 11$$

11 is odd because
you can't count by
2's to 11.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Now that you've completed Problem 1, describe another array in terms of rows and columns in which you can skip-count by twos (i.e., 2 rows of ___ or ___ columns of 2).
- In Problems 3(a) and 3(b), what do you notice about all the even numbers? All the odd numbers? Can you find a similarity between these two patterns?
- For Problem 4, what happens to an even number when you add or subtract 1? What number(s) do you need to add or subtract to make another even number?
- In Problem 5(c), Sami argues that 45 is even because it starts with 4, and numbers that have 0, 2, 4, 6, or 8 are even. Is she correct? How do you know?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Roberto Date _____

1. Skip-count the columns in the array. The first one has been done for you.

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
2	4	6	8	10	12	14	16	18	20

2. a. Solve.

$1 + 1 = 2$

$2 + 2 = 4$

$3 + 3 = 6$

$4 + 4 = 8$

$5 + 5 = 10$

$6 + 6 = 12$

$7 + 7 = 14$

$8 + 8 = 16$

$9 + 9 = 18$

$10 + 10 = 20$

b. Explain the connection between the array in Problem 1 and the answers in Problem 2(a).

The answers in Problem 2 are the same numbers as Problem 1 because you can find doubles by counting by 2 or by adding the same number (like 4+4).

3. a. Fill in the missing numbers on the number path.

20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50

b. Fill in the odd numbers on the number path.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

4. Write to identify the bold numbers as even or odd. The first one has been done for you.

a.	b.	c.
$6 + 1 = 7$ <u>even</u> + 1 = <u>odd</u>	$24 + 1 = 25$ <u>even</u> + 1 = <u>odd</u>	$30 + 1 = 31$ <u>even</u> + 1 = <u>odd</u>
d.	e.	f.
$6 - 1 = 5$ <u>even</u> - 1 = <u>odd</u>	$24 - 1 = 23$ <u>even</u> - 1 = <u>odd</u>	$30 - 1 = 29$ <u>even</u> - 1 = <u>odd</u>

5. Are the bold numbers even or odd? Circle the answer, and explain how you know.

a.	28 <u>even</u> / odd	Explanation: it has 8 in the ones
b.	39 even / <u>odd</u>	Explanation: an even + 1 = odd & 38 is even
c.	45 even / <u>odd</u>	Explanation: 5 is odd, so 45 is odd
d.	50 <u>even</u> / odd	Explanation: you can count by 2's to 50

A

Number Correct: _____

Sums to the Teens

1.	$9 + 2 =$	
2.	$9 + 3 =$	
3.	$9 + 4 =$	
4.	$9 + 7 =$	
5.	$7 + 9 =$	
6.	$10 + 1 =$	
7.	$10 + 2 =$	
8.	$10 + 3 =$	
9.	$10 + 8 =$	
10.	$8 + 10 =$	
11.	$8 + 3 =$	
12.	$8 + 4 =$	
13.	$8 + 5 =$	
14.	$8 + 9 =$	
15.	$9 + 8 =$	
16.	$7 + 4 =$	
17.	$10 + 5 =$	
18.	$6 + 5 =$	
19.	$7 + 5 =$	
20.	$9 + 5 =$	
21.	$5 + 9 =$	
22.	$10 + 6 =$	

23.	$4 + 7 =$	
24.	$4 + 8 =$	
25.	$5 + 6 =$	
26.	$5 + 7 =$	
27.	$3 + 8 =$	
28.	$3 + 9 =$	
29.	$2 + 9 =$	
30.	$5 + 10 =$	
31.	$5 + 8 =$	
32.	$9 + 6 =$	
33.	$6 + 9 =$	
34.	$7 + 6 =$	
35.	$6 + 7 =$	
36.	$8 + 6 =$	
37.	$6 + 8 =$	
38.	$8 + 7 =$	
39.	$7 + 8 =$	
40.	$6 + 6 =$	
41.	$7 + 7 =$	
42.	$8 + 8 =$	
43.	$9 + 9 =$	
44.	$4 + 9 =$	

B

Sums to the Teens

Number Correct: _____

Improvement: _____

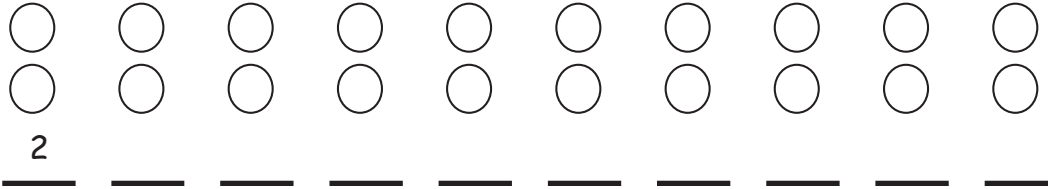
1.	$10 + 1 =$	
2.	$10 + 2 =$	
3.	$10 + 3 =$	
4.	$10 + 9 =$	
5.	$9 + 10 =$	
6.	$9 + 2 =$	
7.	$9 + 3 =$	
8.	$9 + 4 =$	
9.	$9 + 8 =$	
10.	$8 + 9 =$	
11.	$8 + 3 =$	
12.	$8 + 4 =$	
13.	$8 + 5 =$	
14.	$8 + 7 =$	
15.	$7 + 8 =$	
16.	$7 + 4 =$	
17.	$10 + 4 =$	
18.	$6 + 5 =$	
19.	$7 + 5 =$	
20.	$9 + 5 =$	
21.	$5 + 9 =$	
22.	$10 + 8 =$	

23.	$5 + 6 =$	
24.	$5 + 7 =$	
25.	$4 + 7 =$	
26.	$4 + 8 =$	
27.	$4 + 10 =$	
28.	$3 + 8 =$	
29.	$3 + 9 =$	
30.	$2 + 9 =$	
31.	$5 + 8 =$	
32.	$7 + 6 =$	
33.	$6 + 7 =$	
34.	$8 + 6 =$	
35.	$6 + 8 =$	
36.	$9 + 6 =$	
37.	$6 + 9 =$	
38.	$9 + 7 =$	
39.	$7 + 9 =$	
40.	$6 + 6 =$	
41.	$7 + 7 =$	
42.	$8 + 8 =$	
43.	$9 + 9 =$	
44.	$4 + 9 =$	

Name _____

Date _____

1. Skip-count the columns in the array. The first one has been done for you.



2. a. Solve.

$1 + 1 = \underline{\quad}$

$2 + 2 = \underline{\quad}$

$3 + 3 = \underline{\quad}$

$4 + 4 = \underline{\quad}$

$5 + 5 = \underline{\quad}$

$6 + 6 = \underline{\quad}$

$7 + 7 = \underline{\quad}$

$8 + 8 = \underline{\quad}$

$9 + 9 = \underline{\quad}$

$10 + 10 = \underline{\quad}$

- b. Explain the connection between the array in Problem 1 and the answers in Problem 2(a).

3. a. Fill in the missing numbers on the number path.

20, 22, 24, _____, 28, 30, _____, _____, 36, _____, 40, _____, _____, 46, _____, _____

- b. Fill in the odd numbers on the number path.

0, _____, 2, _____, 4, _____, 6, _____, 8, _____, 10, _____, 12, _____, 14, _____, 16, _____, 18, _____, 20, _____

4. Write to identify the **bold** numbers as even or odd. The first one has been done for you.

a. $6 + 1 = 7$ <u>even</u> + 1 = <u>odd</u>	b. $24 + 1 = 25$ _____ + 1 = _____	c. $30 + 1 = 31$ _____ + 1 = _____
d. $6 - 1 = 5$ _____ - 1 = _____	e. $24 - 1 = 23$ _____ - 1 = _____	f. $30 - 1 = 29$ _____ - 1 = _____

5. Are the **bold** numbers even or odd? Circle the answer, and explain how you know.

a. 28 even/odd	Explanation:
b. 39 even/odd	Explanation:
c. 45 even/odd	Explanation:
d. 50 even/odd	Explanation:

Name _____

Date _____

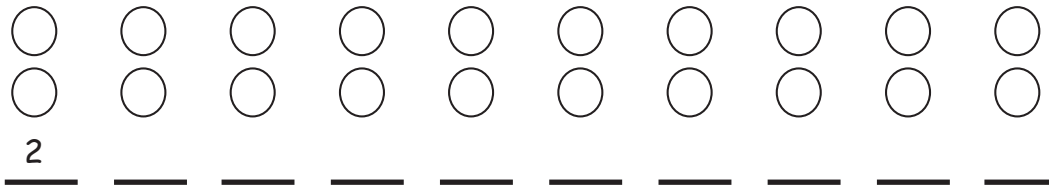
Are the **bold** numbers even or odd? Circle the answer, and explain how you know.

a. 18 even/odd	Explanation:
b. 23 even/odd	Explanation:

Name _____

Date _____

1. Skip-count the columns in the array. The first one has been done for you.



2. a. Solve.

$1 + 1 = \underline{\quad}$

$6 + 6 = \underline{\quad}$

$2 + 2 = \underline{\quad}$

$7 + 7 = \underline{\quad}$

$3 + 3 = \underline{\quad}$

$8 + 8 = \underline{\quad}$

$4 + 4 = \underline{\quad}$

$9 + 9 = \underline{\quad}$

$5 + 5 = \underline{\quad}$

$10 + 10 = \underline{\quad}$

- b. How is the array in Problem 1 related to the answers in Problem 2(a)?

3. Fill in the missing even numbers on the number path.

18, 20, _____, _____, 26, _____ 30, _____, 34, _____, 38, 40, _____, _____

4. Fill in the missing odd numbers on the number path.

0, _____, 2, _____, 4, _____, 6, _____, 8, _____, 10, _____, 12, _____, 14

5. Write to identify the **bold** numbers as even or odd. The first one has been done for you.

a. $4 + 1 = 5$ <u>even</u> + 1 = <u>odd</u>	b. $13 + 1 = 14$ _____ + 1 = _____	c. $20 + 1 = 21$ _____ + 1 = _____
d. $8 - 1 = 7$ _____ - 1 = _____	e. $16 - 1 = 15$ _____ - 1 = _____	f. $30 - 1 = 29$ _____ - 1 = _____

6. Are the **bold** numbers even or odd? Circle the answer, and explain how you know.

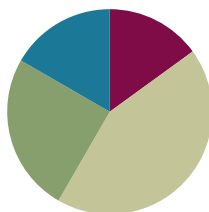
a. 21 even/odd	Explanation:
b. 34 even/odd	Explanation:

Lesson 20

Objective: Use rectangular arrays to investigate odd and even numbers.

Suggested Lesson Structure

■ Fluency Practice	(9 minutes)
■ Concept Development	(26 minutes)
■ Application Problem	(15 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (9 minutes)

- Skip-Counting by Twos **2.OA.3** (4 minutes)
- Grade 2 Core Fluency Practice Sets **2.OA.2** (5 minutes)

Skip-Counting by Twos (4 minutes)

Note: This fluency activity is foundational to understanding the relationship between skip-counting and multiplication and division in Grade 3.

- T: On my signal, count by ones from 0 to 20 in a whisper. Ready? (Tap the desk while students are counting the ones, and knock on the twos. For example, tap, knock, tap, knock, etc.)
- T: Did anyone notice what I was doing while you were counting? I started tapping by ones, but I knocked on every other number. Let's count again, and try knocking and tapping with me.
- S: 1 (tap), 2 (knock), 3 (tap), 4 (knock), 5 (tap), 6 (knock), ...
- T: Now, let's count only when we knock. Ready?
- S: (Tap), 2 (knock), (tap), 4 (knock), (tap), 6 (knock), (tap), 8 (knock), ...

Continue this routine up to 20 and back down again.

Grade 2 Core Fluency Practice Sets (5 minutes)

Materials: (S) Core Fluency Practice Sets (Lesson 12 Core Fluency Practice Sets)

Note: During Topic D and for the remainder of the year, each day's fluency activities include an opportunity for review and mastery of the sums and differences with totals through 20 by means of the Core Fluency Practice Sets or Sprints. Practice Sets, along with details about the process, are provided in Lesson 12.

Concept Development (26 minutes)

Materials: (T) Premade even and odd poster (see image to the right) (S) 25 tiles, personal white board

Part 1: Even + even = even

T: Partner A, make 2 rows of 3 on your personal white board.

T: Partner B, make 2 rows of 4 on your personal white board.

S: (Construct the arrays.)

T: How many tiles are on Partner A's personal white board?

S: 6 tiles!

T: Is 6 even or odd?

S: Even!

T: How many tiles are on Partner B's personal white board?

S: 8 tiles!

T: Is 8 even or odd?

S: Even!

T: Now, let's see what happens when we add two even numbers together. Partners, slide your personal white boards next to each other, and combine the two arrays that you made.

S: (Connect the tiles to show 2 rows of 7.)

T: How many tiles do you have altogether?

S: 14 tiles!

T: Is that even or odd?

S: Even!

Repeat the above process with the following sequence:
2 rows of 5 + 2 rows of 3, and 2 rows of 4 + 2 rows of 8.

T: When we add an even number and an even number, do we get an even or an odd number?

S: Even number!

T: Let's record that on our chart. An even number plus another even number makes an even number. (Record on the chart.)

Part 2: Even + odd = odd

T: Now, let's see what happens when we add an even and an odd!

T: Partner A, make an array with 2 rows of 3 on your board. Partner B, make 2 rows of 3, and then add 1 tile on the top row on the right. (Pause and allow students time to complete the task.)

T: Is 6 even or odd?

S: Even!

$$\text{Even} + \text{Even} = \boxed{}$$

$$\text{Even} + \text{Odd} = \boxed{}$$

$$\text{Odd} + \text{Odd} = \boxed{}$$



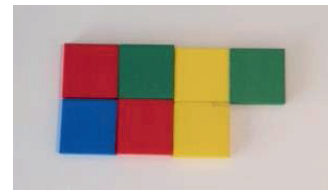
$$\text{Even} + \text{Even} = \boxed{\text{Even}}$$

$$\text{Even} + \text{Odd} = \boxed{}$$

$$\text{Odd} + \text{Odd} = \boxed{}$$



- T: Is 7 even or odd?
 S: Odd!
 T: Now, slide your personal white boards together as you did before.
 S: (Move boards to connect the tiles as pictured.)
 T: How many tiles do you have altogether?
 S: 13 tiles!
 T: Is that even or odd?
 S: Odd!
 T: How do you know?
 S: There is one extra. → There are not 2 equal groups.
 → You can't count by twos to 13.



Repeat the above process using the following possible sequence:

- 2 rows of 5 + 2 rows of 2 (plus 1)
 - 2 rows of 3 (plus 1) + 2 rows of 6
- T: When we add an even and an odd, do we get an even or an odd?
 S: Odd!
 T: (Fill in the chart.)

$$\text{Even} + \text{Even} = \boxed{\text{Even}}$$

$$\text{Even} + \text{Odd} = \boxed{\text{Odd}}$$

$$\text{Odd} + \text{Odd} = \boxed{\phantom{\text{Even}}}$$

Part 3: Odd + odd = even

- T: (Record on the chart.)
 T: Now, let's see what happens when we add an odd number to another odd number!
 T: Partner A, make 2 rows of 3 on your board. Then, add 1 tile to the top row on the right.
 T: Partner B, make 2 rows of 4 on your board. Then, add 1 tile to the bottom row on the left.
 T: Is 7 even or odd?
 S: Odd!
 T: Is 9 even or odd?
 S: Odd!
 T: Partners, slide your boards together to connect the arrays.
 T: What do you have?
 S: 2 rows of 8.
 T: How many is that?
 S: 16.
 T: Is 16 even or odd?
 S: Even!



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

At other times in the school day, consider relating the mathematical term *even* to the everyday term *even* by asking questions such as the following:

- What does it mean for kickball teams to be *even*?
- When you are playing cards with two people, why do we deal an *even* number?
- When we share our grapes with a friend, do we try to make our shares *even*? What does *even* mean then?

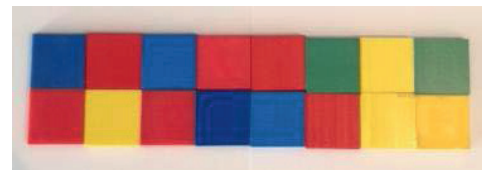
Repeat the above process with the following possible sequence:

- 2 rows of 2 (plus 1) + 2 rows of 4 (plus 1)
- 2 rows of 3 (plus 1) + 2 rows of 5 (plus 1)

T: What do we get when we add an odd and an odd?

S: An even!

T: (Fill in the chart.)



Part 4: Extend the pattern to sums with totals within 50.

T: What do we get when we add an even and an even?

S: An even!

T: What do we get when we add an even and an odd?

S: An odd!

T: Let's see if this is still true when we are adding larger numbers.

T: On your board, write the problem $10 + 12$ and your answer.

T: Is 10 even or odd?

S: Even!

T: Is 12 even or odd?

S: Even!

T: What is $10 + 12$?

S: 22.

T: Turn and talk: Is 22 even or odd, and how do you know?

S: It is even because I can count by 2 to get to 22.
 $\rightarrow$ 22 is even because the ones digit is a 2. $\rightarrow$ It is even because $11 + 11$ makes 22.

Repeat the above process for the following possible problems:

$22 + 4$, $22 + 3$, $21 + 5$

$22 + 14$, $22 + 13$, $21 + 15$

$$\text{Even} + \text{Even} = \boxed{\text{Even}}$$

$$\text{Even} + \text{Odd} = \boxed{\text{Odd}}$$

$$\text{Odd} + \text{Odd} = \boxed{\text{Even}}$$



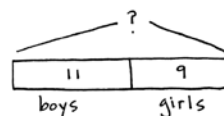
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Provide an extension for students by encouraging them to build other arrays with odd numbers of tiles. Deepen their understanding that an even number can be broken into pairs or groups of 2, but that does not mean that odd numbers cannot be broken into equal groups (9 for example, can be constructed with 3 rows of 3).

Application Problem (15 minutes)

Mrs. Boxer has 11 boys and 9 girls at a Grade 2 party.

- Write the equation to show the total number of people.
- Are the addends even or odd?
- Mrs. Boxer wants to pair everyone up for a game. Does she have the right number of people for everyone to have a partner?



- $11 + 9 = 20$
- 11 - odd 9 - odd
- An odd number plus an odd number makes an even number, so yes, Mrs. Boxer has the right number to make partners.
 $10 + 10 = 20$ so there will be 10 sets of partners.

Note: This Application Problem follows the Concept Development to provide an opportunity for students to apply their understanding from today's lesson. The allotted time period includes 5 minutes to solve the Application Problem and 10 minutes to complete the Problem Set.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use rectangular arrays to investigate odd and even numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problem 1(a), what is the difference between your two drawings? Can you make an array with 2 rows or columns for an odd number of objects? Can you group the circles differently and still make an array?
- For Problem 1(b), must your array show 2 equal rows or columns for a number to be even? What about 4 rows of 3? Can you split that array into groups of 2?
- If you have rows of 3, is it true that the number must be odd? When will the number be even?

2. Solve. Tell if each number is odd (O) or even (E). The first one has been done for you.

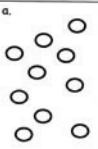
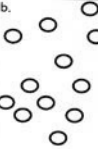
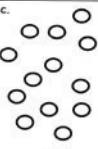
a. $6 + 4 = 10$ E + E = E	d. $14 + 8 = 22$ E + E = E
b. $17 + 2 = 19$ O + E = O	e. $3 + 9 = 12$ O + O = E
c. $11 + 13 = 24$ O + O = E	f. $5 + 14 = 19$ O + E = O

3. Write two examples for each case. Write if your answers are even or odd. The first one has been started for you.

a. Add an even number to an even number. $32 + 8 = 40$ even	$14 + 22 = 36$ even
b. Add an odd number to an even number. $31 + 24 = 55$ odd	$17 + 12 = 29$ odd
c. Add an odd number to an odd number. $25 + 25 = 50$ even	$87 + 13 = 100$ even

Name Roberto Date _____

1. Use the objects to create an array.

a. 	Array 	Redraw your picture with 1 less circle. 
There are an <u>even</u> (circle one) number of circles.	There are an <u>even</u> (circle one) number of circles.	There are an <u>even</u> (circle one) number of circles.
b. 	Array 	Redraw your picture with 1 more circle. 
There are an <u>even</u> (circle one) number of circles.	There are an <u>even</u> (circle one) number of circles.	There are an <u>even</u> (circle one) number of circles.
c. 	Array 	Redraw your picture with 1 less circle. 
There are an <u>odd</u> (circle one) number of circles.	There are an <u>odd</u> (circle one) number of circles.	There are an <u>odd</u> (circle one) number of circles.

- What have you learned about the total when adding different combinations of even and odd addends? How does this connect to the 1 more/1 less circle on the first page of the Problem Set?
- Can you only build rectangular arrays for even numbers? (Think about 15.) How do you know?

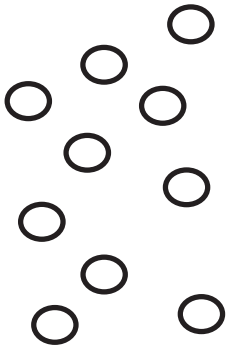
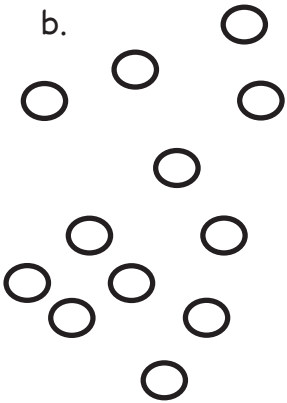
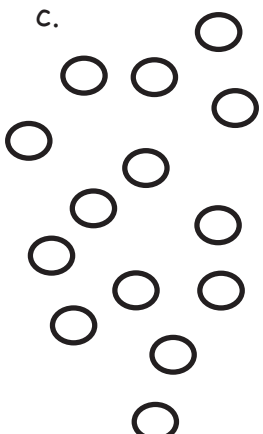
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. Use the objects to create an array.

a. 	Array There are an even/odd (circle one) number of circles.	Redraw your picture with 1 <i>less</i> circle. There are an even/odd (circle one) number of circles.
b. 	Array There are an even/odd (circle one) number of circles.	Redraw your picture with 1 <i>more</i> circle. There are an even/odd (circle one) number of circles.
c. 	Array There are an even/odd (circle one) number of circles.	Redraw your picture with 1 <i>less</i> circle. There are an even/odd (circle one) number of circles.

2. Solve. Tell if each number is odd (O) or even (E). The first one has been done for you.

a. $6 + 4 = 10$
 $\underline{E} + \underline{E} = \underline{E}$

d. $14 + 8 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

b. $17 + 2 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

e. $3 + 9 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

c. $11 + 13 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

f. $5 + 14 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

3. Write two examples for each case. Write if your answers are even or odd. The first one has been started for you.

a. Add an even number to an even number.

$\underline{32 + 8 = 40 \text{ even}}$ $\underline{\hspace{4cm}}$

b. Add an odd number to an even number.

$\underline{\hspace{4cm}}$ $\underline{\hspace{4cm}}$

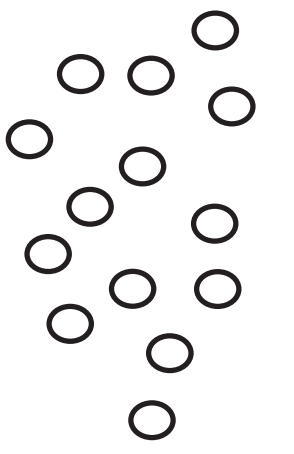
c. Add an odd number to an odd number.

$\underline{\hspace{4cm}}$ $\underline{\hspace{4cm}}$

Name _____

Date _____

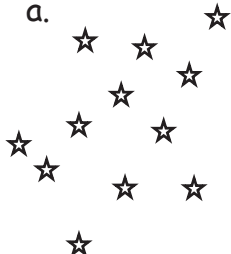
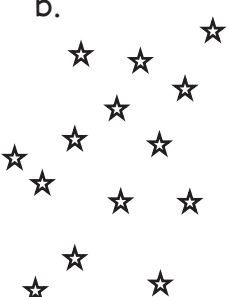
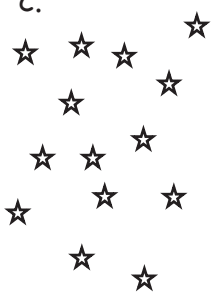
Use the objects to create an array.

	Array There are an even/odd (circle one) number of circles.	Redraw your picture with 1 <i>less</i> circle. There are an even/odd (circle one) number of circles.

Name _____

Date _____

1. Use the objects to create an array with 2 rows.

a. 	Array with 2 rows There are an even/odd (circle one) number of stars.	Redraw your picture with 1 <i>less</i> star. There are an even/odd (circle one) number of stars.
b. 	Array with 2 rows There are an even/odd (circle one) number of stars.	Redraw your picture with 1 <i>more</i> star. There are an even/odd (circle one) number of stars.
c. 	Array with 2 rows There are an even/odd (circle one) number of stars.	Redraw your picture with 1 <i>less</i> star. There are an even/odd (circle one) number of stars.

2. Solve. Tell if each number is odd (O) or even (E) on the line below.

a. $6 + 6 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

e. $7 + 8 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

b. $8 + 13 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

f. $9 + 11 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

c. $9 + 15 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

g. $7 + 14 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

d. $17 + 8 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

h. $9 + 9 = \underline{\hspace{2cm}}$
 $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

3. Write three number sentence examples to prove that each statement is correct.

Even + Even = Even	Even + Odd = Odd	Odd + Odd = Even

4. Write two examples for each case. Next to your answer, write if your answers are even or odd. The first one has been done for you.

a. Add an even number to an even number.

32 + 8 = 40 even _____

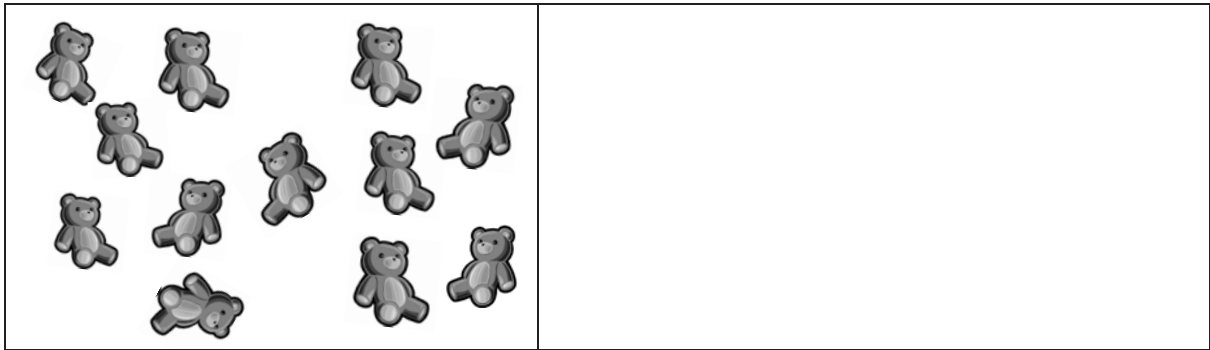
b. Add an odd number to an even number.

c. Add an odd number to an odd number.

Name _____

Date _____

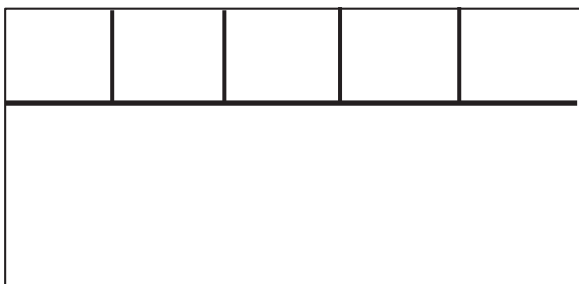
1. a. Does the picture below show an even or an odd number of teddy bears? Explain your thinking using pictures, numbers, or words in the box on the right.



- b. Explain how you know if a number is even.

2. a. Complete the array.

- b. Using the entire rectangle, draw 3 rows of 5 squares. The first row is done for you. Then, write a repeated addition sentence that describes your array.



- c. Henry drew the rectangle below using 12 squares. Draw a different rectangle using 12 squares.



3. Complete each sentence. Explain your thinking using pictures, numbers, or words.

a. 2 groups of 4 make _____.

b. _____ groups of 2 make 6.

4. a. Alex says that 14 is an even number. Do you agree with him? Explain your thinking using pictures, numbers, or words.

- b. Draw an array using 14 squares in 2 rows. The rows have been drawn for you.

- c. Alex has 14 pencils. He gives all of his pencils to his two friends. Each friend gets the same number of pencils. How many pencils did each friend get? Explain your thinking using pictures, numbers, or words.

End-of-Module Assessment Task Standards Addressed

Topics A–D

Work with equal groups of objects to gain foundations for multiplication.

- 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Reason with shapes and their attributes.

- 2.G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 2.OA.3	The student solves zero out of three parts correctly.	The student solves one out of three parts correctly.	The student solves two out of three parts correctly.	The student correctly: - Answers <i>even</i> and explains thinking using pictures, numbers, or words. - Explains that a number is even using at least one of the following reasons: - A number that occurs as we skip-count by twos. - When objects are paired with none left over. - A number that is twice a whole number (double). - A number whose last digit is 0, 2, 4, 6, or 8.
2 2.G.2	The student solves zero out of three parts correctly.	The student solves one out of three parts correctly.	The student solves two out of three parts correctly.	The student correctly: - Completes the array to show 4 rows of 6. - Completes the array to show 3 rows of 5 and gives a repeated addition sentence of $5 + 5 + 5 = 15$ or $3 + 3 + 3 + 3 + 3 = 15$. - Draws a different array using 12 squares.



A Progression Toward Mastery

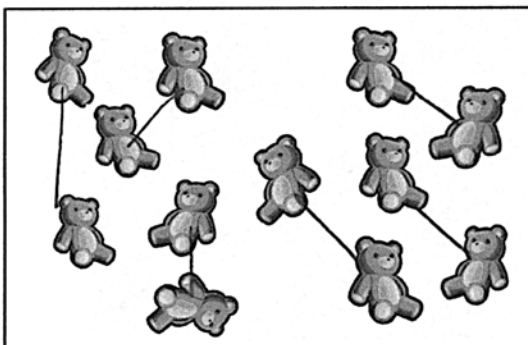
3 2.OA.3 2.OA.4	The student solves one out of four parts correctly.	The student solves two out of four parts correctly.	The student solves three out of four parts correctly.	The student correctly: a. Answers 8 and explains thinking using pictures, numbers, or words. b. Answers 3 and explains thinking using pictures, numbers, or words.
4 2.OA.3 2.G.2	The student solves zero out of three parts correctly.	The student solves one out of three parts correctly.	The student solves two out of three parts correctly.	The student correctly: a. Answers yes and gives an explanation as to why 14 is even (as stated in 1(b)). b. Completes the array to show 2 rows of 7. c. Answers 7 and explains thinking using pictures, numbers, or words.

Name Roberto

Date _____

1.

- a. Does the picture below show an even or an odd number of teddy bears? Explain your thinking using pictures, numbers, or words in the box on the right.

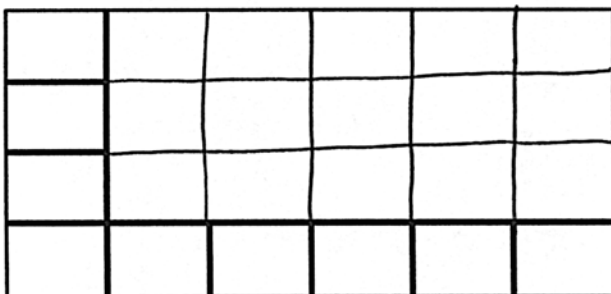
	The number of bears is even because I can pair them up with none left over.

- b. Explain how you know if a number is even.

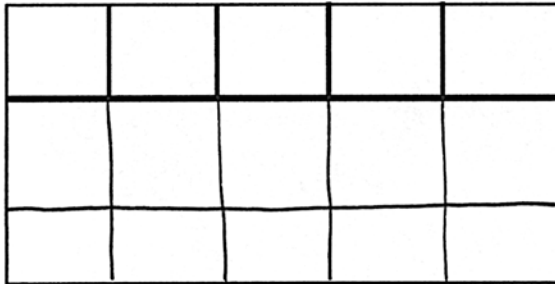
A number is even if you can pair the objects like in Problem 1(a), if you can count by twos to that number, and a number is even if it has 0, 2, 4, 6, or 8 in the ones place.

2.

- a. Complete the array.

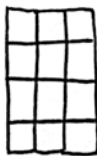


- b. Using the entire rectangle, draw 3 rows of 5 squares. The first row is done for you. Then, write a repeated addition sentence that describes your array.



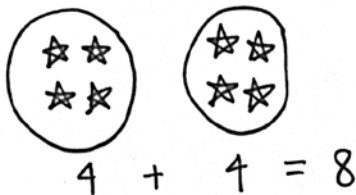
$$5 + 5 + 5 = 15$$

- c. Henry drew the rectangle below using 12 squares. Draw a different rectangle using 12 squares.

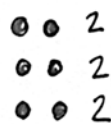


3. Complete each sentence. Explain your thinking using pictures, numbers, or words.

a. 2 groups of 4 make 8.



b. 3 groups of 2 make 6.



$$2 + 2 + 2 = 6$$

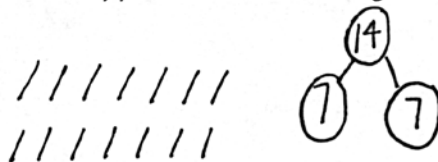
4.

- a. Alex says that 14 is an even number. Do you agree with him? Explain your thinking using pictures, numbers, or words.

I agree. 14 is even because it has a 4 in the ones place. Also $7+7=14$, so it's a double. Doubles are even.

- b. Draw an array using 14 squares in 2 rows. The rows have been drawn for you.

- c. Alex has 14 pencils. He gives all of his pencils to his two friends. Each friend gets the same number of pencils. How many pencils did each friend get? Explain your thinking using pictures, numbers, or words.



$7+7=14$, so each friend got 7 pencils. The array shows how he put the pencils in 2 equal groups. When I counted I found he put 7 pencils in each group.



Answer Key

GRADE 2 • MODULE 6

Foundations of Multiplication and Division

Lesson 1

Core Fluency Practice

Set A

1. 13	11. 19	21. 16	31. 10
2. 16	12. 19	22. 12	32. 15
3. 14	13. 19	23. 14	33. 14
4. 15	14. 16	24. 14	34. 14
5. 18	15. 11	25. 12	35. 15
6. 19	16. 14	26. 13	36. 13
7. 14	17. 16	27. 11	37. 11
8. 17	18. 13	28. 17	38. 12
9. 19	19. 15	29. 11	39. 12
10. 19	20. 16	30. 13	40. 13

Set B

1. 14	11. 19	21. 12	31. 13
2. 19	12. 19	22. 13	32. 5
3. 15	13. 19	23. 7	33. 16
4. 12	14. 11	24. 5	34. 11
5. 15	15. 15	25. 8	35. 12
6. 17	16. 7	26. 7	36. 12
7. 18	17. 7	27. 17	37. 7
8. 5	18. 7	28. 11	38. 8
9. 9	19. 16	29. 15	39. 7
10. 17	20. 12	30. 12	40. 11

Set C

1. 10	11. 9	21. 7	31. 9
2. 10	12. 9	22. 8	32. 7
3. 9	13. 12	23. 8	33. 9
4. 4	14. 9	24. 9	34. 4
5. 10	15. 8	25. 9	35. 3
6. 1	16. 11	26. 9	36. 1
7. 5	17. 8	27. 8	37. 5
8. 10	18. 9	28. 5	38. 3
9. 2	19. 7	29. 8	39. 7
10. 5	20. 7	30. 7	40. 4

Set D

1. 10	11. 8	21. 9	31. 4
2. 2	12. 10	22. 9	32. 6
3. 7	13. 9	23. 9	33. 8
4. 5	14. 7	24. 6	34. 8
5. 5	15. 9	25. 8	35. 7
6. 3	16. 11	26. 3	36. 9
7. 10	17. 9	27. 8	37. 16
8. 10	18. 8	28. 8	38. 3
9. 3	19. 9	29. 5	39. 9
10. 10	20. 7	30. 7	40. 17

Set E

1. 16	11. 8	21. 3	31. 7
2. 20	12. 11	22. 6	32. 2
3. 18	13. 16	23. 7	33. 9
4. 18	14. 13	24. 18	34. 9
5. 15	15. 13	25. 15	35. 19
6. 15	16. 12	26. 20	36. 13
7. 11	17. 11	27. 14	37. 11
8. 8	18. 5	28. 17	38. 15
9. 10	19. 7	29. 12	39. 13
10. 8	20. 6	30. 6	40. 8

Problem Set

1. 5 groups of 2 apples circled; 5	4. 3 groups of 4 oranges circled; 4
2. 5 groups of 3 balls circled; 5	5. 3 groups of 3 flowers drawn; 3, 9
3. 4 groups of 3 oranges circled; 3	6. 2 groups of 10 lemons drawn; 10, 20

Exit Ticket

- 5 groups of 4 hats circled
- 2 groups of 5 smiley faces drawn; 5, 10

Homework

1. 3 groups of 2 shirts circled; 3	4. 4 groups of 3 wheels drawn; 3
2. 4 groups of 3 pants circled; 4	5. 4 groups of 2 apples drawn; 2, 8
3. 3 groups of 4 wheels drawn; 4	6. 3 groups of 5 oranges drawn; 5, 15

Lesson 2

Problem Set

1. a. 2, 2, 2, 6; 2, 6
b. 3, 3, 3, 3, 12; 3, 12
2. 1 group of 4 socks drawn; 4, 4, 4, 4, 4, 20; 4, 20
3. 1 group of 3 presents drawn; 3, 3, 3, 3, 12; 4, 12
4. 2 groups of 2 hearts drawn; 2, 2, 2, 2, 2, 10; 5, 10
5. 3 groups of 5 stars drawn; 5, 5, 5, 15

Exit Ticket

1. 3 rhombuses drawn in the last circle; 3, 3, 3, 3, 12; 3, 12
2. 2 groups of 3 stars drawn; $3 + 3 = 6$

Homework

1. a. 3, 3, 3, 9; 3, 9
b. 2, 2, 2, 2, 8; 2, 8
2. 1 group of 3 cars drawn; 3, 3, 3, 3, 3, 15; 3, 15
3. 1 group of 4 triangles drawn; 4, 4, 4, 4, 16; 4, 16
4. 2 groups of 4 hearts drawn; 4, 4, 4, 4, 4, 20; 5, 20
5. 4 groups of 3 circles drawn; $3 + 3 + 3 + 3 = 12$

Lesson 3

Sprint

Side A

1. 1	12. 3	23. 10	34. 8
2. 2	13. 10	24. 9	35. 10
3. 3	14. 9	25. 8	36. 9
4. 9	15. 8	26. 6	37. 8
5. 10	16. 7	27. 10	38. 7
6. 10	17. 5	28. 9	39. 7
7. 10	18. 10	29. 8	40. 4
8. 10	19. 9	30. 7	41. 9
9. 9	20. 8	31. 5	42. 2
10. 8	21. 7	32. 10	43. 7
11. 7	22. 3	33. 9	44. 6

Side B

1. 10	12. 4	23. 10	34. 8
2. 10	13. 10	24. 9	35. 7
3. 10	14. 9	25. 8	36. 10
4. 10	15. 8	26. 7	37. 9
5. 1	16. 7	27. 5	38. 8
6. 2	17. 4	28. 10	39. 6
7. 3	18. 10	29. 9	40. 9
8. 8	19. 9	30. 8	41. 3
9. 9	20. 8	31. 6	42. 6
10. 8	21. 7	32. 10	43. 3
11. 7	22. 4	33. 9	44. 7

Problem Set

1.
 - a. 3, 3, 3, 3, 12
6, 6, 12
3, 6
 - b. 2, 2, 2, 2, 8
4, 4, 8
2, 4
 - c. 2, 2, 2, 2, 2, 2, 2, 16
4, 4, 4, 4, 16
2, 4
2.
 - a. 3, 3, 3, 3, 3, 15
6, 6, 15
12, 15
 - b. 3, 3, 3, 9
6, 9

Exit Ticket

- 5, 5, 5, 5, 20
10, 10, 20
5, 10

Homework

1.
 - a. 4, 4, 4, 4, 16
8, 8, 16
4, 8
 - b. 2, 2, 2, 2, 8
4, 4, 8
2, 4
 - c. 5, 5, 5, 5, 20
10, 10, 20
5, 10
2.
 - a. 3, 3, 3, 3, 3, 15
6, 6, 15
12, 15
 - b. 2, 2, 2, 2, 2, 10
4, 4, 10
8, 10

Lesson 4

Sprint

Side A

1. 10	12. 17	23. 10	34. 14
2. 11	13. 10	24. 11	35. 10
3. 12	14. 13	25. 12	36. 11
4. 18	15. 14	26. 16	37. 13
5. 10	16. 16	27. 10	38. 10
6. 11	17. 10	28. 11	39. 12
7. 12	18. 16	29. 12	40. 13
8. 17	19. 10	30. 15	41. 10
9. 10	20. 15	31. 10	42. 12
10. 13	21. 10	32. 11	43. 10
11. 14	22. 15	33. 12	44. 11

Side B

1. 10	12. 17	23. 10	34. 13
2. 11	13. 10	24. 11	35. 10
3. 12	14. 13	25. 12	36. 11
4. 16	15. 14	26. 15	37. 12
5. 10	16. 18	27. 10	38. 10
6. 11	17. 10	28. 11	39. 12
7. 12	18. 16	29. 12	40. 14
8. 17	19. 10	30. 14	41. 10
9. 10	20. 15	31. 10	42. 13
10. 13	21. 10	32. 11	43. 10
11. 14	22. 15	33. 12	44. 11

Problem Set

1.
 - a. 2, 2, 2, 2, 8; 8
 - b. 4, 4, 4, 4, 4, 20; 4, 20
 - c. 5, 5, 5, 15; 5, 15
 - d. 3, 3, 3, 3, 3, 18; 6, 3, 18
2.
 - a. 12; tape diagram drawn to show 4 threes
 - b. 12; tape diagram drawn to show 3 fours
 - c. 10; tape diagram drawn to show 5 twos
 - d. 16; tape diagram drawn to show 4 fours
 - e. 15; tape diagram drawn to show 3 fives

Exit Ticket

1. 15; tape diagram drawn to show 3 fives
2. 9; tape diagram drawn to show 3 threes
3. 10; tape diagram drawn to show 5 twos

Homework

1.
 - a. 3, 3, 3, 3, 12; 12
 - b. 5, 5, 5, 5, 5, 25; 5, 25
 - c. 4, 4, 4, 4, 16; 4, 16
 - d. 2, 2, 2, 2, 2, 12; 6, 2, 12
2.
 - a. 20; tape diagram drawn to show 4 fives
 - b. 20; tape diagram drawn to show 5 fours
 - c. 8; tape diagram drawn to show 4 twos
 - d. 15; tape diagram drawn to show 5 threes
 - e. 12; tape diagram drawn to show 3 fours

Lesson 5

Problem Set

- 2 sets of 4 triangles circled; 2 rows of 4 triangles drawn
- 3 sets of 2 smiley faces circled; sets of 2 smiley faces on each row and column drawn
- 4 sets of 3 hearts circled; sets of 3 hearts on each row and column drawn
- 5 rows and 2 columns circled
 - 3 rows and 2 columns circled
- 2 rows of 5 columns of circles drawn; 2 rows of 3 columns of stars drawn
- Answers will vary.
- Answers will vary.

Exit Ticket

- 3 sets of 3 circles circled; 3 rows of 3 circles and 3 columns of 3 circles drawn
- 6 more triangles drawn to complete the array

Homework

- 2 groups of 5 clouds circled; 2 rows of 5 clouds drawn
- 3 groups of 4 diamonds circled; 3 rows and 3 columns of 4 diamonds each drawn
- 4 groups of 4 circles circled; 4 rows and columns of 4 circles each drawn
- 5 rows and 3 columns circled
 - 4 rows and 3 columns circled
- 5 columns of 3 rows of smiley faces drawn; 4 columns of 3 rows of triangles drawn
- Answers will vary.
- Answers will vary.

Lesson 6

Problem Set

1.
 - a. 5 rows of 3 circled; 3, 15; 3, 3, 3, 3, 3, 15
 - b. 3 columns of 5 circled; 5, 15; 5, 5, 5, 15
 - c. 4 rows of 5 circled; 5, 20; 5, 5, 5, 5, 20
 - d. 5 columns of 4 circled; 4, 20; 4, 4, 4, 4, 4, 20
2.
 - a. 3, 4
 - b. 4, 3
 - c. 4, 4, 4, 12
 - d. 1 more row of 4 drawn; 16
 - e. 1 more column of 4 drawn; 20
3.
 - a. 5, 5, 5, 5, 5, 25
 - b. 5, 5, 25
 - c. 5, 5, 25
 - d. 1 row of 5 crossed off; 20
 - e. 1 column of 4 crossed off; 16

Exit Ticket

- a. 3, 4, 12
- b. 4, 3, 12
- c. 3, 3, 3, 3, 12
- d. 1 more row of 4 stars added; 16
- e. 1 more column of 4 stars added; 20

Homework

1.
 - a. 3 rows of 4 circled; 4, 12; 4, 4, 4, 12
 - b. 4 columns of 3 circled; 3, 12; 3, 3, 3, 3, 12
 - c. 5 rows of 3 circled; 3, 15; 3, 3, 3, 3, 3, 15
 - d. 3 columns of 5 circled; 5, 15; 5, 5, 5, 15
2.
 - a. 3, 5, 15
 - b. 5, 3, 15
 - c. 5, 5, 5, 15
 - d. 1 more row of 5 smiley faces added; 20
 - e. 1 more column of 4 smiley faces added; 24
3.
 - a. 3, 3, 3, 3, 12
 - b. 4, 3, 12
 - c. 3, 4, 12
 - d. 1 row of 3 crossed out; 9
 - e. 1 column of 3 crossed out; 6

Lesson 7

Sprint

Side A

1. 11	12. 12	23. 11	34. 13
2. 12	13. 13	24. 12	35. 13
3. 13	14. 17	25. 11	36. 14
4. 16	15. 17	26. 12	37. 14
5. 16	16. 11	27. 11	38. 15
6. 11	17. 15	28. 12	39. 15
7. 12	18. 11	29. 11	40. 12
8. 13	19. 12	30. 15	41. 14
9. 18	20. 14	31. 13	42. 16
10. 18	21. 14	32. 15	43. 18
11. 11	22. 16	33. 15	44. 13

Side B

1. 11	12. 12	23. 11	34. 14
2. 12	13. 13	24. 12	35. 14
3. 13	14. 15	25. 11	36. 15
4. 19	15. 15	26. 12	37. 15
5. 19	16. 11	27. 14	38. 16
6. 11	17. 14	28. 11	39. 16
7. 12	18. 11	29. 12	40. 12
8. 13	19. 12	30. 11	41. 14
9. 17	20. 14	31. 13	42. 16
10. 17	21. 14	32. 13	43. 18
11. 11	22. 18	33. 13	44. 13

Problem Set

1.
 - a. Array of 3 rows of 4 Xs completed
 - b. Array with 3 columns of 4 Xs drawn;
4, 4, 4, 12; 12; 12
2.
 - a. Array with 5 columns of 3 Xs drawn
 - b. Array with 5 rows of 3 Xs drawn;
3, 3, 3, 3, 3, 15; 15; 15
3. Array with 4 rows of 3 Xs drawn; 3, 3, 3, 3, 12; 12
4. Array with 5 rows of 3 Xs drawn;
 $3 + 3 + 3 + 3 + 3 = 15$ or $5 + 5 + 5 = 15$
5. Array with 2 columns of 5 Xs drawn;
 $2 + 2 + 2 + 2 + 2 = 10$ or $5 + 5 = 10$

Exit Ticket

1. Array with 3 rows of 5 Xs drawn; 5, 5, 5, 15; 15
2. Array with 4 rows of 5 Xs drawn; $5 + 5 + 5 + 5 = 20$ or $4 + 4 + 4 + 4 + 4 = 20$

Homework

1.
 - a. Array of 4 rows of 5 Xs completed
 - b. Array with 4 columns of 5 Xs drawn;
5, 5, 5, 5, 20; 20; 20
2.
 - a. Array of 3 columns of 4 Xs drawn
 - b. Array of 3 rows of 4 Xs drawn;
4, 4, 4, 12; 12; 12
3. Array with 3 rows of 3 Xs drawn; 3, 3, 3, 9; 9
4. Array with 5 rows of 3 Xs drawn;
 $3 + 3 + 3 + 3 + 3 = 15$
5. Array with 2 columns of 5 Xs drawn;
 $2 + 2 + 2 + 2 + 2 = 10$ or $5 + 5 = 10$

Lesson 8

Sprint

Side A

1. 1	12. 3	23. 10	34. 8
2. 2	13. 10	24. 9	35. 10
3. 3	14. 9	25. 8	36. 9
4. 9	15. 8	26. 6	37. 8
5. 10	16. 7	27. 10	38. 7
6. 10	17. 5	28. 9	39. 7
7. 10	18. 10	29. 8	40. 4
8. 10	19. 9	30. 7	41. 9
9. 9	20. 8	31. 5	42. 2
10. 8	21. 7	32. 10	43. 7
11. 7	22. 3	33. 9	44. 6

Side B

1. 10	12. 4	23. 10	34. 8
2. 10	13. 10	24. 9	35. 7
3. 10	14. 9	25. 8	36. 10
4. 10	15. 8	26. 7	37. 9
5. 1	16. 7	27. 5	38. 8
6. 2	17. 4	28. 10	39. 6
7. 3	18. 10	29. 9	40. 9
8. 8	19. 9	30. 8	41. 3
9. 9	20. 8	31. 6	42. 6
10. 8	21. 7	32. 10	43. 3
11. 7	22. 4	33. 9	44. 7

Problem Set

1. Array with 2 rows of 5 squares drawn
2. Array with 5 columns of 2 squares drawn
3.
 - a. 5
 - b. 5, 5, 10
 - c. 2
 - d. 2, 2, 2, 2, 2, 10
4.
 - a. 3
 - b. 4
 - c. 4, 4, 4, 12
 - d. 4, 4, 3, 12
5.
 - a. Array with 4 columns of 2 squares drawn
 - b. $2 + 2 + 2 + 2 = 8$ or $4 + 4 = 8$
6.
 - a. Array with 5 columns of 4 squares drawn
 - b. $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$
 - c. Tape diagram drawn

Exit Ticket

1.
 - a. 3
 - b. 5
 - c. 5, 5, 5, 15
 - d. 5, 5, 3, 15
2.
 - a. Array with 2 columns of 5 squares drawn
 - b. $5 + 5 = 10$ or $2 + 2 + 2 + 2 + 2 = 10$

Homework

1. Array with 3 rows of 4 squares drawn
2. Array with 4 columns of 3 squares drawn
3.
 - a. 5
 - b. 5, 5, 5, 15
 - c. 3
 - d. 3, 3, 3, 3, 3, 15
4.
 - a. 2
 - b. 5
 - c. 5, 5, 10
 - d. 5, 5, 2, 10
5.
 - a. Array with 5 columns of 3 squares drawn
 - b. $3 + 3 + 3 + 3 + 3 = 15$ or $5 + 5 + 5 = 15$
6.
 - a. Array with 4 columns of 5 squares drawn
 - b. $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$
 - c. Tape diagram drawn

Lesson 9

Problem Set

1. Array with 5 rows of 3 drawn, $3 + 3 + 3 + 3 + 3 = 15$ or $5 + 5 + 5 = 15$
2. Array with 3 rows of 4 drawn, $4 + 4 + 4 = 12$ or $3 + 3 + 3 + 3 = 12$
3. Array with 3 rows of 5 drawn, $5 + 5 + 5 = 15$ or $3 + 3 + 3 + 3 + 3 = 15$
4. Array with 2 rows of 4 drawn, $4 + 4 = 8$ or $2 + 2 + 2 + 2 = 8$
5. Tape diagram drawn to show 4 fives, $5 + 5 + 5 + 5 = 20$
6. Tape diagram drawn to show 5 fives, $5 + 5 + 5 + 5 + 5 = 25$
7. Tape diagram and array drawn to show 4 fours, $4 + 4 + 4 + 4 = 16$

Exit Ticket

1. Accurate tape diagram or array drawn and repeated addition equation, 12
2. Accurate tape diagram or array drawn and repeated addition equation, 25

Homework

1. Array with 3 columns of 4 drawn, $4 + 4 + 4 = 12$ or $3 + 3 + 3 + 3 = 12$
2. Array with 5 rows of 5 drawn, $5 + 5 + 5 + 5 + 5 = 25$
3. Array with 5 rows of 4 drawn, $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$
4. Array with 4 rows of 4 drawn, $4 + 4 + 4 + 4 = 16$
5. Tape diagram drawn to show 5 fours, $4 + 4 + 4 + 4 + 4 = 20$
6. Tape diagram drawn to show 5 threes, $3 + 3 + 3 + 3 + 3 = 15$
7. Tape diagram and array drawn to show 3 threes, $3 + 3 + 3 = 9$

Lesson 10

Sprint

Side A

1. 10	12. 17	23. 10	34. 14
2. 11	13. 10	24. 11	35. 10
3. 12	14. 13	25. 12	36. 11
4. 18	15. 14	26. 16	37. 13
5. 10	16. 16	27. 10	38. 10
6. 11	17. 10	28. 11	39. 12
7. 12	18. 16	29. 12	40. 13
8. 17	19. 10	30. 15	41. 10
9. 10	20. 15	31. 10	42. 12
10. 13	21. 10	32. 11	43. 10
11. 14	22. 15	33. 12	44. 11

Side B

1. 10	12. 17	23. 10	34. 13
2. 11	13. 10	24. 11	35. 10
3. 12	14. 13	25. 12	36. 11
4. 16	15. 14	26. 15	37. 12
5. 10	16. 18	27. 10	38. 10
6. 11	17. 10	28. 11	39. 12
7. 12	18. 16	29. 12	40. 14
8. 17	19. 10	30. 14	41. 10
9. 10	20. 15	31. 10	42. 13
10. 13	21. 10	32. 11	43. 10
11. 14	22. 15	33. 12	44. 11

Problem Set

1.
 - a. $3 + 3 = 6$
 - b. $3 + 3 = 6$ or $2 + 2 + 2 = 6$
2.
 - a. $2 + 2 + 2 + 2 + 2 = 10$
 - b. $2 + 2 + 2 + 2 + 2 = 10$ or $5 + 5 = 10$
3.
 - a. $3 + 3 + 3 = 9$
 - b. $4 + 4 + 4 + 4 = 16$
4.
 - a. Rectangle
 - b. Shape drawn with one column removed
 - c. Square

Exit Ticket

1.
 - a. Rectangle is appropriately constructed.
 - b. $5 + 5 = 10$
2.
 - a. Rectangle is appropriately constructed.
 - b. $2 + 2 + 2 + 2 + 2 = 10$ or $5 + 5 = 10$

Homework

1.
 - a. $4 + 4 = 8$
 - b. $4 + 4 = 8$ or $2 + 2 + 2 + 2 = 8$
2.
 - a. $2 + 2 + 2 = 6$
 - b. $2 + 2 + 2 = 6$ or $3 + 3 = 6$
3.
 - a. Answers will vary.
 - b. Answers will vary.
4.
 - a. Rectangle
 - b. Shape is appropriately drawn.
 - c. Square
 - d. Answers will vary.

Lesson 11

Sprint

Side A

1. 5	12. 13	23. 8	34. 26
2. 15	13. 23	24. 18	35. 15
3. 25	14. 9	25. 28	36. 25
4. 8	15. 19	26. 8	37. 4
5. 18	16. 29	27. 18	38. 24
6. 28	17. 9	28. 28	39. 13
7. 9	18. 19	29. 7	40. 23
8. 19	19. 29	30. 17	41. 19
9. 29	20. 3	31. 27	42. 19
10. 2	21. 12	32. 6	43. 29
11. 3	22. 21	33. 16	44. 29

Side B

1. 9	12. 12	23. 8	34. 26
2. 19	13. 22	24. 18	35. 5
3. 29	14. 9	25. 28	36. 15
4. 7	15. 19	26. 8	37. 4
5. 17	16. 29	27. 18	38. 24
6. 27	17. 9	28. 28	39. 13
7. 9	18. 19	29. 7	40. 23
8. 19	19. 29	30. 17	41. 29
9. 29	20. 4	31. 27	42. 29
10. 1	21. 13	32. 6	43. 19
11. 2	22. 22	33. 16	44. 19

Problem Set

1.
 - a. 8 square tiles in a row
 - b. Array appropriately constructed
 - c. Answers will vary.
2.
 - a. Array appropriately constructed
 - b. Answers will vary.
 - c. Different array appropriately constructed
 - d. Answers will vary.
3.
 - a. Array appropriately constructed
 - b. Answers will vary.
 - c. Different array appropriately constructed
 - d. Answers will vary.
4.
 - a. 3, 6
 - b. 2, 3
5.
 - a. 5, 10
 - b. 2, 5

Exit Ticket

- a. Array appropriately constructed
- b. Answers will vary.

Homework

1.
 - a. Array appropriately constructed
 - b. Answers will vary.
2.
 - a. Array appropriately constructed
 - b. Answers will vary.
 - c. Different array appropriately constructed
 - d. Answers will vary.
3.
 - a. Array appropriately constructed
 - b. Answers will vary.
 - c. Different array appropriately constructed
 - d. Answers will vary.
4.
 - a. 7, 14
 - b. 7, 2

Lesson 12

Core Fluency Practice

Set A

1. 12	11. 20	21. 16	31. 15
2. 17	12. 18	22. 13	32. 18
3. 15	13. 19	23. 12	33. 12
4. 14	14. 14	24. 14	34. 12
5. 17	15. 11	25. 11	35. 11
6. 14	16. 13	26. 14	36. 16
7. 17	17. 15	27. 12	37. 14
8. 18	18. 17	28. 11	38. 11
9. 19	19. 15	29. 13	39. 17
10. 18	20. 16	30. 15	40. 14

Set B

1. 16	11. 19	21. 11	31. 12
2. 19	12. 16	22. 13	32. 6
3. 17	13. 18	23. 5	33. 14
4. 13	14. 12	24. 4	34. 16
5. 16	15. 15	25. 6	35. 16
6. 20	16. 9	26. 8	36. 12
7. 17	17. 6	27. 12	37. 9
8. 6	18. 4	28. 17	38. 10
9. 3	19. 15	29. 10	39. 5
10. 17	20. 13	30. 12	40. 7

Set C

1. 10	11. 8	21. 8	31. 4
2. 10	12. 11	22. 9	32. 5
3. 5	13. 12	23. 9	33. 7
4. 8	14. 9	24. 8	34. 4
5. 10	15. 9	25. 9	35. 4
6. 1	16. 7	26. 9	36. 3
7. 4	17. 7	27. 9	37. 1
8. 10	18. 9	28. 5	38. 6
9. 3	19. 9	29. 8	39. 6
10. 4	20. 8	30. 7	40. 8

Set D

1. 10	11. 9	21. 9	31. 5
2. 4	12. 8	22. 9	32. 9
3. 8	13. 8	23. 11	33. 7
4. 6	14. 9	24. 8	34. 5
5. 5	15. 7	25. 9	35. 8
6. 2	16. 11	26. 4	36. 9
7. 9	17. 8	27. 7	37. 6
8. 9	18. 9	28. 7	38. 13
9. 7	19. 8	29. 8	39. 9
10. 8	20. 7	30. 8	40. 12

Set E

1. 20	11. 8	21. 6	31. 6
2. 18	12. 13	22. 3	32. 3
3. 17	13. 18	23. 9	33. 8
4. 19	14. 13	24. 19	34. 9
5. 14	15. 12	25. 17	35. 5
6. 16	16. 13	26. 20	36. 13
7. 7	17. 13	27. 12	37. 11
8. 8	18. 5	28. 17	38. 15
9. 9	19. 7	29. 12	39. 13
10. 8	20. 6	30. 7	40. 12

Problem Set

1. Array appropriately drawn; 10; 5, 5, 10	3. a. Array appropriately completed
2. Array appropriately drawn; 12; 3, 3, 3, 3, 12	b. Array appropriately completed
	c. Array appropriately completed

Exit Ticket

Array appropriately drawn

Homework

1. Array appropriately drawn; 8; 4, 4, 8	3. a. Array appropriately completed
2. Array appropriately drawn; 15; 5, 5, 5, 15	b. Array appropriately completed
	c. Array appropriately completed

Lesson 13

Problem Set

1. Number bond shown; $3 + 3 = 6$, $3 + 3 = 6$
2. Number bond shown; $2 + 2 = 4$, $2 + 2 + 2 = 6$
3. Number bond shown; $3 + 3 + 3 = 9$, $3 + 3 = 6$
4.
 - a. 3, 4
 - b. 8
 - c. 6
5.
 - a. Answers will vary.
 - b. Answers will vary.
 - c. Answers will vary.
6.
 - a. Answers will vary.
 - b. Answers will vary.
 - c. Answers will vary.

Exit Ticket

Number bond shown; $4 + 4 = 8$, 4

Homework

1. Number bond shown; $2 + 2 = 4$, $2 + 2 + 2 = 6$
2. Number bond shown; $3 + 3 = 6$, $3 + 3 = 6$
3.
 - a. 3, 3, 9
 - b. 6
 - c. 4
4.
 - a. Answers will vary.
 - b. Answers will vary.
 - c. Answers will vary.

Lesson 14

Sprint

Side A

1. 1	12. 3	23. 10	34. 8
2. 2	13. 10	24. 9	35. 10
3. 3	14. 9	25. 8	36. 9
4. 9	15. 8	26. 6	37. 8
5. 10	16. 7	27. 10	38. 7
6. 10	17. 5	28. 9	39. 7
7. 10	18. 10	29. 8	40. 4
8. 10	19. 9	30. 7	41. 9
9. 9	20. 8	31. 5	42. 2
10. 8	21. 7	32. 10	43. 7
11. 7	22. 3	33. 9	44. 6

Side B

1. 10	12. 4	23. 10	34. 8
2. 10	13. 10	24. 9	35. 7
3. 10	14. 9	25. 8	36. 10
4. 10	15. 8	26. 7	37. 9
5. 1	16. 7	27. 5	38. 8
6. 2	17. 4	28. 10	39. 6
7. 3	18. 10	29. 9	40. 9
8. 8	19. 9	30. 8	41. 3
9. 9	20. 8	31. 6	42. 6
10. 8	21. 7	32. 10	43. 3
11. 7	22. 4	33. 9	44. 7

Problem Set

1.
 - a. 2
 - b. 4
 - c. 2, 4, 8
 - d. 8
2.
 - a. 4
 - b. 2
 - c. 4, 2, 8
 - d. 8
3.
 - a. Rectangle appropriately constructed
 - b. Answers will vary.
 - c. Answers will vary.
 - d. Answers will vary.
4.
 - a. Answers will vary.
 - b. Answers will vary.
 - c. Answers will vary.

Exit Ticket

Answers will vary.

Homework

1.
 - a. Picture drawn appropriately; 4
 - b. Picture drawn appropriately; 3
2. Answers will vary.
3.
 - a. Picture drawn appropriately; 5
 - b. Picture drawn appropriately; 4
4. Answers will vary.

Lesson 15

Sprint

Side A

1. 9	12. 58	23. 15	34. 9
2. 8	13. 6	24. 85	35. 79
3. 18	14. 7	25. 3	36. 8
4. 38	15. 17	26. 4	37. 68
5. 8	16. 67	27. 24	38. 8
6. 9	17. 5	28. 2	39. 78
7. 19	18. 6	29. 3	40. 7
8. 49	19. 16	30. 33	41. 87
9. 7	20. 76	31. 1	42. 9
10. 8	21. 4	32. 2	43. 39
11. 18	22. 5	33. 42	44. 59

Side B

1. 8	12. 68	23. 15	34. 9
2. 18	13. 6	24. 35	35. 89
3. 28	14. 7	25. 3	36. 8
4. 48	15. 17	26. 4	37. 38
5. 8	16. 77	27. 44	38. 8
6. 9	17. 5	28. 2	39. 58
7. 19	18. 6	29. 3	40. 7
8. 59	19. 16	30. 53	41. 77
9. 7	20. 86	31. 1	42. 9
10. 8	21. 4	32. 2	43. 69
11. 18	22. 5	33. 22	44. 49

Problem Set

1. Array appropriately shaded; $3 + 3 = 6$ or $2 + 2 + 2 = 6$
2. Array appropriately shaded; $3 + 3 + 3 + 3 = 12$ or $4 + 4 + 4 = 12$
3. Array appropriately shaded; $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$
4. Column appropriately drawn; $2 + 2 + 2 + 2 + 2 = 10$ or $5 + 5 = 10$
5. Row and column appropriately drawn; $3 + 3 + 3 + 3 + 3 = 15$ or $5 + 5 + 5 = 15$
6. Row and columns appropriately drawn; $5 + 5 + 5 + 5 + 5 = 25$

Exit Ticket

Array appropriately shaded; $5 + 5 + 5 = 15$ or $3 + 3 + 3 + 3 + 3 = 15$

Homework

1. Array appropriately shaded; $2 + 2 + 2 = 6$ or $3 + 3 = 6$
2. Array appropriately shaded; $4 + 4 = 8$ or $2 + 2 + 2 + 2 = 8$
3. Array appropriately shaded; $5 + 5 + 5 + 5 = 20$ or $4 + 4 + 4 + 4 + 4 = 20$
4. Column appropriately drawn; $2 + 2 + 2 + 2 = 8$ or $4 + 4 = 8$
5. Row and column appropriately drawn; $4 + 4 + 4 = 12$ or $3 + 3 + 3 + 3 = 12$
6. Row and columns appropriately drawn; $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$

Lesson 16

Problem Set

1. 9 square tiles and 2 triangles appropriately used to create a design; design appropriately shaded on grid paper
2. 14 square tiles and 4 triangles appropriately used to create a design; design appropriately shaded on grid paper
3. 3 by 3 design appropriately created and copied on grid paper

Exit Ticket

- a. Design appropriately created with paper tiles
- b. Design appropriately shaded on grid paper

Homework

1.
 - a. Design appropriately copied and shaded on the empty grid
 - b. Design appropriately copied and shaded on the empty grid
 - c. Design appropriately copied and shaded on the empty grid
2. Two different designs appropriately created
3. Design appropriately created in the bold square section; design repeated throughout to create a tessellation

Lesson 17

Problem Set

1.
 - a. 1 cloud drawn; 1; 1, 1, 2
 - b. 2 clouds drawn; 2; 2, 2, 4
 - c. 3 clouds drawn; 3; 3, 3, 6
 - d. 4 clouds drawn; 4; 4, 4, 8
 - e. 5 clouds drawn; 5; 5, 5, 10
2.
 - a. 12; 6, 6, 12; 12
 - b. Array appropriately drawn; 14; 7, 7, 14; 14
 - c. Array appropriately drawn; 16; 8, 8, 16; 16
 - d. Array appropriately drawn; 18; 9, 9, 18; 18
 - e. Array appropriately drawn; 20; 10, 10, 20; 20
3.
 - a. 2, 4, 6, 8, 10
 - b. 12, 14, 16, 18, 20
 - c. Even
 - d. Answers will vary.

Exit Ticket

- a. Array appropriately drawn; 10; 5, 5, 10; even circled
- b. Array appropriately drawn; 6; 3, 3, 6; even circled

Homework

1.
 - a. 2 stars drawn; 2; 2, 2, 4
 - b. 4 stars drawn; 4; 4, 4, 8
 - c. 1 star drawn; 1; 1, 1, 2
 - d. 3 stars drawn; 3; 3, 3, 6
 - e. 5 stars drawn; 5; 5, 5, 10
2.
 - a. 12; 6, 6, 12; 12
 - b. Array appropriately drawn; 14; 7, 7, 14; 14
 - c. Array appropriately drawn; 2, 8, 16; 8, 16; 16
 - d. Array appropriately drawn; 18; 9, 9, 18; 18
 - e. Array appropriately drawn; 2, 10, 20; 10, 20; 20
3.
 - a. 4, 6, 2, 8, 10
 - b. 12, 14, 16, 18, 20
 - c. Even
 - d. Answers will vary.

Lesson 18

Sprint

Side A

1. 7	12. 8	23. 2	34. 6
2. 8	13. 9	24. 3	35. 7
3. 9	14. 3	25. 8	36. 9
4. 8	15. 4	26. 2	37. 2
5. 9	16. 5	27. 3	38. 5
6. 5	17. 8	28. 4	39. 6
7. 6	18. 4	29. 8	40. 9
8. 7	19. 5	30. 4	41. 1
9. 9	20. 6	31. 7	42. 4
10. 6	21. 8	32. 9	43. 5
11. 7	22. 1	33. 3	44. 9

Side B

1. 8	12. 6	23. 4	34. 8
2. 9	13. 9	24. 5	35. 3
3. 6	14. 5	25. 9	36. 6
4. 7	15. 6	26. 1	37. 8
5. 8	16. 7	27. 2	38. 2
6. 9	17. 9	28. 3	39. 5
7. 7	18. 2	29. 9	40. 6
8. 8	19. 3	30. 5	41. 8
9. 9	20. 4	31. 8	42. 1
10. 4	21. 9	32. 4	43. 7
11. 5	22. 3	33. 7	44. 8

Problem Set

1. Pairs appropriately circled; even
Pairs appropriately circled; not even
Pairs appropriately circled; even
2. Pattern appropriately drawn
3. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
4. Array appropriately circled
5. Array appropriately boxed
6. a. Dots appropriately redrawn; 10; 10; yes
b. Dots appropriately redrawn; 15; 15; no
7. a. 9; 0
b. 2, 4, 6, 8, 10, 12, 14, 16, 18
c. True

Exit Ticket

1. Dots appropriately redrawn; 13; 13; no
2. Dots appropriately redrawn; 16; 16; yes

Homework

1. Pairs appropriately circled; even
Pairs appropriately circled; not even
Pairs appropriately circled; even
2. Pattern appropriately drawn
3. 18, 16, 14, 12, 10, 8, 6, 4, 2
4. Array appropriately circled
5. Array appropriately boxed
6. Stars appropriately redrawn; 12; 12; yes
7. a. 8; 0
b. 2, 4, 6, 8, 10, 12, 14, 16
c. True

Lesson 19

Sprint

Side A

1. 11	12. 12	23. 11	34. 13
2. 12	13. 13	24. 12	35. 13
3. 13	14. 17	25. 11	36. 14
4. 16	15. 17	26. 12	37. 14
5. 16	16. 11	27. 11	38. 15
6. 11	17. 15	28. 12	39. 15
7. 12	18. 11	29. 11	40. 12
8. 13	19. 12	30. 15	41. 14
9. 18	20. 14	31. 13	42. 16
10. 18	21. 14	32. 15	43. 18
11. 11	22. 16	33. 15	44. 13

Side B

1. 11	12. 12	23. 11	34. 14
2. 12	13. 13	24. 12	35. 14
3. 13	14. 15	25. 11	36. 15
4. 19	15. 15	26. 12	37. 15
5. 19	16. 11	27. 14	38. 16
6. 11	17. 14	28. 11	39. 16
7. 12	18. 11	29. 12	40. 12
8. 13	19. 12	30. 11	41. 14
9. 17	20. 14	31. 13	42. 16
10. 17	21. 14	32. 13	43. 18
11. 11	22. 18	33. 13	44. 13

Problem Set

1. 4, 6, 8, 10, 12, 14, 16, 18, 20
2. a. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
b. Answers will vary.
3. a. 26; 32, 34; 38; 42, 44; 48, 50
b. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21
4. a. Even, odd
b. Even, odd
c. Even, odd
d. Even, odd
e. Even, odd
f. Even, odd
5. a. Even; answers will vary.
b. Odd; answers will vary.
c. Odd; answers will vary.
d. Even; answers will vary.

Exit Ticket

- a. Even; explanation will vary.
- b. Odd; explanation will vary.

Homework

1. 4, 6, 8, 10, 12, 14, 16, 18, 20
2. a. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
b. Answers will vary.
3. 22, 24; 28; 32; 36; 42, 44
4. 1, 3, 5, 7, 9, 11, 13
5. a. Answer provided
b. Odd, even
c. Even, odd
d. Even, odd
e. Even, odd
f. Even, odd
6. a. Odd, explanations will vary.
b. Even, explanations will vary.

Lesson 20

Problem Set

1.
 - a. Array appropriately drawn with 10 circles; even
Picture appropriately drawn with 9 circles; odd
 - b. Array appropriately drawn with 12 circles; even
Picture appropriately drawn with 13 circles; odd
 - c. Array appropriately drawn with 14 circles; even
Picture appropriately drawn with 13 circles; odd
2.
 - a. Answer provided
 - b. 19; O; E; O
 - c. 24; O; O; E
 - d. 22; E; E; E
 - e. 12; O; O; E
 - f. 19; O; E; O
3. Terms and sums will vary.
 - a. Answers are even.
 - b. Answers are odd.
 - c. Answers are even.

Exit Ticket

Array appropriately drawn with 14 circles; even
Picture appropriately drawn with 13 circles; odd

Homework

1.
 - a. Array appropriately drawn with 12 stars; even
Picture appropriately drawn with 11 stars; odd
 - b. Array appropriately drawn with 14 stars; even
Picture appropriately drawn with 15 stars; odd
 - c. Array appropriately drawn with 14 stars; even
Picture appropriately drawn with 13 stars; odd
2.
 - a. 12; E; E; E
 - b. 21; E; O; O
 - c. 24; O; O; E
 - d. 25; O; E; O
 - e. 15; O; E; O
 - f. 20; O; O; E
 - g. 21; O; E; O
 - h. 18; O; O; E
3. Answers will vary.
4. Terms and sums will vary.
 - a. Answers are even.
 - b. Answers are odd.
 - c. Answers are even.