Eureka Math™

Grade 5, Module 4

Teacher Edition

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Printed in the U.S.A.
This book may be purchased from the publisher at eureka-math.org
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Grade 5 • Module 4

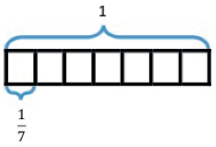
Multiplication and Division of Fractions and Decimal Fractions

OVERVIEW

In Module 4, students learn to multiply fractions and decimal fractions and begin working with fraction division. Topic A opens the 38-day module with an exploration of fractional measurement. Students construct line plots by measuring the same objects using three different rulers accurate to $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch (**5.MD.2**).

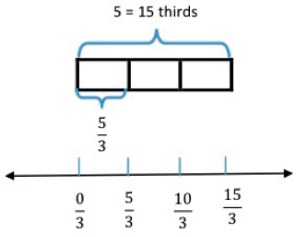
Students compare the line plots and explain how changing the accuracy of the unit of measure affects the distribution of points. This is foundational to the understanding that measurement is inherently imprecise because it is limited by the accuracy of the tool at hand. Students use their knowledge of fraction operations to explore questions that arise from the plotted data. The interpretation of a fraction as division is inherent in this exploration. For measuring to the quarter inch, one inch must be divided into four equal parts, or $1 \div 4$. This reminder of the meaning of a fraction as a point on a number line, coupled with the embedded, informal exploration of fractions as division, provides a bridge to Topic B's more formal treatment of fractions as division.

Topic B focuses on interpreting fractions as division. Equal sharing with area models (both concrete and pictorial) provides students with an opportunity to understand division of whole numbers with answers in the form of fractions or mixed numbers (e.g., seven brownies shared by three girls, three pizzas shared by four people). Discussion also includes an interpretation of remainders as a fraction (**5.NF.3**). Tape diagrams provide a linear model of these problems. Moreover, students see that, by renaming larger units in terms of smaller units, division resulting in a fraction is similar to whole number division.



1 week ÷ 7 = 7 days ÷ 7 = 1 day

1 ÷ 7 = 7 sevenths ÷ 7 = 1 seventh



15 thirds ÷ 3 = 5 thirds

Topic B continues as students solve real-world problems (**5.NF.3**) and generate story contexts for visual models. The topic concludes with students making connections between models and equations while reasoning about their results (e.g., between what two whole numbers does the answer lie?).

$1 \div 7 = \frac{7}{7} \div 7 = \frac{1}{7}$ $5 \div 3 = \frac{5}{3}$

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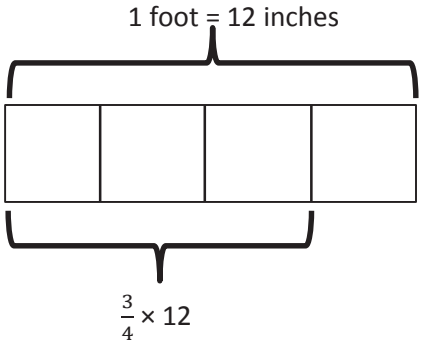
In Topic C, students interpret finding a fraction of a set ($\frac{3}{4}$ of 24) as multiplication of a whole number by a fraction ($\frac{3}{4} \times 24$) and use tape diagrams to support their understandings (**5.NF.4a**). This, in turn, leads students to see division by a whole number as being equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by $\frac{1}{2}$. Students also use the commutative property to relate a fraction of a set to the Grade 4 repeated addition interpretation of multiplication by a fraction. This offers opportunities for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of a fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g., $\frac{1}{3}$ minutes = 20 seconds and $2\frac{1}{4}$ feet = 27 inches).

Interpreting numerical expressions opens Topic D as students learn to evaluate expressions with parentheses, such as $3 \times (\frac{2}{3} - \frac{1}{5})$ or $\frac{2}{3} \times (7 + 9)$ (**5.OA.1**). They then learn to interpret numerical expressions, such as 3 *times the difference between* $\frac{2}{3}$ *and* $\frac{1}{5}$ or *two-thirds the sum of 7 and 9* (**5.OA.2**). Students generate word problems that lead to the same calculation (**5.NF.4a**), such as "Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank $\frac{2}{3}$ of the mixture. How much did Jack drink?" Solving word problems (**5.NF.6**) allows students to apply new knowledge of fraction multiplication in context, and tape diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

Topic E introduces students to multiplication of fractions by fractions—both in fraction and decimal form (**5.NF.4a**, **5.NBT.7**). The topic starts with multiplying a unit fraction by a unit fraction and progresses to multiplying two non-unit fractions. Students use area models, rectangular arrays, and tape diagrams to model the multiplication. These familiar models help students draw parallels between whole number and fraction multiplication, as well as solve word problems. This intensive work with fractions positions students to extend their previous work with decimal-by-whole number multiplication to decimal-by-decimal multiplication. Just as students used unit form to multiply fractional units by wholes in Module 2 (e.g., 3.5 × 2 = 35 tenths × 2 ones = 70 tenths), they connect fraction-by-fraction multiplication to multiply fractional units-by-fractional units (3.5 × 0.2 = 35 tenths × 2 tenths = 70 hundredths).

$\frac{3}{4}$ of a foot = $\frac{3}{4} \times 12$ inches



Express $5\frac{3}{4}$ feet as inches.

$5\frac{3}{4}$ feet = (5×12) inches + $(\frac{3}{4} \times 12)$ inches

= 60 + 9 inches

= 69 inches

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Reasoning about decimal placement is an integral part of these lessons. Finding fractional parts of customary measurements and measurement conversion (**5.MD.1**) concludes Topic E. Students convert smaller units to fractions of a larger unit (e.g., 6 inches = $\frac{1}{2}$ foot). The inclusion of customary units provides a meaningful context for many common fractions (e.g., $\frac{1}{2}$ pint = 1 cup, $\frac{1}{3}$ yard = 1 foot, $\frac{1}{4}$ gallon = 1 quart). This topic, together with the fraction concepts and skills learned in Module 3, opens the door to a wide variety of application word problems (**5.NF.6**).

Students interpret multiplication in Grade 3 as equal groups, and in Grade 4 students begin understanding multiplication as comparison. Here, in Topic F, students once again extend their understanding of multiplication to include scaling (**5.NF.5**). Students compare the product to the size of one factor, given the size of the other factor (**5.NF.5a**) without calculation (e.g., 486 × 1,327.45 is twice as large as 243 × 1,327.45 because 486 = 2 × 243). This reasoning, along with the other work of this module, sets the stage for students to reason about the size of products when quantities are multiplied by numbers larger than 1 and smaller than 1. Students relate their previous work with equivalent fractions to interpreting multiplication by $\frac{n}{n}$ as multiplication by 1 (**5.NF.5b**). Students build on their new understanding of fraction equivalence as multiplication by $\frac{n}{n}$ to convert fractions to decimals and decimals to fractions. For example, $\frac{3}{25}$ is easily renamed in hundredths as $\frac{12}{100}$ using multiplication of $\frac{4}{4}$. The word form of *twelve hundredths* is then used to notate this quantity as a decimal. Conversions between fractional forms are limited to fractions whose denominators are factors of 10, 100, or 1,000. Students apply the concepts of the topic to real-world, multi-step problems (**5.NF.6**).

Topic G begins the work of division with both fractions and decimal fractions. Students use tape diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number (**5.NF.7**). Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering such cases as $5 \div \frac{1}{4}$. They also reason about the size of the unit when $\frac{1}{4}$ is partitioned into 5 equal parts: $\frac{1}{4} \div 5$. Using this thinking as a backdrop, students are introduced to decimal fraction divisors and use equivalent fraction and place value thinking to reason about the size of quotients, calculate quotients, and sensibly place the decimal in quotients (**5.NBT.7**).

The module concludes with Topic H, in which numerical expressions involving fraction-by-fraction multiplication are interpreted and evaluated (**5.OA.1**, **5.OA.2**). Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

The Mid-Module Assessment is administered after Topic D, and the End-of-Module Assessment follows Topic H.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Omit Lesson 4, and use tape diagrams to model fractions as division in Lesson 5. Lessons 11 and 12 are both word problem lessons involving addition, subtraction, and multiplication with fractions. Omit Lesson 11, but include Problems 1 and 4 as part of Lesson 12. In Lesson 12, use Problems 4 and 5 as an extension or challenge for early finishers, and omit Problems 5 and 6 from the Homework. If students have demonstrated success during Lesson 13, consider omitting Problems 1 and 2 of the Concept Development in Lesson 14. Similarly, in Lesson 15, omit Problems 2 and 3(c) from the Concept Development. Omit Lesson 21, and instead, provide regular practice

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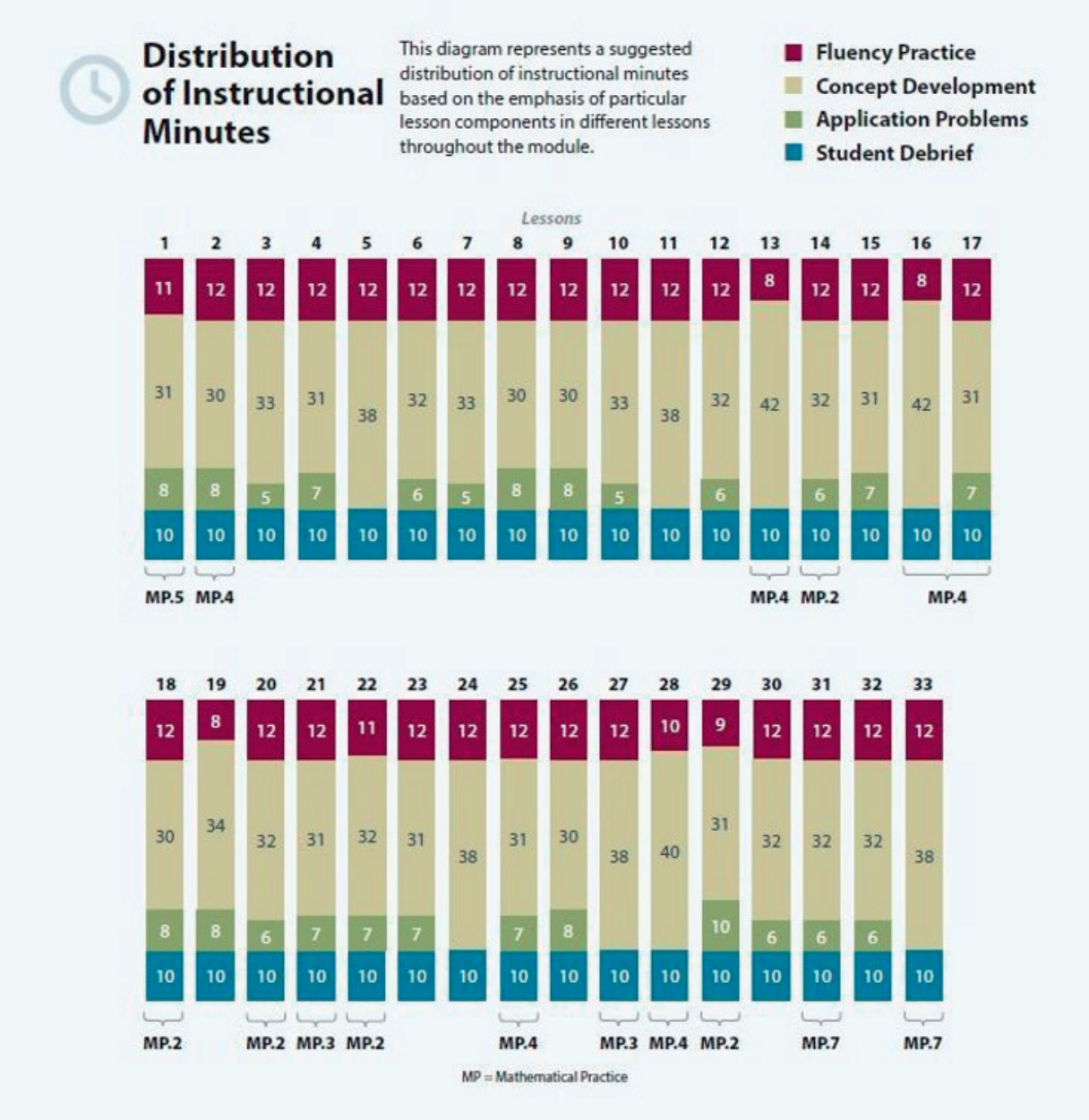
with the "Write Fractions as Decimals" fluency activity, found in Lessons 23, 24, and 25. Lastly, omit Lesson 28.

Note: Looking ahead, Topic D of Module 5 includes drawing in 5 of the 6 geometry lessons. These drawings with the protractor are not included in the Grade 5 CCSS but are critical to the coherence of the geometry standards of Grade 4 and those of middle school. These drawings could be completed during Module 4 but at a different time of the day, such as art class or for morning work. It is best that drawing with the protractor be taught by the math teacher. This modification allows for the later consolidation of Lessons 16, 17, 18, and 19 in Module 5.



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Focus Grade Level Standards

Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 +7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product of $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

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5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students capable of multiplying fractions can generally develop strategies to divide fractions by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade level.)

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

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Foundational Standards

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students capable of generating equivalent fractions can generally develop strategies for adding fractions with unlike denominators. However, addition and subtraction with unlike denominators generally is not a requirement at this grade.) *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

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Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students reason abstractly and quantitatively as they interpret the size of a product in relation to the size of a factor, as well as interpret terms in a multiplication sentence as a quantity and scaling factor. Then, students create a coherent representation of the problem at hand while attending to the meaning of the quantities.

MP.4 **Model with mathematics.** Students model with mathematics as they solve word problems involving multiplication and division of fractions and decimals, as well as identify important quantities in a practical situation and map their relationships using diagrams. Students use a line plot to model measurement data and interpret their results with respect to context of the situation, reflecting on whether results make sense, and possibly improve the model if it has not served its purpose.

MP.5 **Use appropriate tools strategically.** Students use rulers to measure objects to the $\frac{1}{2}$-, $\frac{1}{4}$-, and $\frac{1}{8}$-inch increments, recognizing both the insight to be gained and limitations of this tool as they learn that the actual object may not match the mathematical model precisely.

Overview of Module Topics and Lesson Objectives

Standards		Topics and Objectives	Days
5.MD.2	A	**Line Plots of Fraction Measurements** Lesson 1: Measure and compare pencil lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch, and analyze the data through line plots.	1
5.NF.3	B	**Fractions as Division** Lessons 2–3: Interpret a fraction as division. Lesson 4: Use tape diagrams to model fractions as division. Lesson 5: Solve word problems involving the division of whole numbers with answers in the form of fractions or whole numbers.	4
5.NF.4a **5.MD.1**	C	**Multiplication of a Whole Number by a Fraction** Lesson 6: Relate fractions as division to fraction of a set. Lesson 7: Multiply any whole number by a fraction using tape diagrams. Lesson 8: Relate a fraction of a set to the repeated addition interpretation of fraction multiplication. Lesson 9: Find a fraction of a measurement, and solve word problems.	4

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Standards		Topics and Objectives	Days
5.OA.1 **5.OA.2** **5.NF.4a** **5.NF.6**	D	**Fraction Expressions and Word Problems** Lesson 10: Compare and evaluate expressions with parentheses. Lessons 11–12: Solve and create fraction word problems involving addition, subtraction, and multiplication.	3
		Mid-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
5.NBT.7 **5.NF.4a** **5.NF.6** **5.MD.1** 5.NF.4b	E	**Multiplication of a Fraction by a Fraction** Lesson 13: Multiply unit fractions by unit fractions. Lesson 14: Multiply unit fractions by non-unit fractions. Lesson 15: Multiply non-unit fractions by non-unit fractions. Lesson 16: Solve word problems using tape diagrams and fraction-by-fraction multiplication. Lessons 17–18: Relate decimal and fraction multiplication. Lesson 19: Convert measures involving whole numbers, and solve multi-step word problems. Lesson 20: Convert mixed unit measurements, and solve multi-step word problems.	8
5.NF.5 **5.NF.6**	F	**Multiplication with Fractions and Decimals as Scaling and Word Problems** Lesson 21: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1. Lessons 22–23: Compare the size of the product to the size of the factors. Lesson 24: Solve word problems using fraction and decimal multiplication.	4
5.OA.1 **5.NBT.7** **5.NF.7**	G	**Division of Fractions and Decimal Fractions** Lesson 25: Divide a whole number by a unit fraction. Lesson 26: Divide a unit fraction by a whole number. Lesson 27: Solve problems involving fraction division. Lesson 28: Write equations and word problems corresponding to tape and number line diagrams. Lessons 29: Connect division by a unit fraction to division by 1 tenth and 1 hundredth. Lessons 30–31: Divide decimal dividends by non-unit decimal divisors.	7

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Standards		Topics and Objectives	Days
5.OA.1 **5.OA.2**	H	**Interpretation of Numerical Expressions** Lesson 32: Interpret and evaluate numerical expressions including the language of scaling and fraction division. Lesson 33: Create story contexts for numerical expressions and tape diagrams, and solve word problems.	2
		End-of-Module Assessment: Topics A–H (assessment ½ day, return ½ day, remediation or further applications 2 days)	3
Total Number of Instructional Days			**38**

Terminology

New or Recently Introduced Terms

- Decimal divisor (the number that divides the whole and has units of tenths, hundredths, thousandths, etc.)
- Simplify (using the largest fractional unit possible to express an equivalent fraction)

Familiar Terms and Symbols[1]

- Commutative property (e.g., $4 \times \frac{1}{2} = \frac{1}{2} \times 4$)
- Conversion factor
- Decimal fraction
- Denominator (denotes the fractional unit, e.g., fifths in 3 fifths, which is abbreviated to the 5 in $\frac{3}{5}$)
- Distribute (with reference to the distributive property, e.g., in $1\frac{2}{5} \times 15 = (1 \times 15) + (\frac{2}{5} \times 15)$)
- Divide, division (partitioning a total into equal groups to show how many units in a whole, e.g., $5 \div \frac{1}{5} = 25$)
- Equation (a statement that two expressions are equal, e.g., $3 \times 4 = 6 \times 2$)
- Equivalent fraction
- Expression
- Factors (numbers that are multiplied to obtain a product)
- Foot, mile, yard, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- Fraction greater than or equal to 1 (e.g., $\frac{7}{2}$, $3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)

[1]These are terms and symbols students have seen previously.

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- Fraction written in the largest possible unit (e.g., $\frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ or 1 three out of 2 threes = $\frac{1}{2}$)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in $\frac{3}{5}$)
- Hundredth ($\frac{1}{100}$ or 0.01)
- Line plot
- Mixed number ($3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Numerator (denotes the count of fractional units, e.g., 3 in 3 fifths or 3 in $\frac{3}{5}$)
- Parentheses (symbols ( ) used around a fact or numbers within an equation or expression)
- Quotient (the answer when one number is divided by another)
- Tape diagram (method for modeling problems)
- Tenth ($\frac{1}{10}$ or 0.1)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)
- Whole unit (any unit partitioned into smaller, equally sized fractional units)

Suggested Tools and Representations

- Area models
- Number lines
- Tape diagrams

Scaffolds[2]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

[2]Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

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Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Constructed response with rubric	5.OA.1 5.OA.2 5.NF.3 5.NF.4a 5.NF.6 5.MD.1 5.MD.2
End-of-Module Assessment Task	After Topic H	Constructed response with rubric	5.OA.1 5.OA.2 5.NBT.7 5.NF.3 5.NF.4a 5.NF.5 5.NF.6 5.NF.7 5.MD.1 5.MD.2

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Topic A

Line Plots of Fraction Measurements

5.MD.2

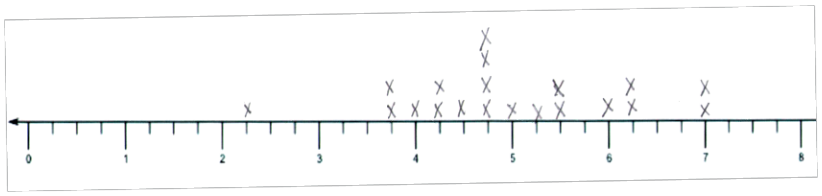
Focus Standard:	5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Instructional Days:	1	
Coherence -Links from:	G4–M5	Fraction Equivalence, Ordering, and Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions

Topic A begins the 38-day module with an exploration of fractional measurement. Students construct line plots by measuring the same objects using three different rulers accurate to $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch (**5.MD.2**). Students compare the line plots and explain how changing the accuracy of the unit of measure affects the distribution of points (see line plots at the end of this page). This is foundational to the understanding that measurement is inherently imprecise because it is limited by the accuracy of the tool at hand.

Students use their knowledge of fraction operations to explore questions that arise from the plotted data, such as "What is the total length of the five longest pencils in our class? Can the half-inch line plot be reconstructed using only data from the quarter-inch line plot? Why or why not?" The interpretation of a fraction as division is inherent within this exploration. To measure to the quarter inch, one inch must be divided into 4 equal parts, or 1 ÷ 4. This reminder of the meaning of a fraction as a point on a number line, coupled with the embedded, informal exploration of fractions as division, provides a bridge to Topic B's more formal treatment of fractions as division.

Pencils measured to $\frac{1}{2}$ inch

Pencils measured to $\frac{1}{4}$ inch



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A Teaching Sequence Toward Mastery of Line Plots of Fraction Measurements

Objective 1: Measure and compare pencil lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch, and analyze the data through line plots.
(Lesson 1)

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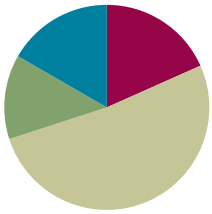
Lesson 1

Objective: Measure and compare pencil lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch, and analyze the data through line plots.

Suggested Lesson Structure

Fluency Practice	(11 minutes)
Application Problem	(8 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (11 minutes)

- Compare Fractions **4.NF.2** (4 minutes)
- Decompose Fractions **4.NF.3** (4 minutes)
- Equivalent Fractions **4.NF.1** (3 minutes)

Compare Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity review prepares students for this lesson's Concept Development.

T: (Project a tape diagram labeled as one whole and partitioned into 2 equal parts. Shade 1 of the parts.) Say the fraction.

S: 1 half.

T: (Write $\frac{1}{2}$ to the right of the tape diagram. Directly below the tape diagram, project another tape diagram partitioned into 4 equal parts. Shade 1 of the parts.) Say this fraction.

S: 1 fourth.

T: (Write $\frac{1}{2}$ _____ $\frac{1}{4}$ to the right of the tape diagrams.) On your personal white board, use the greater than, less than, or equal sign to compare.

S: (Write $\frac{1}{2} > \frac{1}{4}$.)

Continue with the following possible suggestions: $\frac{1}{2}$ — $\frac{1}{8}$, $\frac{1}{8}$ — $\frac{1}{4}$, $\frac{1}{2}$ — $\frac{1}{4}$, $\frac{1}{3}$ — $\frac{1}{4}$, $\frac{2}{4}$ — $\frac{3}{6}$, $\frac{3}{4}$ — $\frac{3}{8}$, $\frac{2}{5}$ — $\frac{2}{3}$, $\frac{3}{10}$ — $\frac{3}{8}$, and $\frac{2}{3}$ — $\frac{6}{9}$.

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Decompose Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity review prepares students for this lesson's Concept Development.

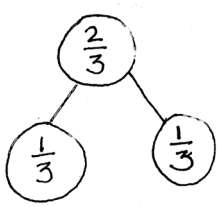
T: (Write a number bond with $\frac{2}{3}$ as the whole and $\frac{1}{3}$ as the part.) Say the whole.

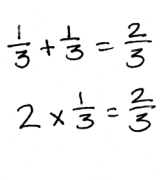
S: 2 thirds.

T: Say the given part.

S: 1 third.

T: On your personal white board, write the number bond. Fill in the missing part.





S: (Write $\frac{1}{3}$ as the missing part.)

T: Write an addition sentence to match the number bond.

S: $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$.

T: Write a multiplication sentence to match the number bond.

S: $2 \times \frac{1}{3}=\frac{2}{3}$.

Continue with the following possible suggestions: $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$, $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$, and $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$.

Equivalent Fractions (3 minutes)

T: (Write $\frac{1}{2}$.)

T: Say the fraction.

S: 1 half.

T: (Write $\frac{1}{2}=\frac{\ }{4}$.)

T: 1 half is equal to how many fourths?

S: 2 fourths.

Continue with the following possible sequence: $\frac{1}{2}=\frac{\ }{6}$, $\frac{1}{3}=\frac{\ }{6}$, $\frac{2}{3}=\frac{\ }{12}$, $\frac{3}{4}=\frac{\ }{16}$, and $\frac{3}{5}=\frac{\ }{25}$.

T: (Write $\frac{1}{2}$.)

T: Say the fraction.

S: 1 half.

T: (Write $\frac{1}{2}=\frac{2}{\ }$.)

T: 1 half, or 1 part of 2, is the same as 2 parts of what unit?

S: Fourths.

Continue with the following possible sequence: $\frac{1}{2}=\frac{2}{\ }$, $\frac{1}{5}=\frac{2}{\ }$, $\frac{2}{5}=\frac{8}{\ }$, $\frac{3}{4}=\frac{9}{\ }$, and $\frac{4}{5}=\frac{16}{\ }$.

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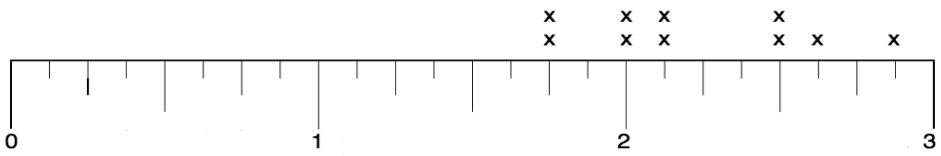
Application Problem (8 minutes)

The following line plot shows the growth, in inches, of 10 bean plants during their second week after sprouting:

Bean Growth in Inches During Week Two



a) $1\frac{3}{4}$ in.
b) 2 plants
c) $2\frac{7}{8}$ in.
d) $2\frac{7}{8} - 1\frac{3}{4}$
$= 1\frac{7}{8} - \frac{3}{4}$
$= 1\frac{7}{8} - \frac{6}{8}$
$= 1\frac{1}{8}$ in.

a. What is the measurement of the shortest plant?
b. How many plants measure $2\frac{1}{2}$ inches?
c. What is the measurement of the tallest plant?
d. What is the difference between the longest and shortest measurements?

Note: This Application Problem provides an opportunity for a quick, formative assessment of students' ability to read a customary ruler and simple line plot. Because today's lesson is time intensive, the analysis of this plot data is simple.

Concept Development (31 minutes)

Materials: (S) Inch ruler, Problem Set, $8\frac{1}{2}$" × 1" strip of paper (with straight edges) per student

Note: Before beginning the lesson, draw three number lines, one beneath the other, on the board. The lines should be marked 0–8 with increments of halves, fourths, and eighths, respectively. Leave plenty of room to position the three line plots directly beneath each other. Students compare these line plots later during the lesson.

T: Cut the strip of paper so that it is the same length as your pencil.
S: (Measure and cut.)
T: Estimate the length of your pencil strip to the nearest inch, and record your estimate on the first line in your Problem Set.
T: If I ask you to measure your pencil strip to the nearest half inch, what do I mean?
S: I should measure my pencil and see which half-inch or whole-inch mark is closest to the length of my strip. → When I look at the ruler, I have to pay attention to the marks that split the inches into 2 equal parts and then look for the one that is closest to the length of my strip. → I know that I will give a measurement that is either a whole number or a measurement with a half in it.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Use colored paper for the pencil measurements to help students see where their pencil paper aligns on the rulers.

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Discuss with students what they should do if their pencil strips are between two marks (e.g., 6 and $6\frac{1}{2}$). Remind students to round up any measurement that is more than halfway.

T: Use your ruler to measure your strip to the nearest half inch. Record your measurement by placing an X on the picture of the ruler in Problem 2 on your Problem Set.

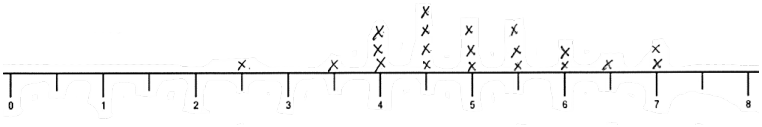
T: Was the measurement to the nearest half inch accurate? Let's find out. Raise your hand if your actual length was on or very close to one of the half-inch markings on your ruler.

S: (Raise hands.)

T: It seems that most of us had to round our measurement to mark it on the sheet. Let's record everyone's measurements on a line plot. As each person calls out his measurement, I'll record the measurements on the board as you record the measurements on your Problem Set. (Poll the students.)

A typical class line plot might look similar to the following:

Class Pencil Lengths (Nearest Half Inch)



T: Which pencil measurement is the most common, or frequent, in our class? Turn and talk.

Answers will vary by class. In the plot above, $4\frac{1}{2}$ inches is most frequent.

T: Are all of the pencils used for these measurements *exactly* the same length? (Point to the X's above the most frequent data point: $4\frac{1}{2}$ inches on the exemplar line plot.) Are they exactly $4\frac{1}{2}$ inches long?

S: No. These measurements are to the nearest half inch. → The pencils are different sizes. We had to round the measurement of some of them. → My partner and I had pencils that were different lengths, but they were close to the same mark. We had to put our marks on the same place on the sheet even though they weren't really the same length.

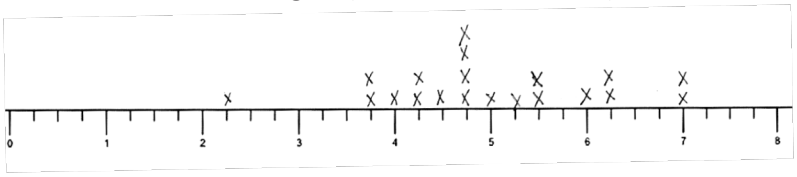
MP.5

T: Now, let's measure our strips to the nearest quarter inch. How is measuring to the quarter inch different from measuring to the half inch? Turn and talk.

S: The whole is divided into 4 equal parts instead of just 2 equal parts. → Quarter inches are smaller than half inches. → Measuring to the nearest quarter inch gives us more choices about where to put our X's on the ruler.

Follow the same sequence of measuring and recording the strips to the nearest quarter inch. The line plot might look similar to the following:

Class Pencil Lengths (Nearest Quarter Inch)



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T: Which pencil measurement is the most frequent this time?

Answers will vary by class. The most frequent above is $4\frac{3}{4}$ inches.

T: If the length of our strips didn't change, why is the most frequent measurement different this time? Turn and talk.

S: The unit on the ruler we used to measure and record was different. → The smaller units made it possible for me to get closer to the real length of my strip. → I rounded to the nearest quarter inch, so I had to move my X to a different mark on the ruler. Other people probably had to do the same thing.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Write math vocabulary words on sentence strips, and display as they are used in context (e.g., *precise, accurate*).

T: Yes, the ruler with smaller units (every quarter inch instead of every half inch) allowed us to be more precise with our measurement. This ruler (point to the $\frac{1}{4}$-inch plot on the board) has more fractional units in a given length, which allows for a more precise measurement. It's a bit like when we round a number by hundreds or tens. Which rounded number will be closer to the actual number? Why? Turn and talk.

S: When we round to the tens place, we can be closer to the actual number because we are using smaller units.

T: That's exactly what's happening here when we measure to the nearest quarter inch versus the nearest half inch. How did your measurements change or not change?

S: My first was 4 inches, but my second was closer to 4 and a quarter inches. → My first measurement was 4 and a half inches. My second was 4 and 2 quarter inches, but that's the same as 4 and a half inches. → When I measured with the half-inch ruler, my first was closer to 4 inches than 3 and a half inches, but when I measured with the fourth-inch ruler, it was closer to 3 and 3 quarter inches. This was because it was a little closer to 3 and 3 quarter inches than 4 inches.

T: Our next task is to measure our strips to the nearest eighth of an inch and record our data in a third line plot. Look at the first two line plots. What do you think the shape of the third line plot will look like? Turn and talk.

S: The line plot will be flatter than the first two. → There are more choices for our measurements on the ruler, so I think that there will be more places where there will only be one X compared to the other line plots. → The eighth-inch ruler will show the differences between pencil lengths more than the half-inch or fourth-inch rulers.

Follow a similar sequence for measuring and recording this line plot.

T: Let's find out how accurate our measurements are. Raise your hand if your actual strip length was on or very close to one of the eighth-inch markings on the ruler. (It is likely that many more students raise their hands than before.)

S: (Raise hands.)

T: Work with your partner to answer Problem 5 on your Problem Set.

Recording the line plots on the board for later analysis with the class is highly recommended.

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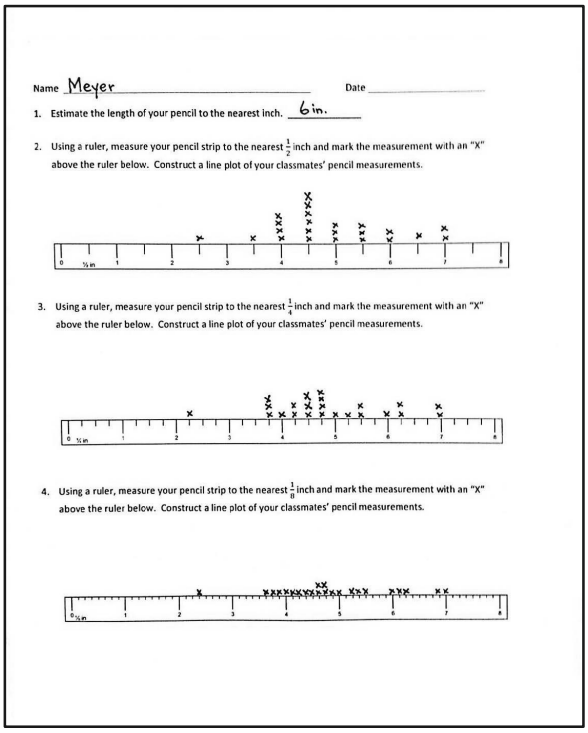
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.



Name Meyer Date

1. Estimate the length of your pencil to the nearest inch. 6in.
2. Using a ruler, measure your pencil strip to the nearest $\frac{1}{2}$ inch and mark the measurement with an "X" above the ruler below. Construct a line plot of your classmates' pencil measurements.
3. Using a ruler, measure your pencil strip to the nearest $\frac{1}{4}$ inch and mark the measurement with an "X" above the ruler below. Construct a line plot of your classmates' pencil measurements.
4. Using a ruler, measure your pencil strip to the nearest $\frac{1}{8}$ inch and mark the measurement with an "X" above the ruler below. Construct a line plot of your classmates' pencil measurements.

Student Debrief (10 minutes)

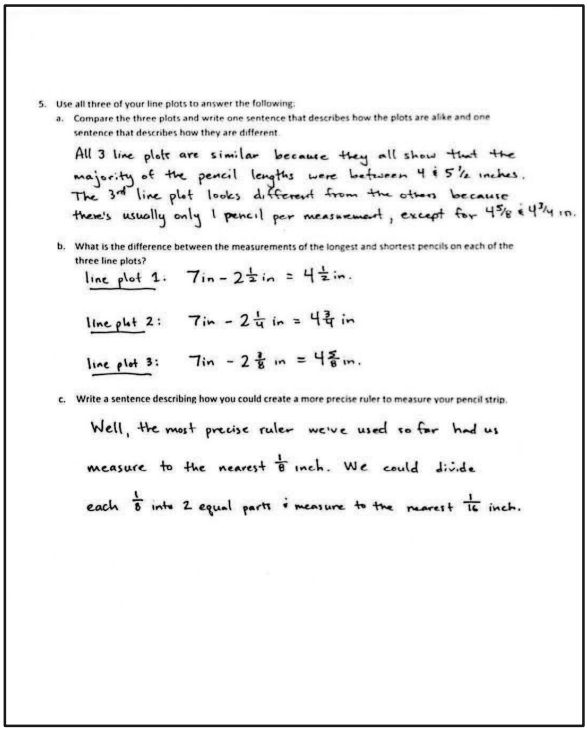
Lesson Objective: Measure and compare pencil lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an inch, and analyze the data through line plots.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion. However, it is recommended that the first bullet be a focus for this lesson's discussion.

- How many of you had a pencil length that didn't fall directly on an inch, a half-inch, a quarter-inch, or an eighth-inch marking?
  - If you wanted a more precise measurement of your pencil's length, what could you do? (Guide students to see that they could choose smaller fractional units.)



5. Use all three of your line plots to answer the following:
   a. Compare the three plots and write one sentence that describes how the plots are alike and one sentence that describes how they are different.

   All 3 line plots are similar because they all show that the majority of the pencil lengths were between 4 & 5 ½ inches. The 3rd line plot looks different from the others because there's usually only 1 pencil per measurement, except for 4⅝ & 4¾ in.

   b. What is the difference between the measurements of the longest and shortest pencils on each of the three line plots?

   line plot 1: 7in - 2½ in = 4½ in.

   line plot 2: 7in - 2¼ in = 4¾ in

   line plot 3: 7in - 2⅜ in = 4⅝ in.

   c. Write a sentence describing how you could create a more precise ruler to measure your pencil strip.

   Well, the most precise ruler we've used so far had us measure to the nearest ⅛ inch. We could divide each ⅛ into 2 equal parts & measure to the nearest 1/16 inch.

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- When someone tells you, "My pencil is 5 and 3 quarters inches long," is it reasonable to assume that her pencil is *exactly* that long? (Guide students to see that, in practice, all measurements are approximations, even when assuming they are exact for the sake of calculation.)

- How does the most frequent pencil length change with each line plot? How does the number of each pencil length for each data point change with each line plot? Which line plot had the most repeated lengths? Which had the fewest repeated lengths?
- What is the effect of changing the precision of the ruler? What happens when you split the wholes on the ruler into smaller and smaller units?
- If you only know the data from the second line plot, can you reconstruct the first line plot? (No. An X at $3\frac{3}{4}$ inches on the second line plot could represent a pencil as short as $3\frac{1}{2}$ inches or as long as 4 inches on the first line plot. However, if an X is on a half-inch mark—3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, etc.,—on the second line plot, then we know that it is at the same half-inch mark on the first line plot.)
- Can the first line plot be completely reconstructed knowing only the data from the third line plot? (Generally, no. However, more of the first line plot can be reconstructed from the third line plot rather than the second line plot.)
- The following can accommodate high-performing students: Which points on the third line plot can be used, and which ones cannot be used to reconstruct the first line plot?
- Which line plot contains the most accurate measurements? Why? Why are smaller units generally more accurate?
- Are smaller units always the better choice when measuring? (Guide students to see that different applications require varying degrees of accuracy. Smaller units allow for greater accuracy, but greater accuracy is not always required.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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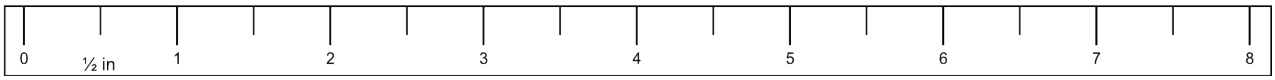
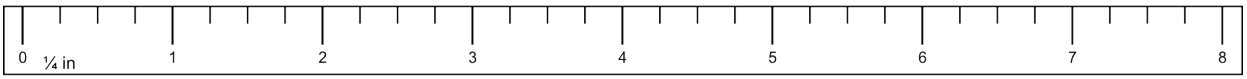
Name _____ Date _____

1. Estimate the length of your pencil to the nearest inch. _____

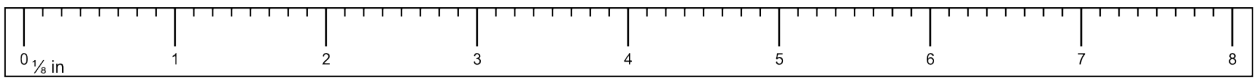
2. Using a ruler, measure your pencil strip to the nearest $\frac{1}{2}$ inch, and mark the measurement with an X above the ruler below. Construct a line plot of your classmates' pencil measurements.



3. Using a ruler, measure your pencil strip to the nearest $\frac{1}{4}$ inch, and mark the measurement with an X above the ruler below. Construct a line plot of your classmates' pencil measurements.



4. Using a ruler, measure your pencil strip to the nearest $\frac{1}{8}$ inch, and mark the measurement with an X above the ruler below. Construct a line plot of your classmates' pencil measurements.



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5. Use all three of your line plots to complete the following:
   a. Compare the three plots, and write one sentence that describes how the plots are alike and one sentence that describes how they are different.

   b. What is the difference between the measurements of the longest and shortest pencils on each of the three line plots?

   c. Write a sentence describing how you could create a more precise ruler to measure your pencil strip.

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Name _____ Date _____

1. Draw a line plot for the following data measured in inches:

   $1\frac{1}{2}, 2\frac{3}{4}, 3, 2\frac{3}{4}, 2\frac{1}{2}, 2\frac{3}{4}, 3\frac{3}{4}, 3, 3\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$

2. Explain how you decided to divide your wholes into fractional parts and how you decided where your number scale should begin and end.

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Name _____ Date _____

A meteorologist set up rain gauges at various locations around a city and recorded the rainfall amounts in the table below. Use the data in the table to create a line plot using $\frac{1}{8}$ inches.

a. Which location received the most rainfall?

b. Which location received the least rainfall?

c. Which rainfall measurement was the most frequent?

d. What is the total rainfall in inches?

Location	Rainfall Amount (inches)
1	$\frac{1}{8}$
2	$\frac{3}{8}$
3	$\frac{3}{4}$
4	$\frac{3}{4}$
5	$\frac{1}{4}$
6	$1\frac{1}{4}$
7	$\frac{1}{8}$
8	$\frac{1}{4}$
9	1
10	$\frac{1}{8}$

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Topic B

Fractions as Division

5.NF.3

Focus Standard:	5.NF.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
Instructional Days:	4	
Coherence -Links from:	G4–M5	Fraction Equivalence, Ordering, and Operations
	G4–M6	Decimal Fractions
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions

Topic B focuses on interpreting fractions as division. Equal sharing with area models (both concrete and pictorial) provides students with an opportunity to understand the division of whole numbers with answers in the form of fractions or mixed numbers (e.g., seven brownies shared by three girls, three pizzas shared by four people). Discussion also includes an interpretation of remainders as a fraction (**5.NF.3**). Tape diagrams provide a linear model of these problems. Moreover, students see that, by renaming larger units in terms of smaller units, division resulting in a fraction is similar to whole number division.

Topic B continues as students solve real-world problems (**5.NF.3**) and generate story contexts for visual models. The topic concludes with students making connections between models and equations while reasoning about their results (e.g., between what two whole numbers does the answer lie?).

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A Teaching Sequence Toward Mastery of Fractions as Division

Objective 1: Interpret a fraction as division.
(Lessons 2–3)

Objective 2: Use tape diagrams to model fractions as division.
(Lesson 4)

Objective 3: Solve word problems involving the division of whole numbers with answers in the form of fractions or whole numbers.
(Lesson 5)

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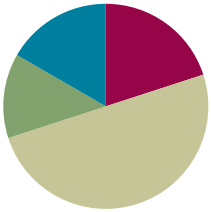
Lesson 2

Objective: Interpret a fraction as division.

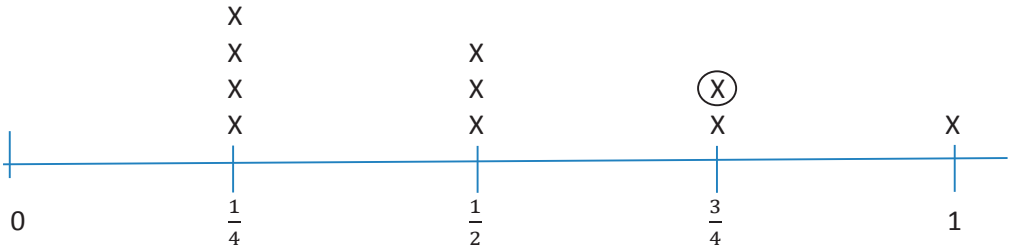
Suggested Lesson Structure



Application Problem	(8 minutes)
Fluency Practice	(12 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Application Problem (8 minutes)

The line plot shows the number of miles run by Noland in his PE class last month, which is rounded to the nearest quarter mile.



a. If Noland ran once a day, how many days did he run?

b. How many miles did Noland run altogether last month?

c. Look at the circled data point. The actual distance Noland ran that day was at least ____ mile and less than ____ mile.

Note: This Application Problem reinforces the work of the previous lesson. Part (c) provides an extension for early finishers.

a. He ran for 10 days.

b. $(4 \times \frac{1}{4}) + (3 \times \frac{1}{2}) + (2 \times \frac{3}{4}) + 1$
$= \frac{4}{4} + \frac{3}{2} + \frac{6}{4} + 1$
$= 1 + 1\frac{1}{2} + 1\frac{1}{2} + 1$
$= 5$
Noland ran 5 miles altogether.

c. The actual distance Noland ran that day was at least $\frac{3}{4}$ mile and less than 1 mile.

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Fluency Practice (12 minutes)

- Factors of 100 **4.NF.5** (2 minutes)
- Compare Fractions **4.NF.2** (4 minutes)
- Decompose Fractions **4.NF.3** (3 minutes)
- Divide with Remainders **5.NF.3** (3 minutes)

Factors of 100 (2 minutes)

Note: This fluency activity prepares students for fractions with denominators of 4, 20, 25, and 50 in Topic G.

T: (Write 50 × ____ = 100.) Say the equation, filling in the missing factor.
S: 50 × 2 = 100.

Continue with the following possible sequence: 25 × ____ = 100, 4 × ____ = 100, 20 × ____ = 100, and 50 × ____ = 100.

T: I'm going to say a factor of 100. You say the other factor that will make 100.
T: 20.
S: 5.

Continue with the following possible sequence: 25, 50, 5, 10, and 4.

Compare Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews concepts from Grade 4 and Grade 5 Module 3.

T: (Project a tape diagram partitioned into 2 equal parts. Shade 1 of the parts.) Say the fraction.
S: 1 half.
T: (Write $\frac{1}{2}$ to the right of the tape diagram. Directly below the first tape diagram, project another tape diagram partitioned into 4 equal parts. Shade 3 of the parts.) Say this fraction.
S: 3 fourths.
T: What's a common unit that we could use to compare these fractions?
S: Fourths. → Eighths. → Twelfths.
T: Let's use fourths. (Below the tape, write $\frac{1}{2}$ ____ $\frac{3}{4}$ and ____ $\frac{3}{4}$.) On your personal white board, write in the unknown numerator and a greater than or less than symbol.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For English language learners or students who must review the relative size of fractional units, folding square paper into various units of halves, thirds, fourths, and eighths can be beneficial. Allow students time to fold, cut, label, and compare the units in relation to the whole and each other.

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S: (Write $\frac{2}{4} < \frac{3}{4}$.)

Continue with, and compare, the following possible sequence: $\frac{1}{2}$ and $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{4}$, and $\frac{3}{4}$ and $\frac{7}{8}$.

Decompose Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews concepts from Grade 4 and Grade 5 Module 3.

T: (Write a number bond with $\frac{3}{5}$ as the whole and 3 missing parts.) On your personal white board, break apart 3 fifths into unit fractions.

S: (Write $\frac{1}{5}$ for each missing part.)

T: Say the multiplication equation for this bond.

S: $3 \times \frac{1}{5} = \frac{3}{5}$.

Continue with the following possible sequence: $\frac{2}{3}$, $\frac{3}{10}$, and $\frac{5}{8}$.

Divide with Remainders (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for this lesson's Concept Development.

T: (Write 8 ÷ 2 = ____.) Say the quotient.

S: 4.

T: Say the remainder.

S: There isn't one. → 0.

T: (Write 9 ÷ 2 = ____.) Quotient?

S: 4.

T: Remainder?

S: 1.

Continue with the following possible sequence: 25 ÷ 5, 27 ÷ 5, 9 ÷ 3, 10 ÷ 3, 16 ÷ 4, 19 ÷ 4, 12 ÷ 6, and 11 ÷ 6.

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Concept Development (30 minutes)

Materials: (S) Personal white board, 15 square pieces of paper per pair of students

Problem 1

2 ÷ 2

1 ÷ 2

1 ÷ 3

2 ÷ 3

T: Imagine we have 2 crackers. Use two pieces of your paper to represent the crackers. Share the crackers equally between 2 people.

S: (Distribute 1 cracker per person.)

T: How many crackers did each person get?

S: 1 cracker.

T: Say a division sentence that tells what you just did with the crackers.

S: 2 ÷ 2 = 1.

T: I'll record that with a drawing. (Draw the 2 ÷ 2 = 1 image on the board.)

T: Now, imagine that there is only 1 cracker to share between 2 people. Use your paper and scissors to show how you would share the cracker.

S: (Cut the paper into halves.)

T: How much will each person get?

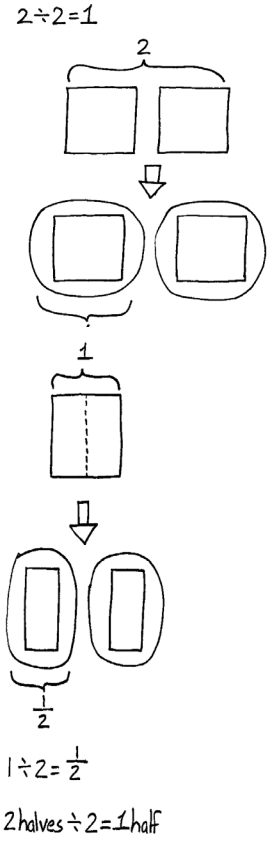
MP.4

S: 1 half of a cracker.

T: Work with your partner to write a number sentence that shows how you shared the cracker equally.

S: $1 \div 2 = \frac{1}{2}$. → $\frac{2}{2} \div 2 = \frac{1}{2}$. → 2 halves ÷ 2 = 1 half.

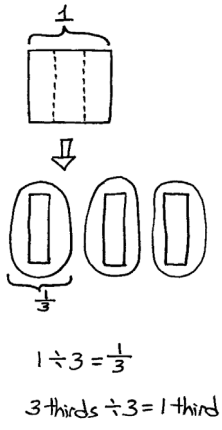
T: I'll record your thinking on the board with another drawing. (Draw the 1 ÷ 2 model, and write the number sentence beneath it.)



Repeat this sequence with 1 ÷ 3.

T: (Point to both division sentences on the board.) Look at these two number sentences. What do you notice? Turn and talk.

S: Both problems start with 1 whole, but it gets divided into 2 parts in the first problem and 3 parts in the second one. → I noticed that both of the answers are fractions, and the fractions have the same digits in them as the division expressions. → When you share the same size whole with 2 people, you get more than when you share it with 3 people. → The fraction looks a lot like the division expression, but it's the amount that each person receives out of the whole.



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T: (Point to the number sentences.) We can write the division expression as a fraction. 1 divided by 2 is the same as 1 half. 1 divided by 3 is the same as 1 third.

T: Let's consider sharing 2 crackers with 3 people. Thinking about 1 divided by 3, how much do you think each person would receive? Turn and talk.

S: It's double the amount of crackers shared with the same number of people. Each person should receive twice as much as before, so they should receive 2 thirds. → The division sentence can be written similarly to a fraction, so 2 divided by 3 would be the same as 2 thirds.

T: Use your materials to show how you would share 2 crackers with 3 people.

S: (Work.)

NOTES ON MULTIPLE MEANS OF EXPRESSION:

Students with fine motor deficits may find the folding and cutting of the concrete materials difficult. Consider allowing them to serve as reporters for their learning groups to share the findings or allowing them to use online virtual manipulatives.

Problem 2

3 ÷ 2

T: Now, let's take 3 crackers and share them equally with 2 people. (Draw 3 squares on the board. Underneath the squares, draw 2 circles to represent the portion each person receives.) Turn and talk about how you can share these crackers. Use your materials to show your thinking.

S: I have 3 crackers, so I can give 1 whole cracker to both people. Then, I'll just have to split the third cracker into halves and share it. → Since there are 2 people, we could cut each cracker into 2 parts and then share them equally that way.

T: Let's record these ideas by drawing. We have 3 crackers. I heard someone say that there is enough for each person to receive a whole cracker. Draw a whole cracker in each circle.

S: (Draw.)

T: How many crackers remain?

S: 1 cracker.

T: What must we do with the remaining cracker if we want to continue sharing equally?

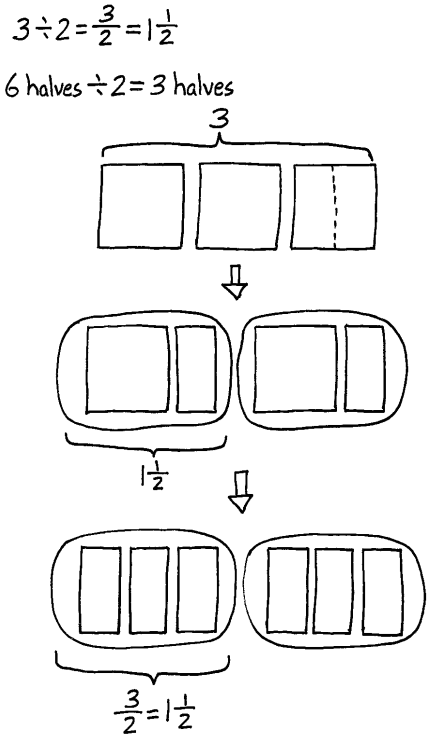
S: Divide it into 2 equal parts. → Split it in half.

T: How many halves will each person receive?

S: 1 half.

T: Record that by drawing one-half of the cracker within each circle. How many crackers did each person receive?

S: 1 and $\frac{1}{2}$ crackers.



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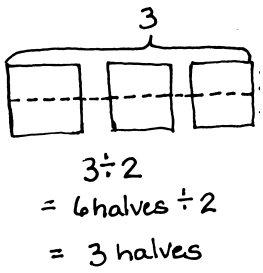
T: (Write $3 \div 2 = 1\frac{1}{2}$ beneath the drawing.) How many halves are in 1 and 1 half?



S: 3 halves.

T: (Write $\frac{3}{2}$ next to the equation.) I noticed that some of you cut the crackers into 2 equal parts before you began sharing. Let's draw that way of sharing. (Redraw 3 wholes. Divide them into halves horizontally.) How many halves were in 3 crackers?

S: 6 halves.

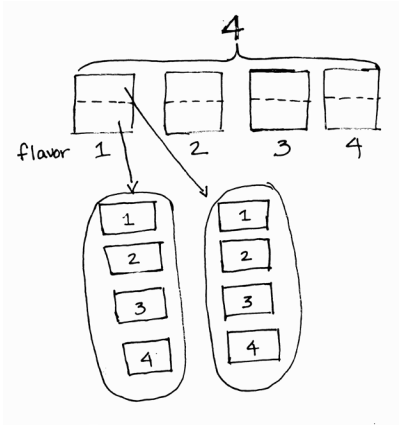
T: What's 6 halves divided by 2? Draw it.

S: (Draw.) 3 halves.

Problem 3

$4 \div 2$

$5 \div 2$



T: Imagine 4 crackers shared with 2 people. How many would each person receive?

S: 2 crackers.

T: (Write $4 \div 2 = 2$ on the board.) Let's now imagine that all four crackers are different flavors, and both people would like to taste all of the flavors. How could we share the crackers equally to make that possible? Turn and talk.

S: To be sure everyone got a taste of all 4 crackers, we would need to split all of the crackers in half first and then share.

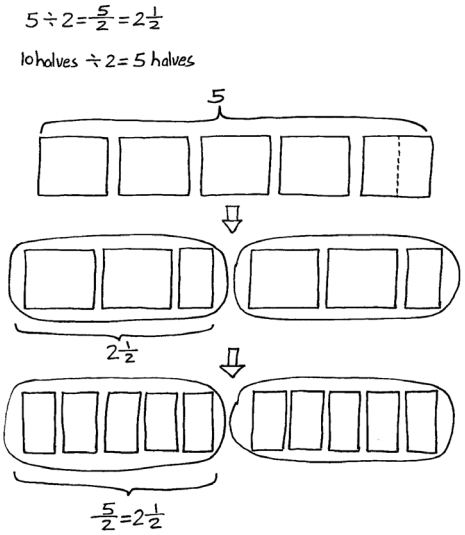
T: How many halves would we have to share in all? How many would each person get?

S: 8 halves in all. Each person would receive 4 halves.

T: Let me record that. (Write 8 halves ÷ 2 = 4 halves.) Although the crackers were shared in units of one-half, what is the total amount of crackers each person receives?

S: 2 whole crackers.

Follow the sequence above to discuss $5 \div 2$ using 5 crackers of the same flavor, followed by 5 differently flavored crackers. Discuss the two ways of sharing.



T: (Point to the division equations that have been recorded.) Look at all the division problems we just solved. Talk to your neighbor about the patterns you see in the quotients.

S: The numbers in the problems are the same as the numbers in the quotients. → The division expressions can be written as fractions with the same digits. → The numerators are the wholes that we shared. The denominators show how many

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equal parts we made. → The numerators are like the dividends, and the denominators are like the divisors. → Even the division symbol looks like a fraction. The dot on top could be a numerator, and the dot on the bottom could be a denominator.

T: Will this always be true? Let's test a few. Since 1 divided by 4 equals 1 fourth, what is 1 divided by 5?

S: 1 fifth.

T: (Write $1 \div 5 = \frac{1}{5}$.) What is 1 ÷ 7?

S: 1 seventh.

T: 3 divided by 7?

S: 3 sevenths.

T: Let's try expressing fractions as division. Say a division expression that is equal to 3 eighths.

S: 3 divided by 8.

T: 3 tenths?

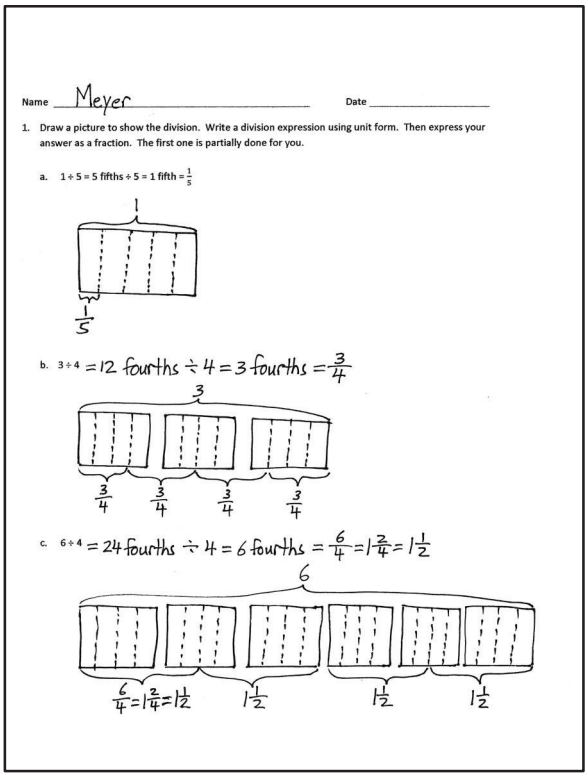
S: 3 divided by 10.

T: 3 hundredths?

S: 3 divided by 100.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Meyer Date

1. Draw a picture to show the division. Write a division expression using unit form. Then express your answer as a fraction. The first one is partially done for you.

a. 1 ÷ 5 = 5 fifths ÷ 5 = 1 fifth = $\frac{1}{5}$

b. 3 ÷ 4 = 12 fourths ÷ 4 = 3 fourths = $\frac{3}{4}$

c. 6 ÷ 4 = 24 fourths ÷ 4 = 6 fourths = $\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

Student Debrief (10 minutes)

Lesson Objective: Interpret a fraction as division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

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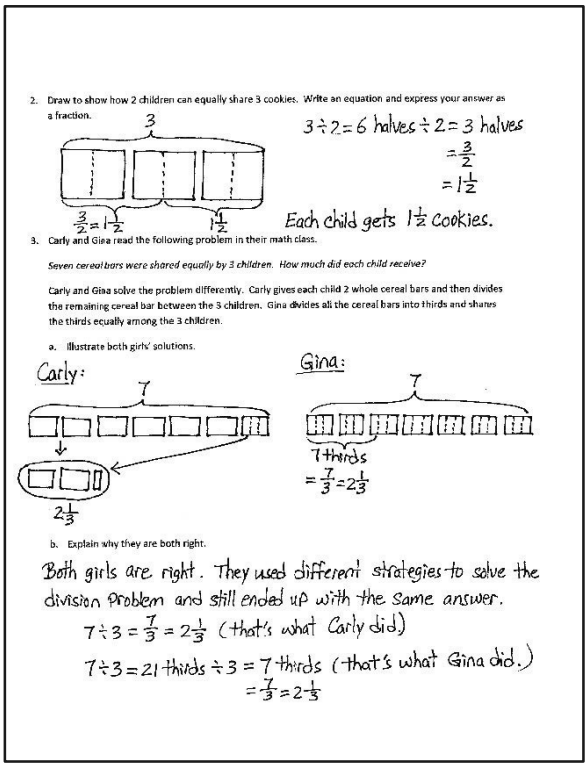
Any combination of the questions below may be used to lead the discussion.

- What did you notice about Problems 4(a) and (b)? What were the wholes, or dividends, and what were the divisors?
- What was your strategy to solve Problem 1(c)?
- What pattern did you notice between 1(b) and (c)? What was the relationship between the size of the dividends and quotients?
- Discuss the division sentence for Problem 2. What number is the whole, and what number is the divisor? How is the division sentence different from $2 \div 3$?
- Explain to your partner the two sharing approaches in Problem 3. (The first approach is to give each girl 2 wholes and then partition the remaining bars. The second approach is to partition all 7 bars, 21 thirds, and share the thirds equally.) When might one approach be more appropriate? (If the cereal bars were different flavors, and each person wanted to try each flavor.)
- True or false? Dividing by 2 is the same as multiplying by $\frac{1}{2}$. (If needed, revisit the fact that $3 \div 2 = \frac{3}{2} = 3 \times \frac{1}{2}$.)



2. Draw to show how 2 children can equally share 3 cookies. Write an equation and express your answer as a fraction.

3 ÷ 2 = 6 halves ÷ 2 = 3 halves
= 3/2
= 1 1/2

Each child gets 1 1/2 cookies.

3. Carly and Gina read the following problem in their math class.

Seven cereal bars were shared equally by 3 children. How much did each child receive?

Carly and Gina solve the problem differently. Carly gives each child 2 whole cereal bars and then divides the remaining cereal bar between the 3 children. Gina divides all the cereal bars into thirds and shares the thirds equally among the 3 children.

a. Illustrate both girls' solutions.

Carly: 7 → 2 1/3

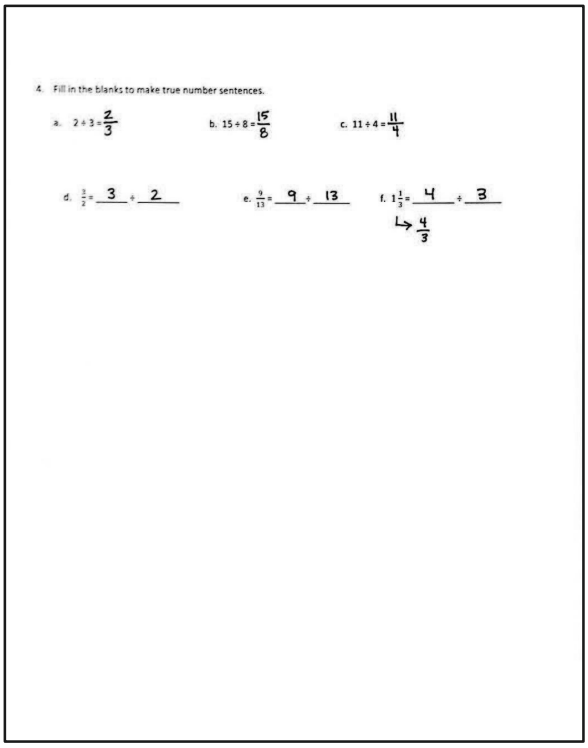
Gina: 7 → 7 thirds = 7/3 = 2 1/3

b. Explain why they are both right.

Both girls are right. They used different strategies to solve the division problem and still ended up with the same answer.

7 ÷ 3 = 7/3 = 2 1/3 (that's what Carly did.)

7 ÷ 3 = 21 thirds ÷ 3 = 7 thirds (that's what Gina did.)
= 7/3 = 2 1/3



4. Fill in the blanks to make true number sentences.

a. 2 ÷ 3 = 2/3

b. 15 ÷ 8 = 15/8

c. 11 ÷ 4 = 11/4

d. 3/2 = 3 ÷ 2

e. 9/13 = 9 ÷ 13

f. 1 1/3 = 4 ÷ 3 → 4/3

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. Draw a picture to show the division. Write a division expression using unit form. Then, express your answer as a fraction. The first one is partially done for you.

   a. 1 ÷ 5 = 5 fifths ÷ 5 = 1 fifth = $\frac{1}{5}$

   b. 3 ÷ 4

   c. 6 ÷ 4

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2. Draw to show how 2 children can equally share 3 cookies. Write an equation, and express your answer as a fraction.

3. Carly and Gina read the following problem in their math class:

   Seven cereal bars were shared equally by 3 children. How much did each child receive?

   Carly and Gina solve the problem differently. Carly gives each child 2 whole cereal bars and then divides the remaining cereal bar among the 3 children. Gina divides all the cereal bars into thirds and shares the thirds equally among the 3 children.

   a. Illustrate both girls' solutions.

   b. Explain why they are both right.

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4. Fill in the blanks to make true number sentences.

a. $2 \div 3 =$ ___

b. $15 \div 8 =$ ___

c. $11 \div 4 =$ ___

d. $\frac{3}{2} =$ _____ ÷ _____

e. $\frac{9}{13} =$ _____ ÷ _____

f. $1\frac{1}{3} =$ _____ ÷ _____

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Name _____ Date _____

1. Draw a picture that shows the division expression. Then, write an equation and solve.

   a. $3 \div 9$

   b. $4 \div 3$

2. Fill in the blanks to make true number sentences.

   a. $21 \div 8 =$ ___

   b. $\frac{7}{4} =$ _____ ÷ _____

   c. $4 \div 9 =$ ___

   d. $1\frac{2}{7} =$ _____ ÷ _____

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Name _____ Date _____

1. Draw a picture to show the division. Express your answer as a fraction.

   a. $1 \div 4$

   b. $3 \div 5$

   c. $7 \div 4$

2. Using a picture, show how six people could share four sandwiches. Then, write an equation and solve.

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3. Fill in the blanks to make true number sentences.

a. $2 \div 7 =$ ___

b. $39 \div 5 =$ ___

c. $13 \div 3 =$ ___

d. $\frac{9}{5} =$ _____ ÷ _____

e. $\frac{19}{28} =$ _____ ÷ _____

f. $1\frac{3}{5} =$ _____ ÷ _____

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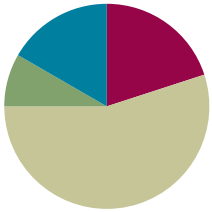
Lesson 3

Objective: Interpret a fraction as division.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Convert to Hundredths **4.NF.5** (3 minutes)
- Compare Fractions **4.NF.2** (4 minutes)
- Fractions as Division **5.NF.3** (3 minutes)
- Write Fractions as Decimals **4.NF.5** (2 minutes)

Convert to Hundredths (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for decimal fractions later in the module.

T: I'll say a factor, and then you'll say the factor you need to multiply it by to get 100. 50.

S: 2.

Continue with the following possible sequence: 25, 20, and 4.

T: (Write $\frac{1}{4} = \frac{\quad}{100}$.) How many fours are in 100?

S: 25.

T: Write the equivalent fraction.

S: (Write $\frac{1}{4} = \frac{25}{100}$.)

Continue with the following possible sequence: $\frac{3}{4} = \frac{\quad}{100}$, $\frac{1}{50} = \frac{\quad}{100}$, $\frac{3}{50} = \frac{\quad}{100}$, $\frac{1}{20} = \frac{\quad}{100}$, $\frac{3}{20} = \frac{\quad}{100}$, $\frac{1}{25} = \frac{\quad}{100}$, and $\frac{2}{25} = \frac{\quad}{100}$.

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Compare Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews concepts from Grade 4 and Grade 5 Module 3.

T: (Write $\frac{1}{2}$ ___ $\frac{1}{6}$.) Compare these fractions, and write a greater than or less than symbol.

S: (Write $\frac{1}{2} > \frac{1}{6}$.)

T: Why is this true?

S: Both have 1 unit, but halves are larger than sixths.

Continue with the following possible sequence: $\frac{2}{3}$ and $\frac{1}{8}$, $\frac{3}{4}$ and $\frac{3}{8}$, $\frac{2}{5}$ and $\frac{9}{10}$, and $\frac{5}{8}$ and $\frac{5}{7}$.

Students should be able to reason about these comparisons without the need for common units. Reasoning, such as *greater* or *less than* half or *the same number* of different sized units, should be the focus.

Fractions as Division (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 2 content.

T: (Write 1 ÷ 3.) Write a complete number sentence using the expression.

S: (Write $1 \div 3 = \frac{1}{3}$.)

Continue with the following possible sequence: 1 ÷ 4 and 2 ÷ 3.

T: (Write 5 ÷ 2.) Write a complete number sentence using the expression.

S: (Write $5 \div 2 = \frac{5}{2}$ or $5 \div 2 = 2\frac{1}{2}$.)

Continue with the following possible sequence: 13 ÷ 5, 7 ÷ 6, and 17 ÷ 4.

T: (Write $\frac{4}{3}$.) Say the fraction.

S: 4 thirds.

T: Write a complete number sentence using the fraction.

S: (Write $4 \div 3 = \frac{4}{3}$ or $4 \div 3 = 1\frac{1}{3}$.)

Continue with the following possible sequence: $\frac{13}{2}$, $\frac{23}{4}$, and $\frac{32}{5}$.

Write Fractions as Decimals (2 minutes)

Note: This fluency activity prepares students for fractions with denominators of 4, 20, 25, and 50 in Topic G.

T: (Write $\frac{1}{10}$.) Say the fraction.

S: 1 tenth.

T: Say it as a decimal.

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S: Zero point one.

Continue with the following possible sequence: $\frac{2}{10}, \frac{3}{10}, \frac{7}{10}, \frac{5}{10}$, and $\frac{9}{10}$.

T: (Write 0.1 = ____.) Write the decimal as a fraction.

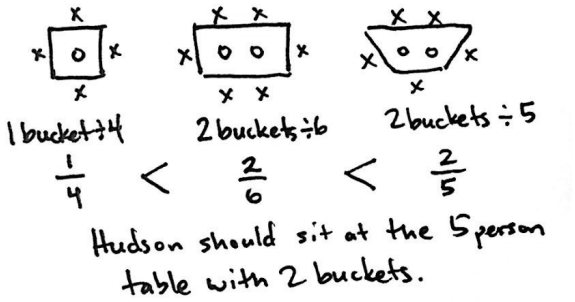
S: (Write 0.1 = $\frac{1}{10}$.)

Continue with the following possible sequence: 0.2, 0.4, 0.8, and 0.6.

Application Problem (5 minutes)

Hudson is choosing a seat in art class. He scans the room and sees a 4-person table with 1 bucket of art supplies, a 6-person table with 2 buckets of supplies, and a 5-person table with 2 buckets of supplies. Which table should Hudson choose if he wants the largest share of art supplies? Support your answer with pictures.

Note: Students must first use division to see which fractional portion of art supplies is available at each table. Then, students compare the fractions and determine which one represents the largest value.



Concept Development (33 minutes)

Materials: (S) Personal white board

Problem 1

A baker poured 4 kilograms of oats equally into 3 bags. What is the weight of each bag of oats?

T: In our story, which operation is needed to find the weight of each bag of oats?

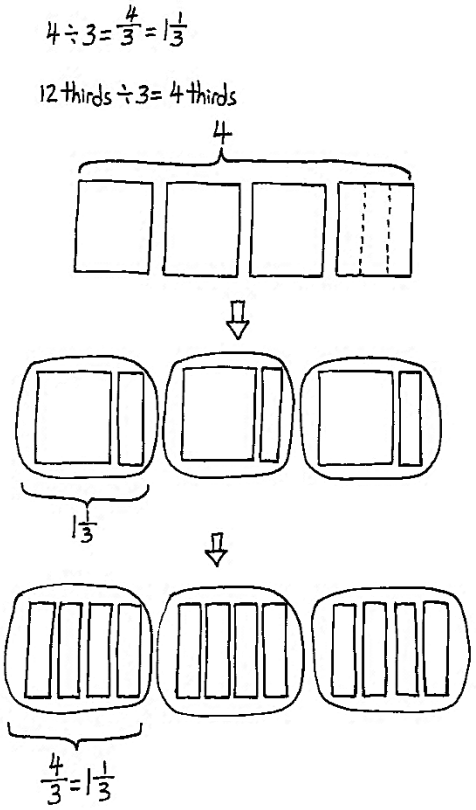
S: Division.

T: Turn and discuss with your partner how you know, as well as what the division expression would be.

S: When you share equally, it means taking what you have and dividing it into equal groups. → The total is 4 kilograms of oats being divided into 3 bags, so the division expression is 4 divided by 3. → The whole is 4, and the divisor is 3.

T: Say the division expression.

S: 4 ÷ 3.



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T: (Write 4 ÷ 3, and draw 4 squares on the board.) Let's represent the kilograms with squares like we did yesterday. Squares are easier to cut into equal shares than circles.

T: Turn and talk about how you'll share the 4 kilograms of oats equally in 3 bags. Draw a picture to show your thinking.

S: Every bag will get a whole kilogram of oats, and then we will split the last kilogram equally into 3 thirds to share. So, each bag gets a whole kilogram and one-third of another one. → I can cut all 4 kilograms into thirds and then split them into the 3 bags. Each bag will get 4 thirds of a kilogram. → I know the answer is 4 over 3, or 4 thirds, because that is just another way to write 4 divided by 3.

T: As we saw yesterday, there are two ways of dividing the oats. Let me record your approaches. (Draw the approaches on the board and restate.) Let's say the division sentence with the quotient.

S: 4 ÷ 3 = 4 thirds. → 4 ÷ 3 = 1 and 1 third.

T: (Point to the diagram on the board.) When we cut them all into thirds, how many thirds were there to share?

S: 12 thirds.

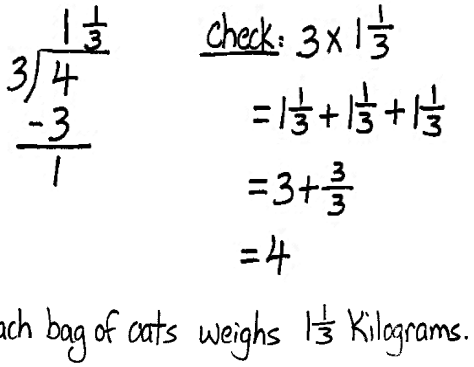
T: Say the division sentence in unit form, starting with 12 thirds.

S: 12 thirds ÷ 3 = 4 thirds.

T: (Write 12 thirds ÷ 3 = 4 thirds on the board.) What is 4 thirds as a mixed number?

S: 1 and 1 third.

T: (Write the algorithm on the board.) Let's show how we divided the oats using the division algorithm.



T: How many groups of 3 can I make with 4 kilograms?

S: 1 group of three.

T: (Record 1 in the quotient.) What's 1 group of three?

S: 3.

T: (Record 3 under 4.) How many whole kilograms are left to share?

S: 1.

T: What did we do with this last kilogram? Turn and discuss with your partner.

S: This one remaining kilogram was split into 3 equal parts to continue sharing it. → I had to split the last kilogram into thirds to share it equally. → The quotient is 1 whole kilogram, and the remainder is 1. → The quotient is 1 whole kilogram and 1 third kilogram. → Each of the 3 bags gets 1 and 1 third kilogram of oats.

T: Let's record what you said. (Point to the remainder of 1.) This remainder is 1 kilogram. To keep sharing it, we split it into 3 parts (point to the divisor), so each bag gets 1 third. I'll write 1 third next to the 1 in the quotient. (Write $\frac{1}{3}$ next to the quotient of 1.)

T: Use the quotient to answer the question.

S: Each bag of oats weighs $1\frac{1}{3}$ kilograms.

T: Let's check our answer. How can we know if we put the right amount of oats in each bag?

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S: We can total up the 3 parts that we put into each bag when we divided the kilograms. → The total should be the same as our original whole. → The sum of the equal parts should be the same as our dividend.

T: We have 3 groups of $1\frac{1}{3}$. Say the multiplication expression.

S: $3 \times 1\frac{1}{3}$.

T: Express 3 copies of $1\frac{1}{3}$ using repeated addition.

S: $1\frac{1}{3} + 1\frac{1}{3} + 1\frac{1}{3}$.

T: Is the total the same number of kilograms we had before we shared?

S: The total is 4 kilograms. → It is the same as our whole before we shared. → 3 ones plus 3 thirds is 3 plus 1. That's 4.

T: We've seen more than one way to write down how to share 4 kilograms in 3 bags. Why is the quotient the same using the algorithm?

S: The same thing is happening to the oats. It is being divided into 3 parts. → We are just using another way to write it.

T: Let's use different strategies in our next problem as well.

Problem 2

If the baker doubles the number of kilograms of oats to be poured equally into 3 bags, what is the weight of each bag of oats?

T: What's the whole in this problem? Turn and share with your partner.

S: 4 doubled is 8. → 4 times 2 is 8. → The baker now has 8 kilograms of oats to pour into 3 bags.

T: Say the whole.

S: 8.

T: Say the divisor.

S: 3.

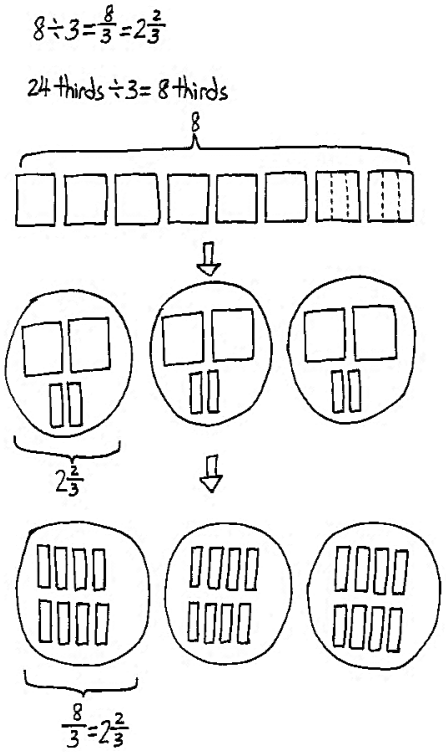
T: Say the division expression for this problem.

S: $8 \div 3$.

T: Compare this expression with the one we just completed. What do you notice?

S: The whole is twice as much as the problem before. → The number of shares is the same.

T: Using that insight, make a prediction about the quotient of this problem.



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S: Since the whole is twice as much shared with the same number of bags, then the answer should be twice as much as the answer to the last problem. → Two times 4 thirds is equal to 8 thirds. → The answer should be double. So, it should be $1\frac{1}{3} + 1\frac{1}{3}$, and that is $2\frac{2}{3}$.

T: Work with your partner to solve, and confirm the predictions you made. Each partner should use a different strategy for sharing the kilograms and draw a picture of his thinking. Then, work together to solve using the standard algorithm.

$2\frac{2}{3}$
$3\overline{)8}$
-6
2

Check: $3 \times 2\frac{2}{3}$
$= 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3}$
$= 6 + \frac{6}{3}$
$= 6 + 2$
$= 8$

Each bag of oats weighs $2\frac{2}{3}$ kilograms.

Circulate as students work.

T: How many kilograms are in each bag this time? Whisper and tell your partner.

S: Each bag gets 2 whole kilograms and $\frac{2}{3}$ of another one. → Each bag gets a third of each kilogram, which would be 8 thirds. → 8 thirds is the same as $2\frac{2}{3}$ kilograms.

T: If we split all of the kilograms into thirds before we share, how many thirds are in all 8 kilograms?

S: 24 thirds.

T: Say the division sentence in unit form.

S: 24 thirds ÷ 3 = 8 thirds.

T: (Set up the standard algorithm on the board, and solve it together.) The quotient is 2 wholes and 2 thirds. Use the quotient to answer the question.

S: Each bag of oats weighs $2\frac{2}{3}$ kilograms.

T: Let's check it now. Say the addition sentence for 3 groups of $2\frac{2}{3}$.

S: $2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} = 8$.

T: So, $8 \div 3 = 2\frac{2}{3}$. How does this quotient compare to our predictions?

S: This answer is what we thought it would be. → It was double the last quotient, which is what we predicted.

T: Great. Now, let's change our whole one more time and see how it affects the quotient.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For students who need the support of concrete materials, continue to use square paper and scissors to represent the equal shares, as well as the pictorial and abstract representations.

Problem 3

If the baker doubles the number of kilograms of oats again, and they are poured equally into 3 bags, what is the weight of each bag of oats?

Repeat the process used in Problem 2. When predicting the quotient for Problem 3, ensure students notice that, this time, the baker has four times the amount of oats as Problem 1 and twice as much as Problem 2. This is important for the scaling interpretation of multiplication.

$16 \div 3 = \frac{16}{3} = 5\frac{1}{3}$

$5\frac{1}{3}$
$3\overline{)16}$
-15
1

Check: $3 \times 5\frac{1}{3}$
$= 5\frac{1}{3} + 5\frac{1}{3} + 5\frac{1}{3}$
$= 15 + \frac{3}{3}$
$= 16$

Each bag of oats weighs $5\frac{1}{3}$ kilograms.

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The closing extension of the dialogue, in which students realize the efficiency of the algorithm, is detailed below.

T: Say the division expression for this problem.
S: 16 ÷ 3.
T: Say the answer as a fraction greater than 1.
S: 16 thirds.
T: Which strategy would be easier to use for solving this problem? Draw out 16 wholes to split into 3 groups, or use the standard algorithm? Turn and discuss with a partner.
S: (Share.)
T: Solve this problem independently using the standard algorithm. If you want, you may also draw.
S: (Work.)
T: Let's solve using the standard algorithm. (Set up the standard algorithm, and solve it on the board.) What is 16 thirds as a mixed number?
S: $5\frac{1}{3}$.
T: Use the quotient to answer the question.
S: Each bag of oats weighs $5\frac{1}{3}$ kilograms.
T: Let's check with repeated addition. Say the entire addition sentence.
S: $5\frac{1}{3} + 5\frac{1}{3} + 5\frac{1}{3} = 16$.
T: So, $16 \div 3 = 5\frac{1}{3}$. How does this quotient compare to our predictions?
S: This answer is what we thought it would be. → It was quadruple the first quotient. → We were right; it was twice as much as the last quotient.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Fractions are generally represented in student materials by equation editing software using a horizontal line to separate the numerator from the denominator (e.g., $\frac{3}{5}$). However, it may be wise to expose students to other formats of notating fractions, such as formats that use a diagonal line to separate the numerator from the denominator (e.g., $^3/_5$).

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

Lesson Objective: Interpret a fraction as division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

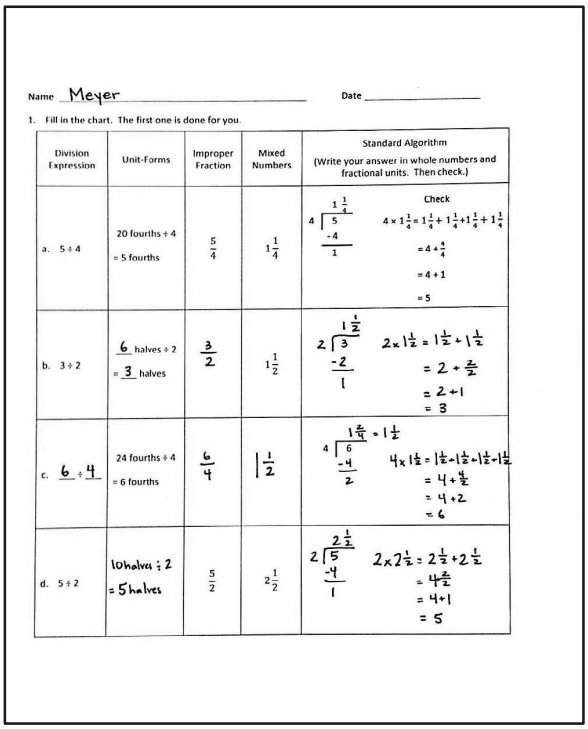
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice between Problems 1(b) and 1(c)? Look at the whole and divisor. Is 3 halves greater than, less than, or equal to 6 fourths? What about the answers?
- What's the relationship between the answers for Problems 2(a) and 2(b)? Explain it to your partner. (Students should note that Problem 2(b) is four times as much as 2(a).) Can you generate a problem where the answer is the same as Problem 2(a) or the same as 2(b)?
- Explain to your partner how you solved Problem 3(a). Why do we need one more warming box than the actual quotient?
- We expressed our remainders today as fractions. Compare this with the way we expressed our remainders as decimals in Module 2. How is it similar? How is it different?

Exit Ticket (3 minutes)

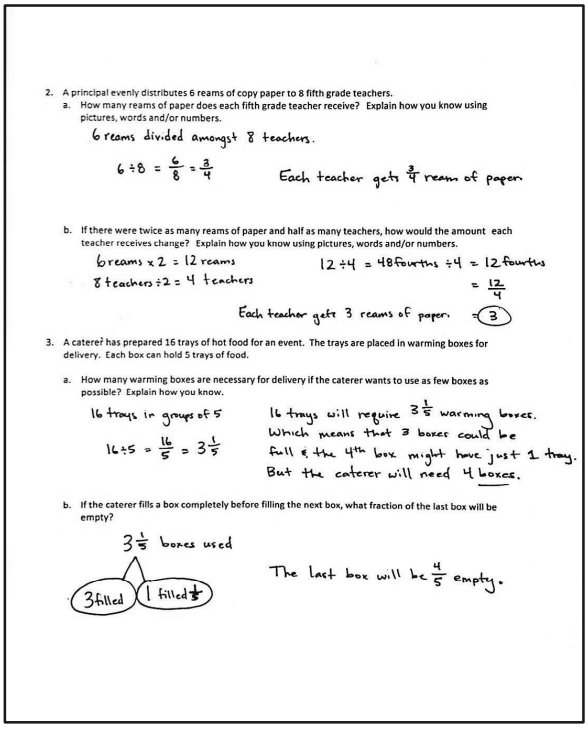
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



Name Meyer Date

1. Fill in the chart. The first one is done for you.

Division Expression	Unit-Forms	Improper Fraction	Mixed Numbers	Standard Algorithm (Write your answer in whole numbers and fractional units. Then check.)
a. $5 \div 4$	20 fourths ÷ 4 = 5 fourths	$\frac{5}{4}$	$1\frac{1}{4}$	$4\overline{)5}$ $1\frac{1}{4}$, -4, 1 — Check: $4 \times 1\frac{1}{4} = 1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{4}$ $= 4+\frac{4}{4}$ $= 4+1$ $= 5$
b. $3 \div 2$	6 halves ÷ 2 = 3 halves	$\frac{3}{2}$	$1\frac{1}{2}$	$2\overline{)3}$ $1\frac{1}{2}$, -2, 1 — $2 \times 1\frac{1}{2} = 1\frac{1}{2}+1\frac{1}{2}$ $= 2+\frac{2}{2}$ $= 2+1$ $= 3$
c. $6 \div 4$	24 fourths ÷ 4 = 6 fourths	$\frac{6}{4}$	$1\frac{1}{2}$	$4\overline{)6}$ $1\frac{2}{4} = 1\frac{1}{2}$, -4, 2 — $4 \times 1\frac{1}{2} = 1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}$ $= 4+\frac{4}{2}$ $= 4+2$ $= 6$
d. $5 \div 2$	10 halves ÷ 2 = 5 halves	$\frac{5}{2}$	$2\frac{1}{2}$	$2\overline{)5}$ $2\frac{1}{2}$, -4, 1 — $2 \times 2\frac{1}{2} = 2\frac{1}{2}+2\frac{1}{2}$ $= 4\frac{2}{2}$ $= 4+1$ $= 5$



2. A principal evenly distributes 6 reams of copy paper to 8 fifth grade teachers.
   a. How many reams of paper does each fifth grade teacher receive? Explain how you know using pictures, words and/or numbers.

6 reams divided amongst 8 teachers.

$6 \div 8 = \frac{6}{8} = \frac{3}{4}$ Each teacher gets $\frac{3}{4}$ ream of paper.

   b. If there were twice as many reams of paper and half as many teachers, how would the amount each teacher receives change? Explain how you know using pictures, words and/or numbers.

6 reams × 2 = 12 reams
8 teachers ÷ 2 = 4 teachers
$12 \div 4 = 48$ fourths ÷ 4 = 12 fourths
$= \frac{12}{4}$
Each teacher gets 3 reams of paper. = 3

3. A caterer has prepared 16 trays of hot food for an event. The trays are placed in warming boxes for delivery. Each box can hold 5 trays of food.
   a. How many warming boxes are necessary for delivery if the caterer wants to use as few boxes as possible? Explain how you know.

16 trays in groups of 5
$16 \div 5 = \frac{16}{5} = 3\frac{1}{5}$
16 trays will require $3\frac{1}{5}$ warming boxes.
Which means that 3 boxes could be full & the 4th box might have just 1 tray.
But the caterer will need 4 boxes.

   b. If the caterer fills a box completely before filling the next box, what fraction of the last box will be empty?

$3\frac{1}{5}$ boxes used
3 filled, 1 filled $\frac{1}{5}$
The last box will be $\frac{4}{5}$ empty.

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Name _____ Date _____

1. Fill in the chart. The first one is done for you.

Division Expression	Unit Forms	Improper Fraction	Mixed Numbers	Standard Algorithm (Write your answer in whole numbers and fractional units. Then check.)
a. $5 \div 4$	20 fourths ÷ 4 = 5 fourths	$\frac{5}{4}$	$1\frac{1}{4}$	$4 \overline{)\,5}$ → $1\frac{1}{4}$; -4; 1 Check $4 \times 1\frac{1}{4} = 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4}$ $= 4 + \frac{4}{4}$ $= 4 + 1$ $= 5$
b. $3 \div 2$	___ halves ÷ 2 = ___ halves		$1\frac{1}{2}$	
c. ___ ÷ ___	24 fourths ÷ 4 = 6 fourths			$4 \overline{)\,6}$
d. $5 \div 2$		$\frac{5}{2}$	$2\frac{1}{2}$	

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2. A principal evenly distributes 6 reams of copy paper to 8 fifth-grade teachers.

   a. How many reams of paper does each fifth-grade teacher receive? Explain how you know using pictures, words, or numbers.

   b. If there were twice as many reams of paper and half as many teachers, how would the amount each teacher receives change? Explain how you know using pictures, words, or numbers.

3. A caterer has prepared 16 trays of hot food for an event. The trays are placed in warming boxes for delivery. Each box can hold 5 trays of food.

   a. How many warming boxes are necessary for delivery if the caterer wants to use as few boxes as possible? Explain how you know.

   b. If the caterer fills a box completely before filling the next box, what fraction of the last box will be empty?

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A baker made 9 cupcakes, each a different type. Four people want to share them equally. How many cupcakes will each person get?

Fill in the chart to show how to solve the problem.

Division Expression	Unit Forms	Fractions and Mixed numbers	Standard Algorithm
Draw to show your thinking:			

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Name _____ Date _____

1. Fill in the chart. The first one is done for you.

Division Expression	Unit Forms	Improper Fractions	Mixed Numbers	Standard Algorithm (Write your answer in whole numbers and fractional units. Then check.)
a. $4 \div 3$	12 thirds ÷ 3 = 4 thirds	$\frac{4}{3}$	$1\frac{1}{3}$	$3 \overline{)\,4}$ → $1\frac{1}{3}$, -3, 1 Check $3 \times 1\frac{1}{3} = 1\frac{1}{3} + 1\frac{1}{3} + 1\frac{1}{3}$ $= 3 + \frac{3}{3}$ $= 3 + 1$ $= 4$
b. ___ ÷ ___	___ fifths ÷ 5 = ___ fifths		$1\frac{2}{5}$	
c. ___ ÷ ___	___ halves ÷ 2 = ___ halves			$2 \overline{)\,7}$
d. $7 \div 4$		$\frac{7}{4}$		

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2. A coffee shop uses 4 liters of milk every day.

   a. If there are 15 liters of milk in the refrigerator, after how many days will more milk need to be purchased? Explain how you know.

   b. If only half as much milk is used each day, after how many days will more milk need to be purchased?

3. Polly buys 14 cupcakes for a party. The bakery puts them into boxes that hold 4 cupcakes each.

   a. How many boxes will be needed for Polly to bring all the cupcakes to the party? Explain how you know.

   b. If the bakery completely fills as many boxes as possible, what fraction of the last box is empty? How many more cupcakes are needed to fill this box?

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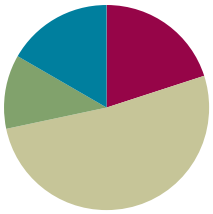
Lesson 4

Objective: Use tape diagrams to model fractions as division.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Write Fractions as Decimals **5.NF.3** (4 minutes)
- Convert to Hundredths **4.NF.5** (4 minutes)
- Fractions as Division **5.NF.3** (4 minutes)

Write Fractions as Decimals (4 minutes)

Note: This fluency activity prepares students for Topic G.

T: (Write $\frac{1}{10}$.) Say the fraction.

S: 1 tenth.

T: Say it as a decimal.

S: Zero point one.

Continue with the following possible sequence: $\frac{2}{10}$, $\frac{3}{10}$, $\frac{8}{10}$, and $\frac{5}{10}$.

T: (Write $\frac{1}{100}$ = ____.) Say the fraction.

S: 1 hundredth.

T: Say it as a decimal.

S: Zero point zero one.

Continue with the following possible sequence: $\frac{2}{100}$, $\frac{3}{100}$, $\frac{9}{100}$, and $\frac{13}{100}$.

T: (Write 0.01 = ____.) Say it as a fraction.

S: 1 hundredth.

T: (Write 0.01 = $\frac{1}{100}$.)

Continue with the following possible sequence: 0.02, 0.09, 0.11, and 0.39.

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Convert to Hundredths (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Topic G.

T: (Write $\frac{1}{4} = \frac{\quad}{100}$.) Write the equivalent fraction.

S: (Write $\frac{1}{4} = \frac{25}{100}$.)

T: (Write $\frac{1}{4} = \frac{25}{100} =$ ____.) Write 1 fourth as a decimal.

S: (Write $\frac{1}{4} = \frac{25}{100} = 0.25$.)

Continue with the following possible sequence: $\frac{3}{4}$, $\frac{1}{50}$, $\frac{7}{50}$, $\frac{12}{50}$, $\frac{1}{20}$, $\frac{7}{20}$, $\frac{11}{20}$, $\frac{1}{25}$, $\frac{2}{25}$, $\frac{9}{25}$, and $\frac{11}{25}$.

Fractions as Division (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews content from Lessons 2 and 3.

T: (Write 1 ÷ 2.) Solve.

S: (Write $1 \div 2 = \frac{1}{2}$.)

Continue with the following possible sequence: 1 ÷ 5 and 3 ÷ 4.

T: (Write 7 ÷ 2.) Solve.

S: (Write $7 \div 2 = \frac{7}{2}$ or $7 \div 2 = 3\frac{1}{2}$.)

Continue with the following possible sequence: 12 ÷ 5, 11 ÷ 6, 19 ÷ 4, 31 ÷ 8, and 49 ÷ 9.

T: (Write $\frac{5}{3}$.) Write the fraction as a whole number division expression.

S: (Write 5 ÷ 3.)

Continue with the following possible sequence: $\frac{11}{2}$, $\frac{15}{4}$, and $\frac{24}{5}$.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If students are comfortable with interpreting fractions as division, consider foregoing the written component of this fluency activity, and ask students to visualize the fractions, making this activity more abstract.

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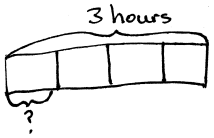
Application Problem (7 minutes)

Four grade levels need equal time for indoor recess, and the gym is available for three hours.

a. How many hours of recess will each grade level receive? Draw a picture to support your answer.

b. How many minutes?

c. If the gym can accommodate two grade levels at once, how many hours of recess will 2 grade levels receive in 3 hours?



Each grade level gets 3/4 of an hour, or 45 min.

For 2 grade levels at once, the time doubles to 90 min or $1\frac{1}{2}$ hours.

4 units = 3 hours
1 unit = $\frac{3}{4}$ hours
$\frac{3}{4}$ hour x 1 hour =
$\frac{3}{4}$ hour x 60 minutes =
$\frac{3}{1}$ x $\frac{60}{1}$ = 45 minutes

45 minutes x 2 = 90 minutes
$\frac{90}{60} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$ hours

Note: Students practice division with fractional quotients, which leads into today's lesson. Note that the whole remains constant in Part (c), while the divisor is cut in half. Guide students to analyze the effect of this halving on the quotient as related to the doubling of the whole from previous problems.

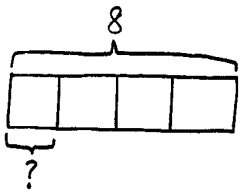
Concept Development (31 minutes)

Materials: (S) Personal white board

Problem 1

Eight tons of gravel is equally divided between 4 dump trucks. How much gravel is in one dump truck?

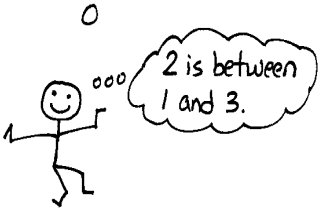
T: Say a division sentence to solve the problem.
S: 8 ÷ 4 = 2.
T: Model this problem with a tape diagram. (Pause as students work.)
T: We know that 4 units are equal to 8 tons. (Write 4 units = 8.) We want to find what 1 unit is equal to.
T: (Write 1 unit = 8 ÷ 4.)
T: How many tons of gravel are in one dump truck?
S: 2.
T: Use your quotient to answer the question.
S: Each dump truck held 2 tons of gravel.



$4\overline{)8}$ = 2
−8
0

Check: 4 x 2 = 8

4 units = 8
1 unit = 8 ÷ 4
= 2



Each dump truck held 2 tons of gravel.

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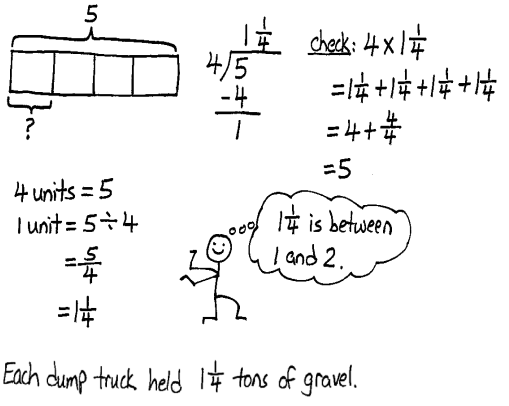
Problem 2

Five tons of gravel is equally divided between 4 dump trucks. How much gravel is in one dump truck?



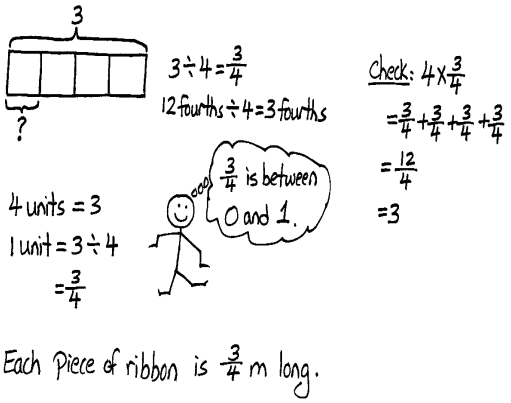
- T: (Change the value from the previous problem to 5 tons on the board.) How would our drawing be different if we had 5 tons of gravel?
- S: Our whole would be different—5, not 8. → The tape diagram is the same, except for the value of the whole. We'll still partition it into fourths because there are still 4 trucks.
- T: (Partition a new bar into 4 equal parts labeled with 5 as the whole.)
- T: We know that these 4 units are equal to 5 tons. (Write 4 units = 5.) We want to find what 1 unit is equal to. (Write a question mark beneath 1 fourth of the bar.) What is the division expression you'll use to find what 1 unit is?
- S: 5 ÷ 4.
- T: (Write 1 unit = 5 ÷ 4.) 5 ÷ 4 is …?
- S: 5 fourths.
- T: So, each unit is equal to 5 fourths tons of gravel. Can we prove this using the standard algorithm?
- T: What is 5 ÷ 4?
- S: One and one-fourth.
- T: (Write $5 \div 4 = 1\frac{1}{4}$.) Use your quotient to answer the question.
- S: Each dump truck held one and one-fourth tons of gravel.
- T: Visualize a number line. Between which two adjacent whole numbers is 1 and one-fourth?
- S: 1 and 2.
- T: Check your work using repeated addition.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide number lines with fractional markings for students who still need support to visualize the placement of the fractions.

Problem 3

A 3-meter ribbon is cut into 4 equal pieces to make flowers. What is the length of each piece?



- T: (Write the word problem on the board.) Work with a partner, and draw a tape diagram to solve.
- T: Say the division expression you solved.
- S: 3 divided by 4.
- T: Say the answer as a fraction.
- S: Three-fourths.

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T: (Write $\frac{3}{4}$ on the board.) In this case, does it make sense to use the standard algorithm to solve? Turn and talk.

S: No. It's just 3 divided by 4, which is $\frac{3}{4}$. → I don't think so. It's really easy. → We could, but the quotient of zero looks strange. It's just easier to say 3 divided by 4 equals 3 fourths.

T: Use your quotient to answer the question.

S: Each piece of ribbon is $\frac{3}{4}$ m long.

T: Let's check the answer. Say the multiplication sentence, starting with 4.

S: $4 \times \frac{3}{4} = \frac{12}{4}$. → $4 \times \frac{3}{4} = 3$.

T: Our answer is correct. If we wanted to place our quotient of $\frac{3}{4}$ on a number line, between which two adjacent whole numbers would we place it?

S: 0 and 1.

Problem 4

14 gallons of water is used to completely fill 3 fish tanks. If each tank holds the same amount of water, how many gallons will each tank hold?

T: Let's read this problem together. (All read.) Work with a partner to solve this problem. Draw a tape diagram, and solve using the standard algorithm.

T: Say the division equation you solved.

S: $14 \div 3 = \frac{14}{3}$.

T: Say the quotient as a mixed number.

S: $4\frac{2}{3}$.

T: Use your quotient to answer the question.

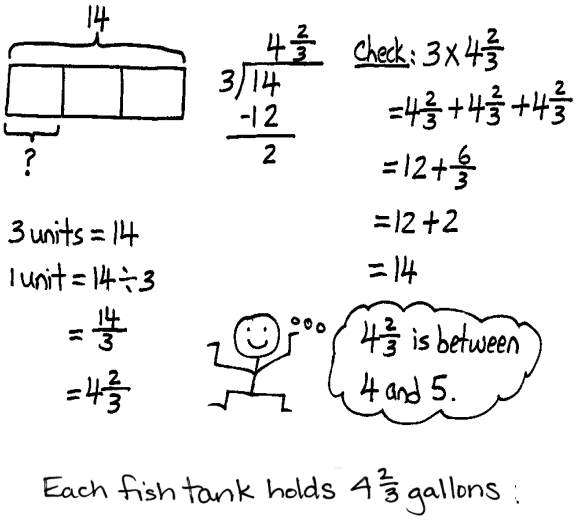
S: The volume of each fish tank is $4\frac{2}{3}$ gallons.

T: Between which two adjacent whole numbers does our answer lie?

S: Between 4 and 5.

T: Check your answers using multiplication.

S: (Check answers.)



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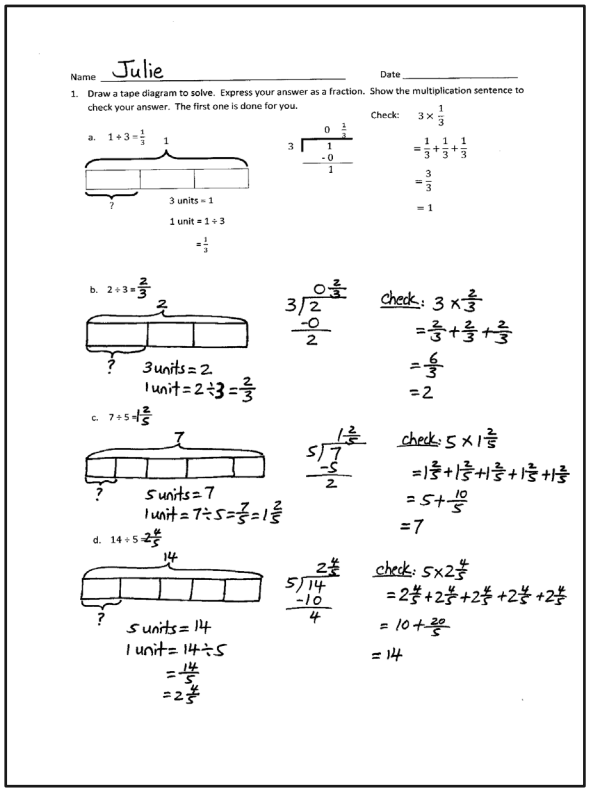
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Julie Date

1. Draw a tape diagram to solve. Express your answer as a fraction. Show the multiplication sentence to check your answer. The first one is done for you.

a. $1 \div 3 = \frac{1}{3}$

3 units = 1

1 unit = $1 \div 3$

$= \frac{1}{3}$

Check: $3 \times \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{3}{3}$

$= 1$

b. $2 \div 3 = \frac{2}{3}$

3 units = 2

1 unit = $2 \div 3 = \frac{2}{3}$

Check: $3 \times \frac{2}{3}$

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

$= \frac{6}{3}$

$= 2$

c. $7 \div 5 = 1\frac{2}{5}$

5 units = 7

1 unit = $7 \div 5 = \frac{7}{5} = 1\frac{2}{5}$

Check: $5 \times 1\frac{2}{5}$

$= 1\frac{2}{5} + 1\frac{2}{5} + 1\frac{2}{5} + 1\frac{2}{5} + 1\frac{2}{5}$

$= 5 + \frac{10}{5}$

$= 7$

d. $14 \div 5 = 2\frac{4}{5}$

5 units = 14

1 unit = $14 \div 5$

$= \frac{14}{5}$

$= 2\frac{4}{5}$

Check: $5 \times 2\frac{4}{5}$

$= 2\frac{4}{5} + 2\frac{4}{5} + 2\frac{4}{5} + 2\frac{4}{5} + 2\frac{4}{5}$

$= 10 + \frac{20}{5}$

$= 14$

Student Debrief (10 minutes)

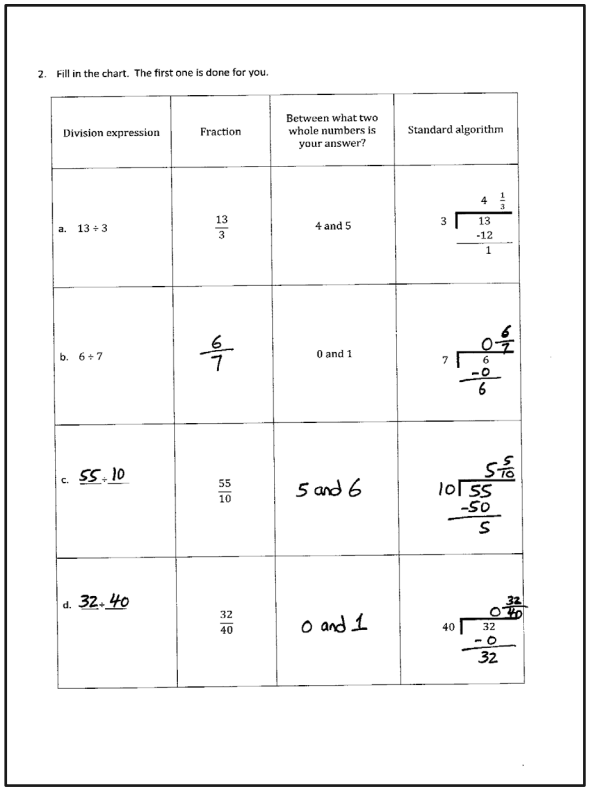
Lesson Objective: Use tape diagrams to model fractions as division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice between Problem 1(a) and Problems 1(b), 1(c), and 1(d)? What did you notice about the wholes or dividends and the divisors?
- In Problem 2(c), can you name the fraction of $\frac{55}{10}$ using a larger fractional unit? In other words, can you simplify it? Are both fractions located at the same point on the number line?
- Compare Problems 3 and 4. What's the division sentence for each problem? What's the whole and divisor for each problem? (Problem 3's division expression is 4 ÷ 5, and Problem 4's division expression is 5 ÷ 4.)



2. Fill in the chart. The first one is done for you.

Division expression	Fraction	Between what two whole numbers is your answer?	Standard algorithm
a. $13 \div 3$	$\frac{13}{3}$	4 and 5	$4\frac{1}{3}$; $3\overline{)13}$, -12, 1
b. $6 \div 7$	$\frac{6}{7}$	0 and 1	$\frac{6}{7}$; $7\overline{)6}$, -0, 6
c. $55 \div 10$	$\frac{55}{10}$	5 and 6	$5\frac{5}{10}$; $10\overline{)55}$, -50, 5
d. $32 \div 40$	$\frac{32}{40}$	0 and 1	$\frac{32}{40}$; $40\overline{)32}$, -0, 32

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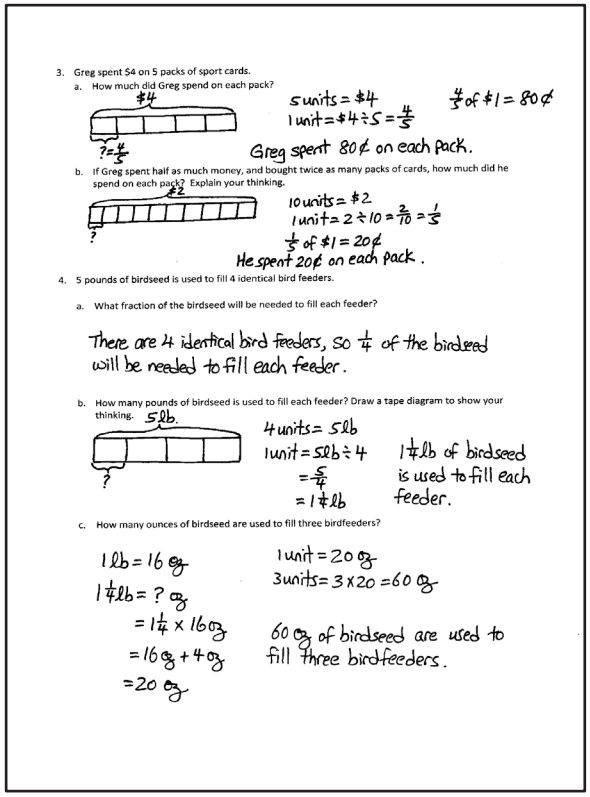
- Explain to your partner the difference between the questions asked in Problems 4(a) and 4(b). (Problem 4(a) is asking *what fraction* of the seed is in each feeder, while 4(b) is asking *the number of pounds* of seed in each feeder.)
- How was our learning today built on what we learned yesterday? (Students may point out that the models used today were more abstract than the concrete materials used previously. Students may also point out that it was easier to see the fractions as division when presented as equations than before.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



3. Greg spent $4 on 5 packs of sport cards.
   a. How much did Greg spend on each pack?

   b. If Greg spent half as much money, and bought twice as many packs of cards, how much did he spend on each pack? Explain your thinking.

4. 5 pounds of birdseed is used to fill 4 identical bird feeders.
   a. What fraction of the birdseed will be needed to fill each feeder?

There are 4 identical bird feeders, so 1/4 of the birdseed will be needed to fill each feeder.

   b. How many pounds of birdseed is used to fill each feeder? Draw a tape diagram to show your thinking.

   c. How many ounces of birdseed are used to fill three birdfeeders?

1 lb = 16 oz
1 1/4 lb = ? oz
= 1 1/4 × 16 oz
= 16 oz + 4 oz
= 20 oz

1 unit = 20 oz
3 units = 3 × 20 = 60 oz
60 oz of birdseed are used to fill three birdfeeders.

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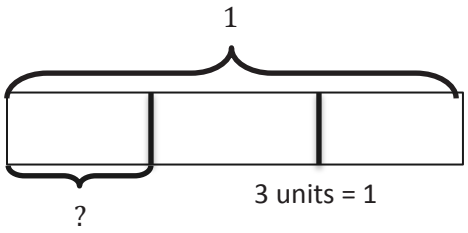
Name _____ Date _____

1. Draw a tape diagram to solve. Express your answer as a fraction. Show the multiplication sentence to check your answer. The first one is done for you.

a. $1 \div 3 = \frac{1}{3}$



3 units = 1

1 unit = $1 \div 3$

$= \frac{1}{3}$

$$\begin{array}{r} 0 \quad \frac{1}{3} \\ 3 \overline{)\,1} \\ -\,0 \\ \hline 1 \end{array}$$

Check: $3 \times \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{3}{3}$

$= 1$

b. $2 \div 3 =$ ___

c. $7 \div 5 =$ ___

d. $14 \div 5 =$ ___

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2. Fill in the chart. The first one is done for you.

Division Expression	Fraction	Between which two whole numbers is your answer?	Standard Algorithm
a. $13 \div 3$	$\frac{13}{3}$	4 and 5	$3 \overline{)13}$ $4\ \frac{1}{3}$, -12, 1
b. $6 \div 7$		0 and 1	$7 \overline{)6}$
c. ____÷____	$\frac{55}{10}$		
d. ___÷____	$\frac{32}{40}$		$40 \overline{)32}$

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3. Greg spent $4 on 5 packs of sport cards.

   a. How much did Greg spend on each pack?

   b. If Greg spent half as much money and bought twice as many packs of cards, how much did he spend on each pack? Explain your thinking.

4. Five pounds of birdseed is used to fill 4 identical bird feeders.

   a. What fraction of the birdseed will be needed to fill each feeder?

   b. How many pounds of birdseed are used to fill each feeder? Draw a tape diagram to show your thinking.

   c. How many ounces of birdseed are used to fill three bird feeders?

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Matthew and his 3 siblings are weeding a flower bed with an area of 9 square yards. If they share the job equally, how many square yards of the flower bed will each child need to weed? Use a tape diagram to show your thinking.

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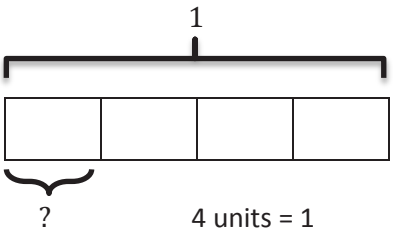
Name _____ Date _____

1. Draw a tape diagram to solve. Express your answer as a fraction. Show the addition sentence to support your answer. The first one is done for you.

a. $1 \div 4 = \frac{1}{4}$



4 units = 1

1 unit = $1 \div 4$

$= \frac{1}{4}$

$$\begin{array}{r} 0 \quad \frac{1}{4} \\ 4 \overline{)\,1} \\ -\,0 \\ \hline 1 \end{array}$$

Check:

$$4 \times \frac{1}{4}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

b. $4 \div 5 =$ ____

c. $8 \div 5 =$ ____

d. $14 \div 3 =$ ____

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2. Fill in the chart. The first one is done for you.

Division Expression	Fraction	Between which two whole numbers is your answer?	Standard Algorithm
a. $16 \div 5$	$\frac{16}{5}$	3 and 4	$5 \overline{)\,16}$ $3\ \frac{1}{5}$ -15 1
b. ____÷____	$\frac{3}{4}$	0 and 1	
c. ____÷____	$\frac{7}{2}$		$2 \overline{)\,7}$
d. ____÷____	$\frac{81}{90}$		

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3. Jackie cut a 2-yard spool into 5 equal lengths of ribbon.

   a. What is the length of each ribbon in yards? Draw a tape diagram to show your thinking.

   b. What is the length of each ribbon in feet? Draw a tape diagram to show your thinking.

4. Baa Baa, the black sheep, had 7 pounds of wool. If he separated the wool equally into 3 bags, how much wool would be in 2 bags?

5. An adult sweater is made from 2 pounds of wool. This is 3 times as much wool as it takes to make a baby sweater. How much wool does it take to make a baby sweater? Use a tape diagram to solve.

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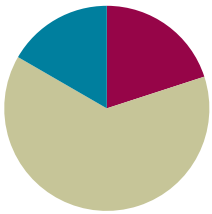
Lesson 5

Objective: Solve word problems involving the division of whole numbers with answers in the form of fractions or whole numbers.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(38 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Fraction of a Set **4.NF.4** (4 minutes)
- Write Division Sentences as Fractions **5.NF.3** (3 minutes)
- Write Fractions as Mixed Numbers **5.NF.3** (5 minutes)

Fraction of a Set (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 6.

T: (Write $10 \times \frac{1}{2}$.) 10 copies of one-half is …?

S: 5.

T: (Write $10 \times \frac{1}{5}$.) 10 copies of one-fifth is …?

S: 2.

Continue with the following possible sequence: $8 \times \frac{1}{2}$, $8 \times \frac{1}{4}$, $6 \times \frac{1}{3}$, $30 \times \frac{1}{6}$, $42 \times \frac{1}{7}$, $42 \times \frac{1}{6}$, $48 \times \frac{1}{8}$, $54 \times \frac{1}{9}$, and $54 \times \frac{1}{6}$.

Write Division Sentences as Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 4.

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T: (Write 9 ÷ 30 = ____.) Write the quotient as a fraction.

S: (Write 9 ÷ 30 = $\frac{9}{30}$.)

T: Express the fraction in its simplest form, and then write it as a decimal.

S: (Write 9 ÷ 30 = $\frac{3}{10}$ = 0.3.)

Continue with the following possible sequence: 28 ÷ 40, 18 ÷ 60, 63 ÷ 70, 24 ÷ 80, and 63 ÷ 90.

Write Fractions as Mixed Numbers (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 4.

T: (Write $\frac{13}{2}$ = ____ ÷ ____ = ____.) Write the fraction as a division problem and mixed number.

S: (Write $\frac{13}{2}$ = 13 ÷ 2 = $6\frac{1}{2}$.)

Continue with the following possible sequence: $\frac{11}{2}$, $\frac{17}{2}$, $\frac{44}{2}$, $\frac{31}{10}$, $\frac{23}{10}$, $\frac{47}{10}$, $\frac{89}{10}$, $\frac{8}{3}$, $\frac{13}{3}$, $\frac{26}{3}$, $\frac{9}{4}$, $\frac{13}{4}$, $\frac{15}{4}$, and $\frac{35}{4}$.

Concept Development (38 minutes)

Materials: (S) Problem Set

Suggested Delivery of Instruction for Solving Lesson 5 Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Appropriate scaffolds help all students feel successful. Students may use translators, interpreters, or sentence frames to present their solutions and respond to feedback. Models shared may include concrete manipulatives. If the pace of the lesson is a consideration, allow presenters to prepare beforehand.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish their work on that question, sharing their work and thinking with a peer. All students should write their equations and statements of the answers.

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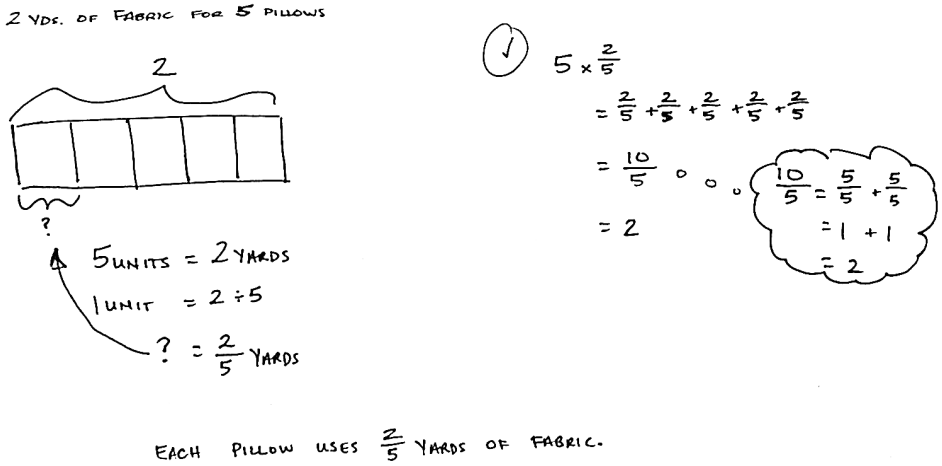
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Problem 1

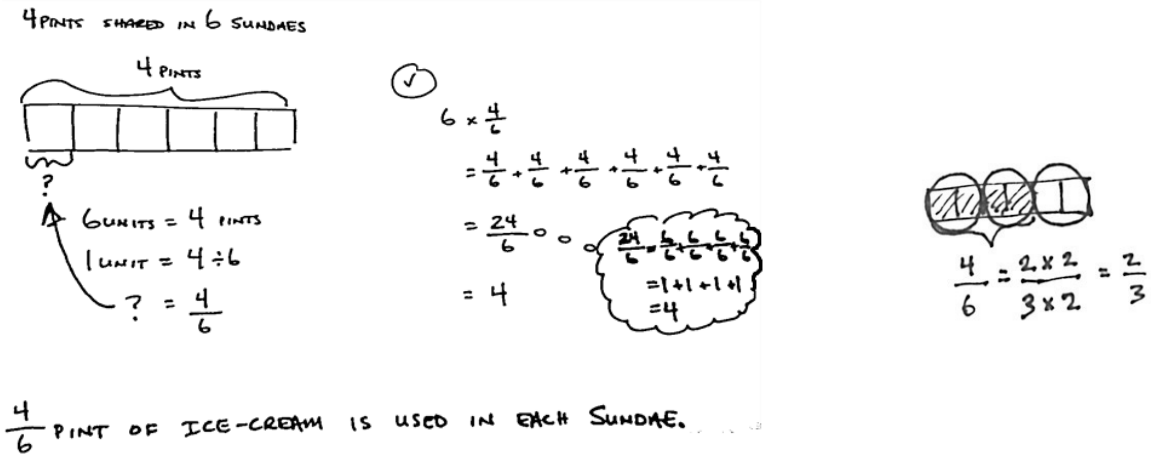
A total of 2 yards of fabric is used to make 5 identical pillows. How much fabric is used for each pillow?



This problem requires understanding of the whole and divisor. The whole of 2 is divided by 5, which results in a quotient of 2 fifths. Circulate, looking for different visuals (tape diagram and the region models from Lessons 2–3) to facilitate a discussion as to how these different models support the solution of $\frac{2}{5}$.

Problem 2

An ice cream shop uses 4 pints of ice cream to make 6 sundaes. How many pints of ice cream are used for each sundae?



This problem also requires students' understanding of the whole versus the divisor. The whole is 4, and it is divided equally into 6 units with the solution of 4 sixths. Students should not have to use the standard algorithm to solve because they should be comfortable interpreting the division expression as a fraction and

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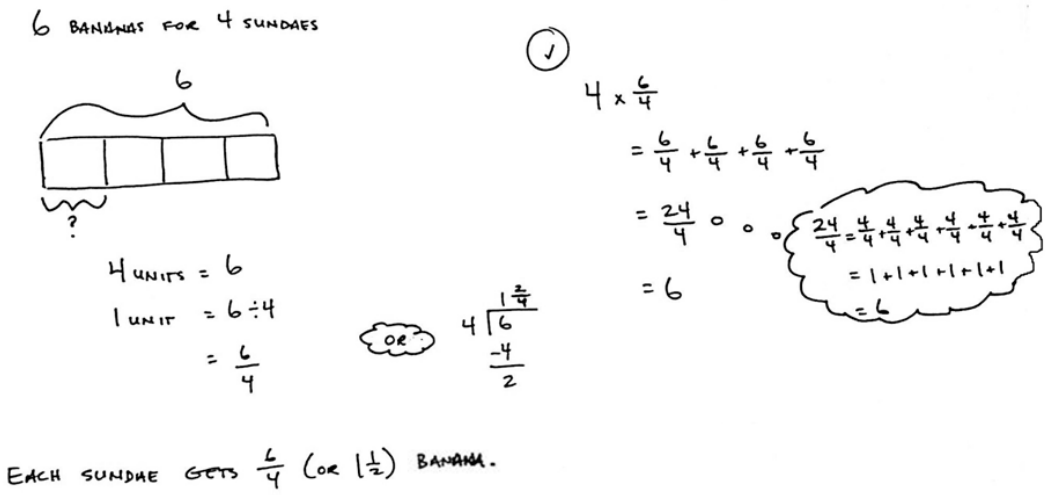
vice versa. Circulate, looking for alternate modeling strategies that can be quickly mentioned or explored more deeply, if desired. Students might express 4 sixths as 2 thirds. The tape diagram illustrates that larger units of 2 can be made. Quickly model a tape with 6 parts (now representing 1 pint), shade 4, and circle sets of 2.

Problem 3

An ice cream shop uses 6 bananas to make 4 identical sundaes. How many bananas are used in each sundae? Use a tape diagram to show your work.



This problem has the same two digits (4 and 6) as the previous problem. However, it is important for students to realize that the digits take on a new role, either as whole or divisor, in this context. Six wholes divided by 4 is equal to 6 fourths or 1 and 2 fourths. Although it is not required that students use the standard algorithm, it can be easily used to find the mixed number value of $1\frac{2}{4}$.

Students may also be engaged in a discussion about the practicality of dividing the remainder of the 2 bananas into fourths and then giving each sundae 2 fourths. Many students may clearly see that the bananas can instead be divided into halves, and each sundae can be given 1 and 1 half. Facilitate a quick discussion with students about which form of the answer makes more sense given the story's context (i.e., should the sundae maker divide all of the bananas in fourths and then give each sundae 6 fourths, or should each sundae be given a whole banana and then divide the remaining bananas?).

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Support English language learners as they explain their thinking. Provide sentence starters and a word bank. Examples are given below.

Sentence starters:

"I had ____ (unit) in all."

"1 unit equals ____."

Word bank:

fraction of divided by remainder

half as much twice as many

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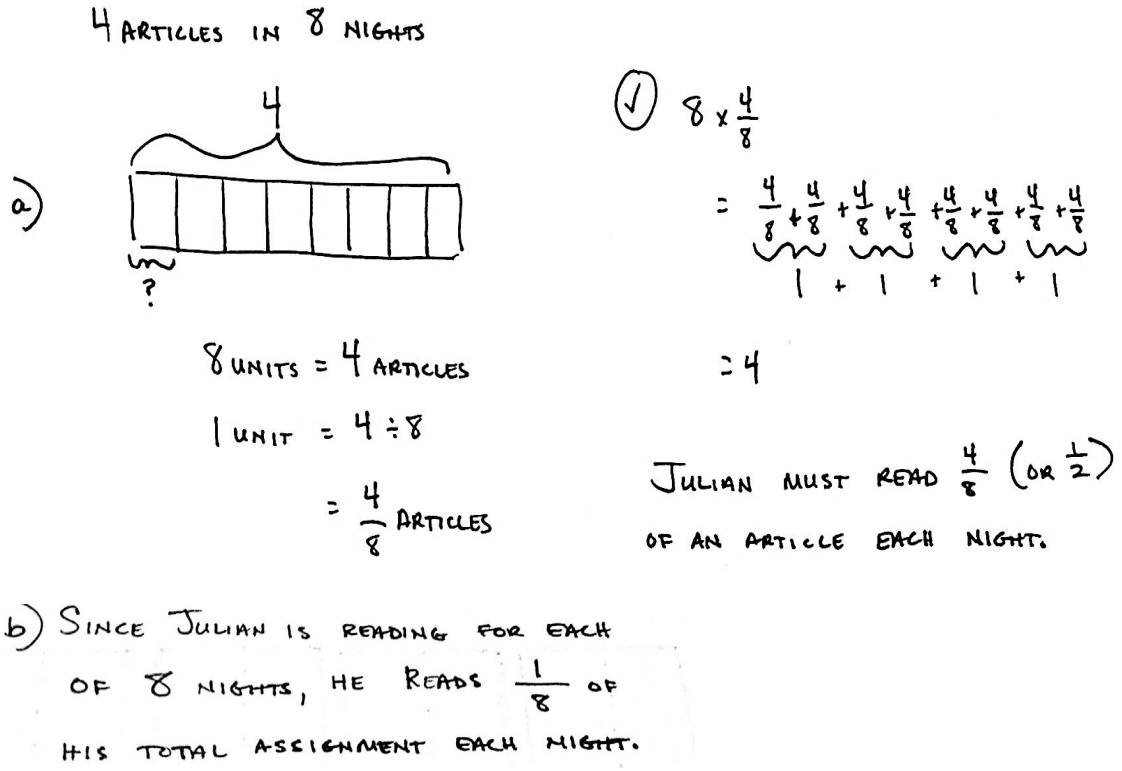
Problem 4

Julian has to read 4 articles for school. He has 8 nights to read them. He decides to read the same number of articles each night.

a. How many articles will he have to read per night?

b. What fraction of the reading assignment will he read each night?



In this problem, Julian must read 4 articles throughout the course of 8 nights. The solution of 4 eighths of an article each night might imply that Julian can simply divide each article into eighths and read any 4 articles on any of the 8 nights. Engage in a discussion allowing students to see that 4 eighths must be interpreted as 4 consecutive eighths or 1 half of an article. It would be most practical for Julian to read the first half of an article one night and the remaining half the following night. In this manner, he will finish his reading assignment within the 8 days. Part (b) provides for deeper thinking about units being considered.

Students must differentiate between the article-as-unit and assignment-as-unit to answer. While 1 half of an article is read each night, the assignment has been split into eight parts. Take the opportunity to discuss with students whether the articles are all equal in length. Since the problem does not specify, a simplifying assumption is created to solve, which finds that, each night, 1 eighth of the total assignment must be read. Discuss how the answer would change if one article were twice the length of the other three.

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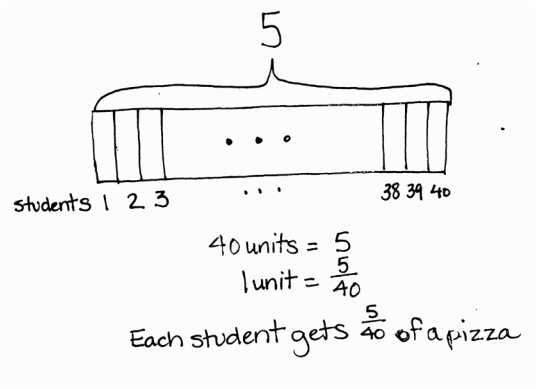
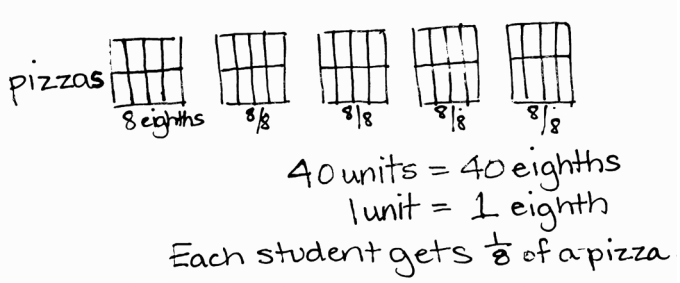
Problem 5

40 students shared 5 pizzas equally. How much pizza did each student receive? What fraction of the pizza did each student receive?





Because this is the fifth problem on the page, students may recognize the division expression very quickly and realize that 5 divided by 40 yields 5 fortieths of the pizza per student. However, in this context, it is interesting to discuss with students the practicality of serving the pizzas in fortieths. Here, one might better ask, "How can I make 40 equal parts out of 5 pizzas?" This question leads to thinking about making the least number of cuts to each pizza—eighths. Now, the simplified answer of 1 eighth of a pizza per student makes more sense. The follow-up question highlights the changing of the unit from *how much pizza per student* (1 eighth of a pizza) to *what fraction of the total* (1 fortieth of the total amount). Because there are so many slices to be made, students may use the *dot, dot, dot* format to show the smaller units in their tape diagram. Others may opt to simply show their work with an equation.

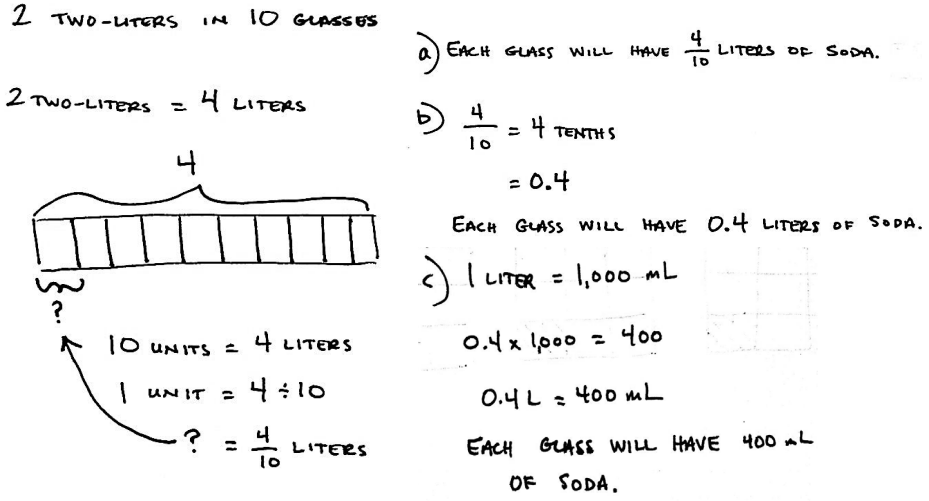
Problem 6

Lillian had 2 two-liter bottles of soda, which she distributed equally between 10 glasses.

a. How much soda was in each glass? Express your answer as a fraction of a liter.

b. Express your answer as a decimal number of liters.

c. Express your answer as a whole number of milliliters.



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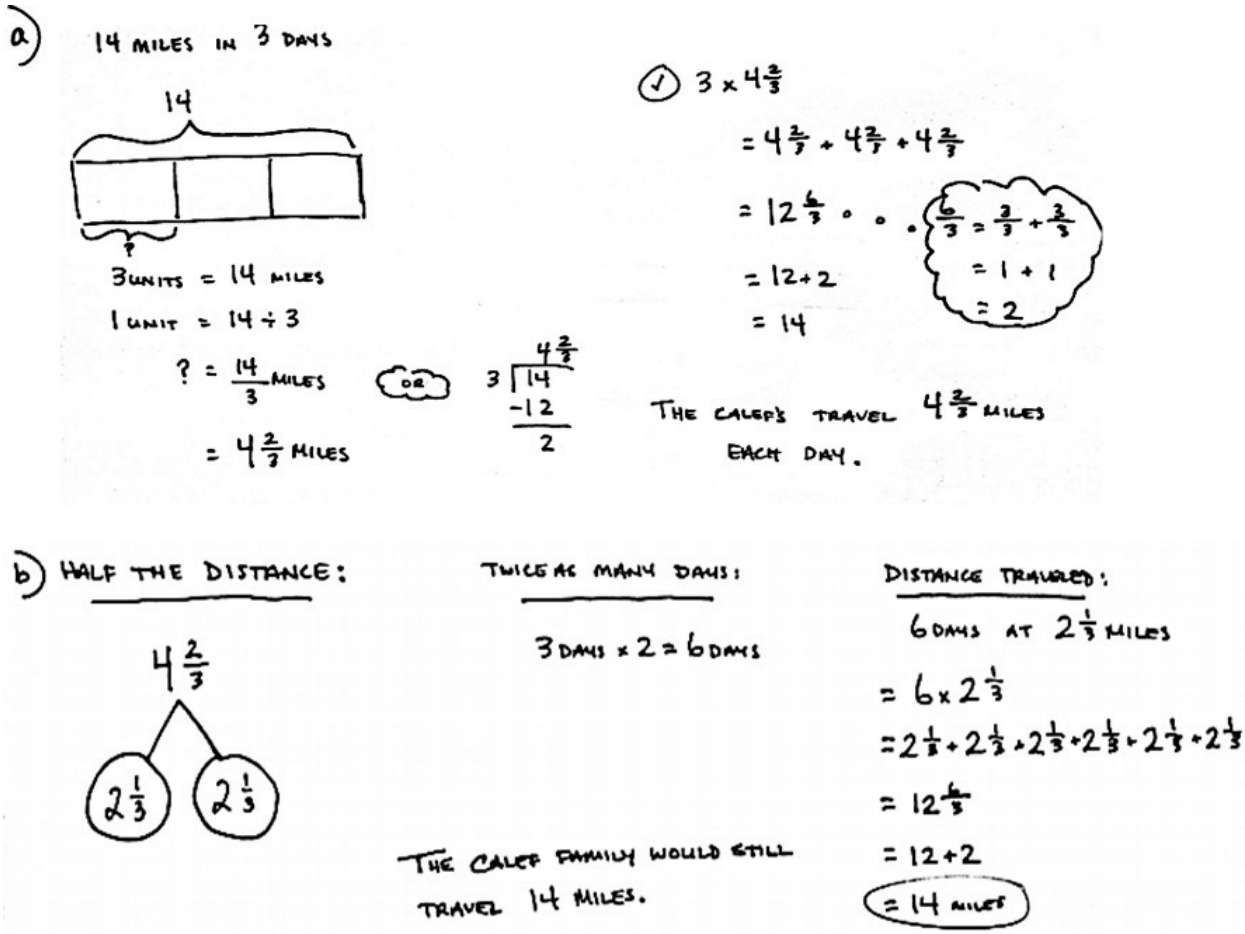
This is a three-part problem that asks students to find the amount of soda in each glass. Carefully guide students when reading the problem so they can interpret that 2 two-liter bottles are equal to 4 liters total. The whole of 4 liters is then divided by 10 glasses to obtain 4 tenths liters of soda per glass. To answer Part (b), students must remember how to express fractions as decimals (i.e., $\frac{1}{10}$ = 0.1, $\frac{1}{100}$ = 0.01, and $\frac{1}{1{,}000}$ = 0.001). For Part (c), students may need to be reminded about the equivalency between liters and milliliters (1 L = 1,000 mL).

Problem 7

The Calef family likes to paddle along the Susquehanna River.

a. They paddled the same distance each day throughout the course of 3 days, traveling a total of 14 miles. How many miles did they travel each day? Show your thinking in a tape diagram.

b. If the Calefs went half their daily distance each day but extended their trip to twice as many days, how far would they travel?



In Part (a), students can easily use the standard algorithm to solve 14 miles divided by 3 days and determine that it is equal to 4 and 2 thirds miles per day. Part (b) requires some deliberate thinking. Guide students to read the question carefully before solving it.

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Student Debrief (10 minutes)

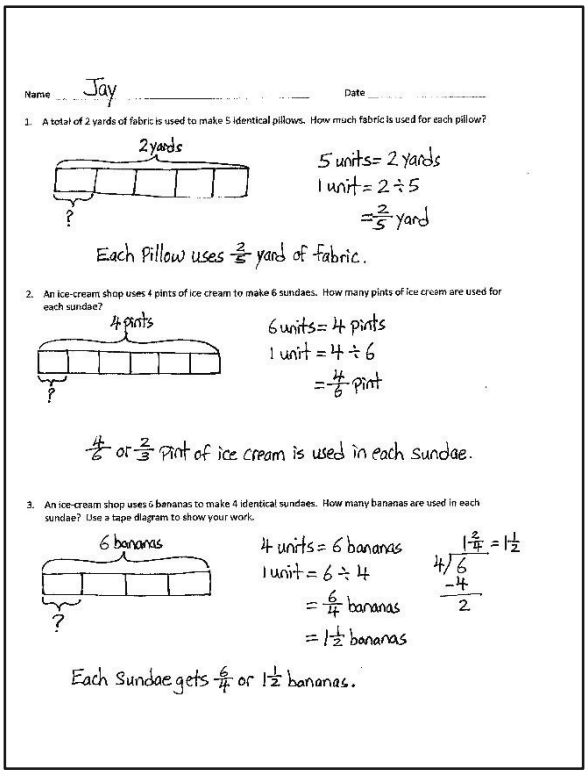
Lesson Objective: Solve word problems involving the division of whole numbers with answers in the form of fractions or whole numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How are the problems similar? How are they different?
- How was your solution the same as and different from those that were demonstrated?
- Did you see other solutions that surprised you or made you see the problems differently?
- Why should we assess reasonableness after solving?
- Were there problems in which it made more sense to express the answer as a fraction rather than a mixed number and vice versa? Give examples.



Name Jay Date

1. A total of 2 yards of fabric is used to make 5 identical pillows. How much fabric is used for each pillow?

2 yards

5 units = 2 yards
1 unit = 2 ÷ 5
= 2/5 yard

Each pillow uses 2/5 yard of fabric.

2. An ice-cream shop uses 4 pints of ice cream to make 6 sundaes. How many pints of ice cream are used for each sundae?

4 pints

6 units = 4 pints
1 unit = 4 ÷ 6
= 4/6 pint

4/6 or 2/3 pint of ice cream is used in each sundae.

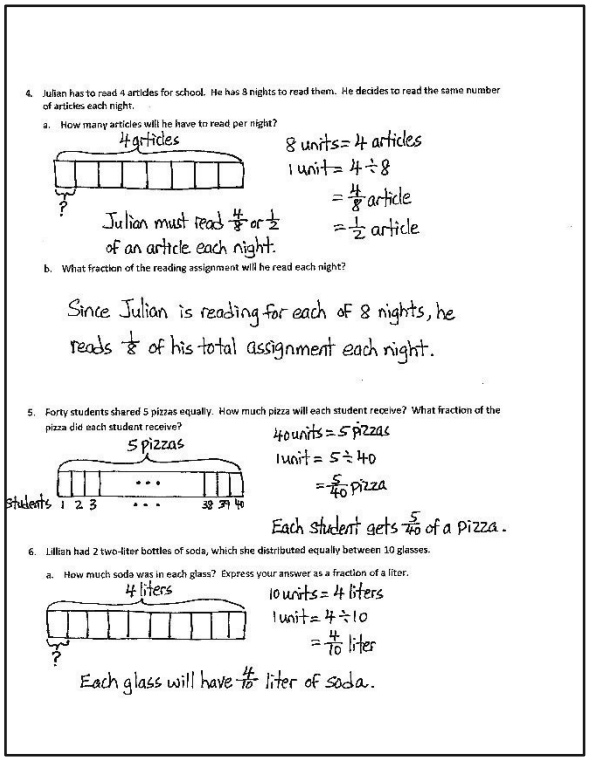
3. An ice-cream shop uses 6 bananas to make 4 identical sundaes. How many bananas are used in each sundae? Use a tape diagram to show your work.

6 bananas

4 units = 6 bananas
1 unit = 6 ÷ 4
= 6/4 bananas
= 1 1/2 bananas

1 2/4 = 1 1/2
4 | 6
−4
2

Each sundae gets 6/4 or 1 1/2 bananas.



4. Julian has to read 4 articles for school. He has 8 nights to read them. He decides to read the same number of articles each night.

a. How many articles will he have to read per night?

4 articles

8 units = 4 articles
1 unit = 4 ÷ 8
= 4/8 article
= 1/2 article

Julian must read 4/8 or 1/2 of an article each night.

b. What fraction of the reading assignment will he read each night?

Since Julian is reading for each of 8 nights, he reads 1/8 of his total assignment each night.

5. Forty students shared 5 pizzas equally. How much pizza will each student receive? What fraction of the pizza did each student receive?

5 pizzas

Students 1 2 3 ... 38 39 40

40 units = 5 pizzas
1 unit = 5 ÷ 40
= 5/40 pizza

Each student gets 5/40 of a pizza.

6. Lillian had 2 two-liter bottles of soda, which she distributed equally between 10 glasses.

a. How much soda was in each glass? Express your answer as a fraction of a liter.

4 liters

10 units = 4 liters
1 unit = 4 ÷ 10
= 4/10 liter

Each glass will have 4/10 liter of soda.

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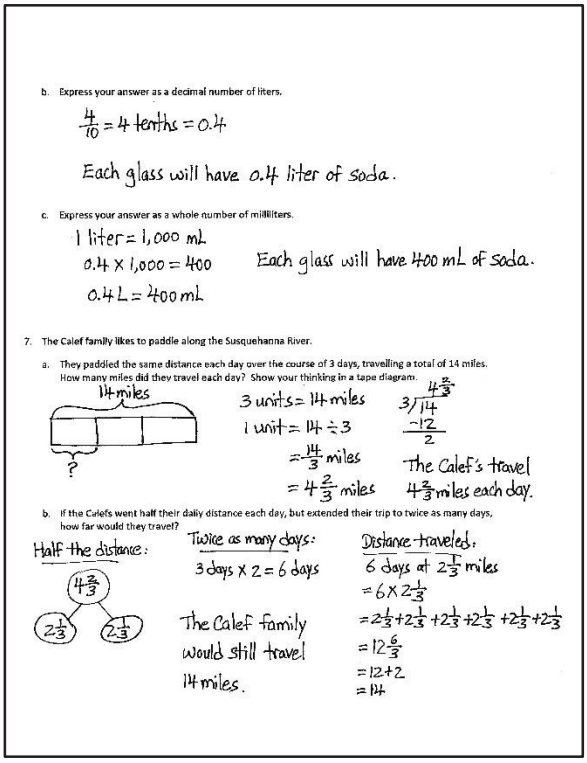
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



b. Express your answer as a decimal number of liters.

$\frac{4}{10} = 4$ tenths $= 0.4$

Each glass will have 0.4 liter of soda.

c. Express your answer as a whole number of milliliters.

1 liter = 1,000 mL

$0.4 \times 1{,}000 = 400$

$0.4\,L = 400$ mL

Each glass will have 400 mL of soda.

7. The Calef family likes to paddle along the Susquehanna River.

a. They paddled the same distance each day over the course of 3 days, travelling a total of 14 miles. How many miles did they travel each day? Show your thinking in a tape diagram.

14 miles

?

3 units = 14 miles

1 unit = $14 \div 3$

$= \frac{14}{3}$ miles

$= 4\frac{2}{3}$ miles

$3\overline{)14}$ $4\frac{2}{3}$, -12, 2

The Calef's travel $4\frac{2}{3}$ miles each day.

b. If the Calefs went half their daily distance each day, but extended their trip to twice as many days, how far would they travel?

Half the distance:

$4\frac{2}{3}$

$2\frac{1}{3}$ $2\frac{1}{3}$

Twice as many days:

3 days $\times 2 = 6$ days

The Calef family would still travel 14 miles.

Distance traveled:

6 days at $2\frac{1}{3}$ miles

$= 6 \times 2\frac{1}{3}$

$= 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3}$

$= 12\frac{6}{3}$

$= 12 + 2$

$= 14$

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Name _____ Date _____

1. A total of 2 yards of fabric is used to make 5 identical pillows. How much fabric is used for each pillow?

2. An ice cream shop uses 4 pints of ice cream to make 6 sundaes. How many pints of ice cream are used for each sundae?

3. An ice cream shop uses 6 bananas to make 4 identical sundaes. How many bananas are used in each sundae? Use a tape diagram to show your work.

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4. Julian has to read 4 articles for school. He has 8 nights to read them. He decides to read the same number of articles each night.

   a. How many articles will he have to read per night?

   b. What fraction of the reading assignment will he read each night?

5. 40 students shared 5 pizzas equally. How much pizza will each student receive? What fraction of the pizza did each student receive?

6. Lillian had 2 two-liter bottles of soda, which she distributed equally between 10 glasses.

   a. How much soda was in each glass? Express your answer as a fraction of a liter.

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b. Express your answer as a decimal number of liters.

c. Express your answer as a whole number of milliliters.

7. The Calef family likes to paddle along the Susquehanna River.

   a. They paddled the same distance each day over the course of 3 days, traveling a total of 14 miles. How many miles did they travel each day? Show your thinking in a tape diagram.

   b. If the Calefs went half their daily distance each day but extended their trip to twice as many days, how far would they travel?

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Name _____ Date _____

A grasshopper covered a distance of 5 yards in 9 equal hops. How many yards did the grasshopper travel on each hop?

a. Draw a picture to support your work.

b. How many yards did the grasshopper travel after hopping twice?

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Name _____ Date _____

1. When someone donated 14 gallons of paint to Rosendale Elementary School, the fifth grade decided to use it to paint murals. They split the gallons equally among the four classes.

   a. How much paint did each class have to paint their mural?

   b. How much paint will three classes use? Show your thinking using words, numbers, or pictures.

   c. If 4 students share a 30-square-foot wall equally, how many square feet of the wall will be painted by each student?

   d. What fraction of the wall will each student paint?

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2. Craig bought a 3-foot-long baguette and then made 4 equally sized sandwiches with it.

   a. What portion of the baguette was used for each sandwich? Draw a visual model to help you solve this problem.

   b. How long, in feet, is one of Craig's sandwiches?

   c. How many inches long is one of Craig's sandwiches?

3. Scott has 6 days to save enough money for a $45 concert ticket. If he saves the same amount each day, what is the minimum amount he must save each day in order to reach his goal? Express your answer in dollars.

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Topic C

Multiplication of a Whole Number by a Fraction

5.NF.4a, 5.MD.1

Focus Standard:	5.NF.4a	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product of *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
	5.MD.1	Convert among different -sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Instructional Days:	4	
Coherence -Links from:	G4–M5	Fraction Equivalence, Ordering, and Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions

In Topic C, students interpret finding a fraction of a set ($\frac{3}{4}$ of 24) as multiplication of a whole number by a fraction ($\frac{3}{4} \times 24$) and use tape diagrams to support their understandings (**5.NF.4a**). This, in turn, leads students to see division by a whole number as being equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by $\frac{1}{2}$.

Students also use the commutative property to relate fraction of a set to the Grade 4 repeated addition interpretation of multiplication by a fraction. This offers opportunities for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g., $\frac{1}{3}$ minutes = 20 seconds and $2\frac{1}{4}$ feet = 27 inches).

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A Teaching Sequence Toward Mastery of Multiplication of a Whole Number by a Fraction	
Objective 1:	**Relate fractions as division to fraction of a set. (Lesson 6)**
Objective 2:	**Multiply any whole number by a fraction using tape diagrams. (Lesson 7)**
Objective 3:	**Relate a fraction of a set to the repeated addition interpretation of fraction multiplication. (Lesson 8)**
Objective 4:	**Find a fraction of a measurement, and solve word problems. (Lesson 9)**

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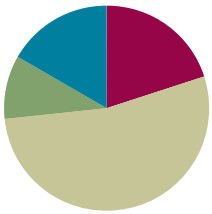
Lesson 6

Objective: Relate fractions as division to fraction of a set.

Suggested Lesson Structure

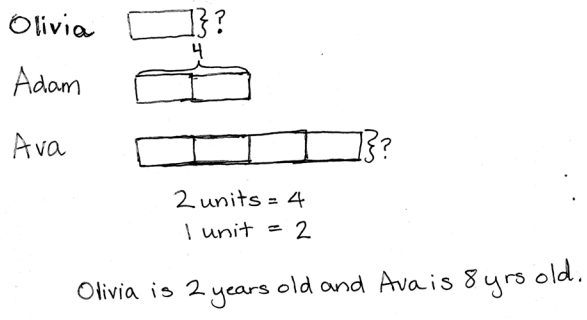


Application Problem	(6 minutes)
Fluency Practice	(12 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Application Problem (6 minutes)

Olivia is half the age of her brother, Adam. Olivia's sister, Ava, is twice as old as Adam. Adam is 4 years old. How old is each sibling? Use tape diagrams to show your thinking.

Note: This Application Problem is intended to activate students' prior knowledge of *half of* in a simple context as a precursor to today's more formalized introduction to fraction of a set.



Fluency Practice (12 minutes)

- Sprint: Divide Whole Numbers **5.NF.3** (8 minutes)
- Fractions as Division **5.NF.3** (4 minutes)

Sprint: Divide Whole Numbers (8 minutes)

Materials: (S) Divide Whole Numbers Sprint

Note: This Sprint reviews Lessons 2–4.

Fractions as Division (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 5.

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T: I'll say a division expression. You write it on your personal white board as a fraction. 4 ÷ 2.

S: $\frac{4}{2}$.

T: 6 ÷ 4.

S: $\frac{6}{4}$.

T: 3 ÷ 4.

S: $\frac{3}{4}$.

T: 2 ÷ 10.

S: $\frac{2}{10}$.

T: Rename this fraction using fifths.

S: $\frac{1}{5}$.

T: (Write $\frac{56}{2}$.) Write the fraction as a division equation and solve.

S: 56 ÷ 2 = 28.

T: Now, I'll say the fraction, and you say the division expression.

Continue with the following possible sequence: 6 thirds, 9 thirds, 18 thirds, 54 thirds, 8 fourths, 12 fourths, 28 fourths, 72 fourths.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If students struggle with the set model of this lesson, consider allowing them to fold a square of paper into the desired fractional parts. Then, have them place the counters in the sections created by the folding.

Concept Development (32 minutes)

Materials: (S) Two-sided counters, drinking straws, personal white board

Problem 1

$\frac{1}{3}$ of 6

$\frac{2}{3}$ of 6

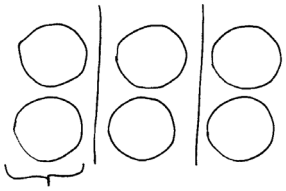
T: Make an array with 6 counters turned to the red side, and use your straws to divide your array into 3 equal parts.

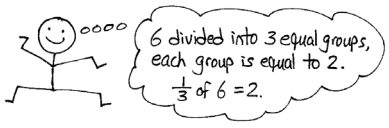
T: Write a division sentence for what you just did.

S: 6 ÷ 3 = 2.

T: Rewrite your division sentence as a fraction, and say it aloud as you write it.

S: (Write $\frac{6}{3}$ = 2.) 6 divided by 3 equals 2.





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T: If I want to show 1 third of this set, how many counters should I turn over to yellow? Turn and talk.

S: Two counters. → Each group is 1 third of all the counters, so we would have to turn over 1 group of 2 counters. → Six divided by 3 tells us there are 2 in each group.

T: 1 third of 6 is equal to …?

S: 2.

T: (Write $\frac{1}{3}$ of 6 = 2.) How many counters should be turned over to show 2 thirds? Whisper to your partner how you know.

S: I can count from our array. 1 third is 2 counters, so 2 thirds is 4 counters. → 6 divided by 3 once is 2 counters. Double that is 4 counters. → I know 1 group out of 3 groups is 2 counters, so 2 groups out of 3 would be 4 counters. → Since $\frac{1}{3}$ of 6 is equal to 2, then $\frac{2}{3}$ of 6 is double that. 2 plus 2 is 4. → (6 ÷ 3) × 2, but I wrote 6 ÷ 3 as a fraction.

T: (Write $\frac{2}{3}$ of 6.) What is 2 thirds of 6 counters?

S: 4 counters.

T: (Write $\frac{3}{3}$ of 6 = ___.) What is 3 thirds of 6 counters?

S: 6 counters.

T: How do you know? Turn and discuss with your partner.

S: I counted 2, 4, 6. → $\frac{3}{3}$ is a whole, and our whole set is 6 counters.

T: Following this pattern, what is 4 thirds of 6?

S: It would be more than 6. → It would be more than the whole set. We would have to add 2 more counters. It would be 8. → 6 divided by 3 times 4 is 8.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

It is acceptable for students to orient their arrays in either direction. For example, in Problem 2, students may arrange their counters in the 3 × 4 arrangement pictured, or they may show a 4 × 3 array that is divided by the straws horizontally.

Problem 2

$\frac{1}{4}$ of 12

T: Make an array using 12 counters turned to the red side. Use your straws to divide the array into fourths. (Draw an array on the board.)

T: How many counters did you place in each fourth?

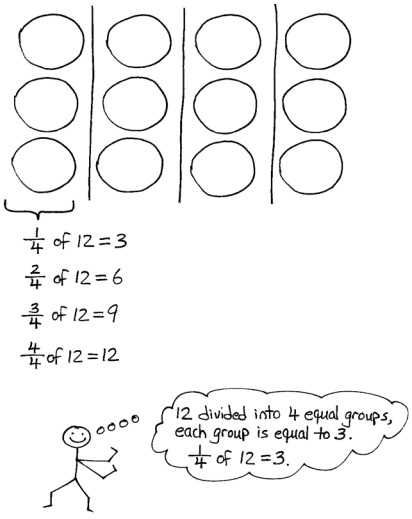
S: 3.

T: Write the division sentence as a fraction on your board.

S: $\frac{12}{4} = 3$.

T: What is 1 fourth of 12?

S: 3.



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T: (Write $\frac{1}{4}$ of 12 = 3.) 1 fourth of 12 is equal to 3. Look at your array. What fraction of 12 is equal to 6 counters? Turn and discuss with your partner.

S: I see 2 groups is equal to 6, so the answer is $\frac{2}{4}$. → Since 1 fourth is equal to 3, and 6 is double that much, I can double 1 fourth to get 2 fourths.

T: (Write $\frac{2}{4}$ of 12 = 6.) 2 fourths of 12 is equal to 6. What is another way to say 2 fourths?

S: 1 half.

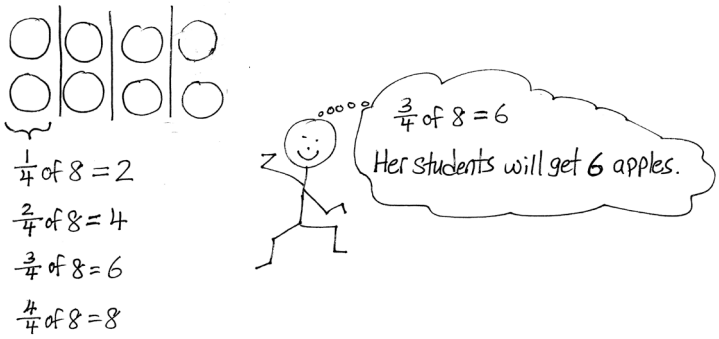
T: Is 1 half of 12 equal to 6?

S: Yes.

Continue with the following possible sequence: $\frac{1}{3}$ of 9, $\frac{1}{6}$ of 12, and $\frac{1}{5}$ of 15.

Problem 3

Mrs. Pham has 8 apples. She wants to give $\frac{3}{4}$ of the apples to her students. How many apples will her students get?



T: Use your counters or draw an array to show how many apples Mrs. Pham has.

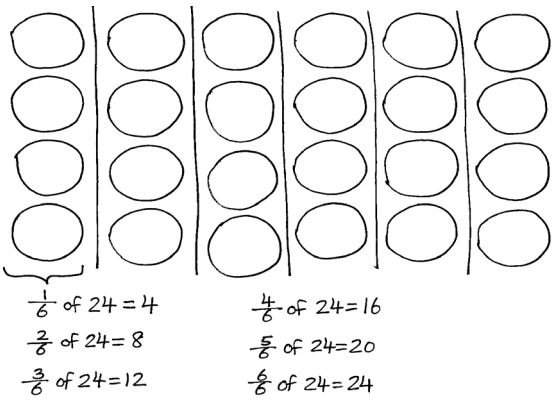
S: (Represent 8 apples.)

T: (Write $\frac{3}{4}$ of 8 = ___.) How will we find 3 fourths of 8? Turn and talk.

S: I divided my counters to make 4 equal parts. Then, I counted the number in 3 of those parts. → I can draw 4 rows of 2 and count 2, 4, 6, so the answer is 6 apples. → I need to make fourths. That's 4 equal parts, but I only want to know about 3 of them. There are 2 in 1 part and 6 in 3 parts. → I know if 1 fourth is equal to 2, then 3 fourths is 3 groups of 2. The answer is 6 apples.

Problem 4

In a class of 24 students, $\frac{5}{6}$ are boys. How many boys are in the class?



T: How many students are in the whole class?

S: 24.

T: What is the question?

S: How many boys are in the class?

T: What fraction of the whole class of 24 is boys?

S: $\frac{5}{6}$.

T: Will our answer be more than half of the class or less than half? How do you know? Turn and talk.

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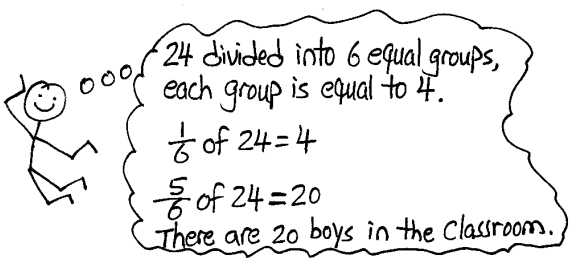
S: 5 sixths is more than half, so the answer should be more than 12. → Half of the class would be 12, which would also be 3 sixths. We need more sixths than that, so our answer will be more than 12.

T: (Write $\frac{5}{6}$ of 24 on the board.) Use your counters or draw to solve. Turn and discuss with a partner.

S: We should draw a total of 24 circles and then split them into 6 equal groups. → We will have 6 columns representing 6 groups, and each group will have 4 circles. → We could draw 6 rows of 4 circles to show 6 equal parts. We only care how many are in 5 of the rows, and 5 × 4 is 20 boys. → We need to find sixths, so we need to divide the set of 24 into 6 equal parts. We only need to know how many are in 5 of the equal parts. That's 4, 8, 12, 16, 20. There are 20 boys in the class.

T: (Point to the drawing on the board.) Let's think of this another way. What is $\frac{1}{6}$ of 24?



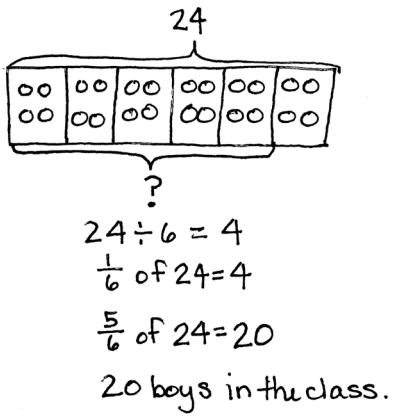
S: 4.

T: How do we know? Say the division sentence.

S: 24 ÷ 6 = 4.

T: How can we use $\frac{1}{6}$ of 24 to help us solve for $\frac{5}{6}$ of 24? Whisper and tell your partner.

S: $\frac{1}{6}$ of 24 is equal to 4. $\frac{5}{6}$ of 24 is just 5 groups of 4. 4 + 4 + 4 + 4 + 4 = 20. → I know each group is 4. To find 5 groups, I can multiply 5 × 4 = 20. → $\frac{24}{6}$ (24 divided by 6) times 5 is 20.



T: I'm going to rearrange the circles a bit. (Draw a bar directly above the array, and label it as 24.) We said we needed to find sixths, so how many units should I cut the whole into?

S: We need 6 units to be the same size.

T: (Cut the bar into 6 equal parts.) If 6 units are 24, how many circles are in one unit? How do you know?

S: 4 because 24 ÷ 6 is 4.

T: (Write $\frac{1}{6}$ of 24 = 4 under the bar.) Let me draw 4 counters into each unit. Count with me as I write.

S: 4, 8, 12, 16, 20, 24.

T: We are only interested in the part of the class that is boys. How many of these units represent the boys in the class?

S: 5 units. → 5 sixths.

T: What are 5 units worth? Or what is 5 sixths of 24? (Draw a bracket around 5 units, and write $\frac{5}{6}$ of 24.)

S: 20.

T: (Write the answer on the board.)

T: Answer the question with a sentence.

S: There are 20 boys in the class.

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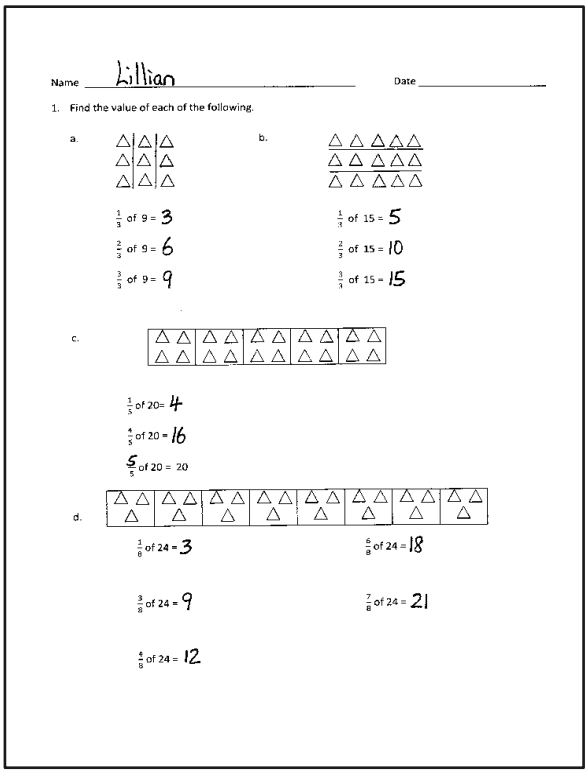
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Lillian Date

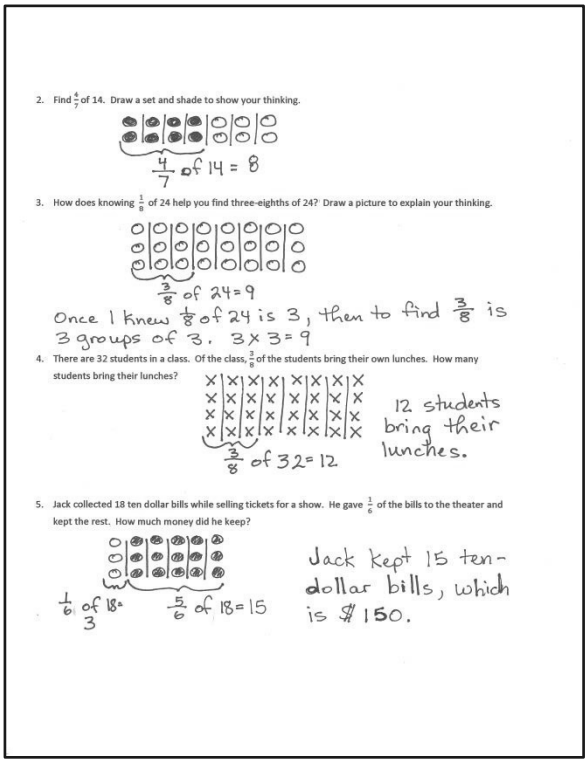
1. Find the value of each of the following.

a. $\frac{1}{3}$ of 9 = 3; $\frac{2}{3}$ of 9 = 6; $\frac{3}{3}$ of 9 = 9

b. $\frac{1}{3}$ of 15 = 5; $\frac{2}{3}$ of 15 = 10; $\frac{3}{3}$ of 15 = 15

c. $\frac{1}{5}$ of 20 = 4; $\frac{4}{5}$ of 20 = 16; $\frac{5}{5}$ of 20 = 20

d. $\frac{1}{8}$ of 24 = 3; $\frac{3}{8}$ of 24 = 9; $\frac{4}{8}$ of 24 = 12; $\frac{6}{8}$ of 24 = 18; $\frac{7}{8}$ of 24 = 21


2. Find $\frac{4}{7}$ of 14. Draw a set and shade to show your thinking.

$\frac{4}{7}$ of 14 = 8

3. How does knowing $\frac{1}{8}$ of 24 help you find three-eighths of 24? Draw a picture to explain your thinking.

$\frac{3}{8}$ of 24 = 9

Once I knew $\frac{1}{8}$ of 24 is 3, then to find $\frac{3}{8}$ is 3 groups of 3. 3 × 3 = 9

4. There are 32 students in a class. Of the class, $\frac{3}{8}$ of the students bring their own lunches. How many students bring their lunches?

$\frac{3}{8}$ of 32 = 12

12 students bring their lunches.

5. Jack collected 18 ten dollar bills while selling tickets for a show. He gave $\frac{1}{6}$ of the bills to the theater and kept the rest. How much money did he keep?

$\frac{1}{6}$ of 18 = 3; $\frac{5}{6}$ of 18 = 15

Jack kept 15 ten-dollar bills, which is $150.

Student Debrief (10 minutes)

Lesson Objective: Relate fractions as division to fraction of a set.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice in Problem 1(a)? (Students may say it skip-counts by threes, or all the answers are multiples of 3.) Based on this pattern, what do you think the answer for $\frac{4}{3}$ of 9 is? Why is this more than 9? (Because 4 thirds is more than a whole, and 9 is the whole.)
- How did you solve for the last question in 1(c)? Explain to your partner.
- In Problem 1(d), what did you notice about the two fractions $\frac{4}{8}$ and $\frac{6}{8}$? Can you name either of them using larger units (simplify them)? What connections did you make about $\frac{4}{8}$ of 24 and $\frac{1}{2}$ of 24, as well as $\frac{6}{8}$ of 24 and $\frac{3}{4}$ of 24?

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- When solving these problems (fraction of a set), how important is it to first determine how many are in each group (unit)? Explain your thinking to a partner.
- Is this a true statement? (Write $\frac{2}{3}$ of 18 = $\frac{18}{3}$ × 2.) Is two-thirds of 18 the same as 18 divided by 3, times 2? Why or why not?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Number Correct: _____

A

Divide Whole Numbers

1.	1 ÷ 2 =	
2.	1 ÷ 3 =	
3.	1 ÷ 8 =	
4.	2 ÷ 2 =	
5.	2 ÷ 3 =	
6.	3 ÷ 3 =	
7.	3 ÷ 4 =	
8.	3 ÷ 10 =	
9.	3 ÷ 5 =	
10.	5 ÷ 5 =	
11.	6 ÷ 5 =	
12.	7 ÷ 5 =	
13.	9 ÷ 5 =	
14.	2 ÷ 3 =	
15.	4 ÷ 4 =	
16.	5 ÷ 4 =	
17.	7 ÷ 4 =	
18.	4 ÷ 2 =	
19.	5 ÷ 2 =	
20.	10 ÷ 5 =	
21.	11 ÷ 5 =	
22.	13 ÷ 5 =	

23.	6 ÷ 2 =	
24.	7 ÷ 2 =	
25.	8 ÷ 8 =	
26.	9 ÷ 8 =	
27.	15 ÷ 8 =	
28.	8 ÷ 4 =	
29.	11 ÷ 4 =	
30.	15 ÷ 2 =	
31.	24 ÷ 5 =	
32.	17 ÷ 4 =	
33.	20 ÷ 3 =	
34.	13 ÷ 6 =	
35.	30 ÷ 7 =	
36.	27 ÷ 8 =	
37.	49 ÷ 9 =	
38.	29 ÷ 6 =	
39.	47 ÷ 7 =	
40.	53 ÷ 8 =	
41.	67 ÷ 9 =	
42.	59 ÷ 6 =	
43.	63 ÷ 8 =	
44.	71 ÷ 9 =	

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B

Number Correct: _____

Improvement: _____

Divide Whole Numbers

1.	1 ÷ 3 =	
2.	1 ÷ 4 =	
3.	1 ÷ 10 =	
4.	5 ÷ 5 =	
5.	5 ÷ 6 =	
6.	3 ÷ 3 =	
7.	3 ÷ 7 =	
8.	3 ÷ 10 =	
9.	3 ÷ 4 =	
10.	4 ÷ 4 =	
11.	5 ÷ 4 =	
12.	2 ÷ 2 =	
13.	3 ÷ 2 =	
14.	4 ÷ 5 =	
15.	10 ÷ 10 =	
16.	11 ÷ 10 =	
17.	13 ÷ 10 =	
18.	10 ÷ 5 =	
19.	11 ÷ 5 =	
20.	13 ÷ 5 =	
21.	4 ÷ 2 =	
22.	5 ÷ 2 =	

23.	15 ÷ 5 =	
24.	16 ÷ 5 =	
25.	6 ÷ 6 =	
26.	7 ÷ 6 =	
27.	11 ÷ 6 =	
28.	6 ÷ 3 =	
29.	8 ÷ 3 =	
30.	13 ÷ 2 =	
31.	23 ÷ 5 =	
32.	15 ÷ 4 =	
33.	19 ÷ 4 =	
34.	19 ÷ 6 =	
35.	31 ÷ 7 =	
36.	37 ÷ 8 =	
37.	50 ÷ 9 =	
38.	17 ÷ 6 =	
39.	48 ÷ 7 =	
40.	51 ÷ 8 =	
41.	68 ÷ 9 =	
42.	53 ÷ 6 =	
43.	61 ÷ 8 =	
44.	70 ÷ 9 =	

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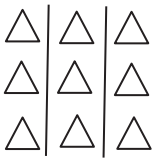
Name _____ Date _____

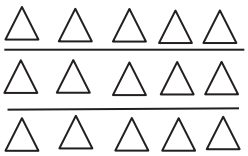
1. Find the value of each of the following.

a. 

$\frac{1}{3}$ of 9 =

$\frac{2}{3}$ of 9 =

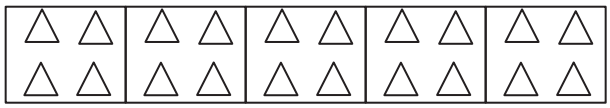
$\frac{3}{3}$ of 9 =

b. 

$\frac{1}{3}$ of 15 =

$\frac{2}{3}$ of 15 =

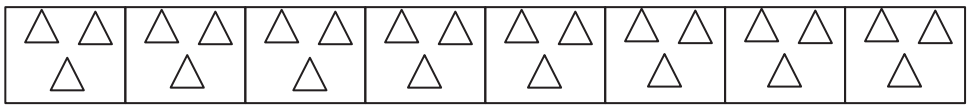
$\frac{3}{3}$ of 15 =

c. 

$\frac{1}{5}$ of 20 =

$\frac{4}{5}$ of 20 =

$\frac{\quad}{5}$ of 20 = 20

d. 

$\frac{1}{8}$ of 24 =

$\frac{6}{8}$ of 24 =

$\frac{3}{8}$ of 24 =

$\frac{7}{8}$ of 24 =

$\frac{4}{8}$ of 24 =

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2. Find $\frac{4}{7}$ of 14. Draw a set, and shade to show your thinking.

3. How does knowing $\frac{1}{8}$ of 24 help you find three-eighths of 24? Draw a picture to explain your thinking.

4. There are 32 students in a class. Of the class, $\frac{3}{8}$ of the students bring their own lunches. How many students bring their lunches?

5. Jack collected 18 ten-dollar bills while selling tickets for a show. He gave $\frac{1}{6}$ of the bills to the theater and kept the rest. How much money did he keep?

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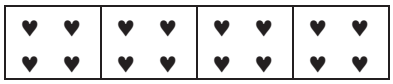
Name _____ Date _____

1. Find the value of each of the following.



a. $\frac{1}{4}$ of 16 =

b. $\frac{3}{4}$ of 16 =

2. Out of 18 cookies, $\frac{2}{3}$ are chocolate chip. How many of the cookies are chocolate chip?

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Name _____ Date _____

1. Find the value of each of the following.

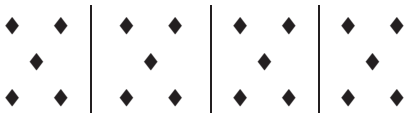
a.

$\frac{1}{3}$ of 12 =

$\frac{2}{3}$ of 12 =

$\frac{3}{3}$ of 12 =

b.



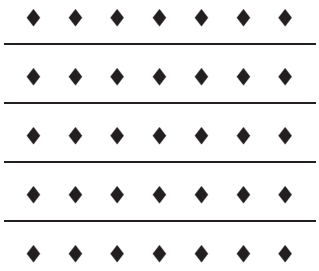
$\frac{1}{4}$ of 20 =

$\frac{2}{4}$ of 20 =

$\frac{3}{4}$ of 20 =

$\frac{4}{4}$ of 20 =

c.



$\frac{1}{5}$ of 35 =

$\frac{2}{5}$ of 35 =

$\frac{3}{5}$ of 35 =

$\frac{4}{5}$ of 35 =

$\frac{5}{5}$ of 35 =

$\frac{6}{5}$ of 35 =

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2. Find $\frac{2}{3}$ of 18. Draw a set and shade to show your thinking.

3. How does knowing $\frac{1}{5}$ of 10 help you find $\frac{3}{5}$ of 10? Draw a picture to explain your thinking.

4. Sara just turned 18 years old. She spent $\frac{4}{9}$ of her life living in Rochester, NY. How many years did Sara live in Rochester?

5. A farmer collected 12 dozen eggs from her chickens. She sold $\frac{5}{6}$ of the eggs at the farmers' market and gave the rest to friends and neighbors.

   a. How many dozen eggs did the farmer give away? How many eggs did she give away?

   b. She sold each dozen for $4.50. How much did she earn from the eggs she sold?

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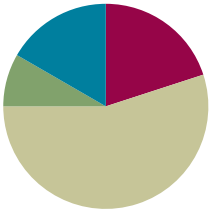
Lesson 7

Objective: Multiply any whole number by a fraction using tape diagrams.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Read Tape Diagrams **5.NF.4** (4 minutes)
- Half of Whole Numbers **5.NF.4** (4 minutes)
- Fractions as Whole Numbers **5.NF.3** (4 minutes)

Read Tape Diagrams (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students to multiply fractions by whole numbers during the Concept Development.

T: (Project a tape diagram with 10 partitioned into 2 equal units.) Say the whole.
S: 10.
T: On your personal white board, write the division sentence.
S: (Write 10 ÷ 2 = 5.)

Continue with the following possible sequence: 6 ÷ 2, 9 ÷ 3, 12 ÷ 3, 8 ÷ 4, 12 ÷ 4, 25 ÷ 5, 40 ÷ 5, 42 ÷ 6, 63 ÷ 7, 64 ÷ 8, and 54 ÷ 9.

Half of Whole Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews content from Lesson 6 and prepares students for multiplying fractions by whole numbers during the Concept Development using tape diagrams.

T: Draw 4 counters. What's half of 4?
S: 2.

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T: (Write $\frac{1}{2}$ of 4 = 2.) Say a division sentence that helps you find the answer.

S: 4 ÷ 2 = 2.

Continue with the following possible sequence: 1 half of 10, 1 half of 8, 1 half of 30, 1 half of 54, 1 fourth of 20, 1 fourth of 16, 1 third of 9, and 1 third of 18.

Fractions as Whole Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 5, as well as denominators that are equivalent to hundredths. Instruct students to use their personal white boards for calculations that they cannot do mentally.

T: I'll say a fraction. You say it as a division problem. 4 halves.

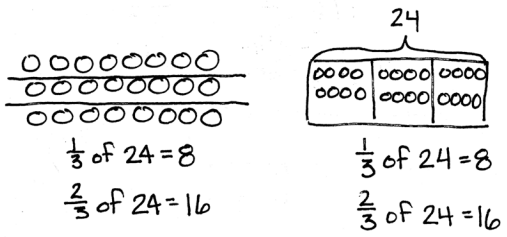
S: 4 ÷ 2 = 2.

Continue with the following possible suggestions:
$\frac{6}{2}, \frac{14}{2}, \frac{54}{2}, \frac{40}{20}, \frac{80}{20}, \frac{180}{20}, \frac{960}{20}, \frac{10}{5}, \frac{15}{5}, \frac{35}{5}, \frac{85}{5}, \frac{100}{50}, \frac{150}{50}, \frac{300}{50}, \frac{900}{50}, \frac{8}{4}, \frac{12}{4}, \frac{24}{4}, \frac{96}{4}, \frac{50}{25}, \frac{75}{25}$, and $\frac{800}{25}$.

Application Problem (5 minutes)

Mr. Peterson bought a case (24 boxes) of fruit juice. One-third of the drinks were grape, and two-thirds were cranberry. How many boxes of each flavor did Mr. Peterson buy? Show your work using a tape diagram or an array.

Note: This Application Problem requires students to use skills explored in Lesson 6. Students are finding fractions of a set and showing their thinking with models.



Mr. Peterson bought 8 boxes of grape juice and 16 boxes of cranberry juice.

Concept Development (33 minutes)

Materials: (S) Personal white board

Problem 1

What is $\frac{3}{5}$ of 35?

T: (Write $\frac{3}{5}$ of 35 = ___ on the board.) We used two different models (counters and arrays) yesterday to find fractions of sets. We will use tape diagrams to help us today.

T: We must find 3 fifths of 35. Draw a bar to represent our whole. What's our whole?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Please note that, throughout the lesson, division sentences are written as fractions to reinforce the interpretation of a fraction as division. When reading the fraction notation, the language of division should be used. For example, in Problem 1, 1 unit = $\frac{35}{5}$ should be read as 1 unit equals 35 divided by 5.

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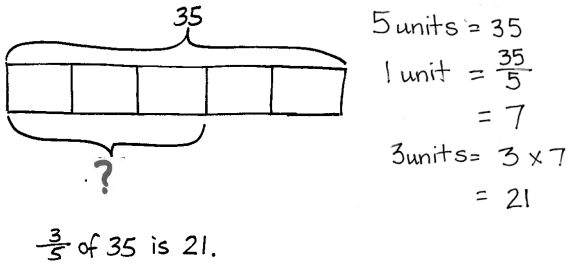
S: (Draw.) 35.

T: (Draw a bar, and label it as 35.) How many units should we cut the whole into?

S: 5.

T: How do you know?

S: The denominator tells us we want fifths. → That is the unit being named by the fraction. → We are asked about fifths, so we know we need 5 equal parts.



T: Divide your bar into fifths.

S: (Work.)

T: (Cut the bar into 5 equal units.) We know 5 units are equal to 35. How do we find the value of 1 unit? Say the division sentence.

S: 35 ÷ 5 = 7.

T: (Write 5 units = 35, 1 unit = 35 ÷ 5 = 7.) Have we answered our question?

S: No. We found 1 unit is equal to 7, but the question is to find 3 units. → We need 3 fifths. When we divide by 5, that's just 1 fifth of 35.

T: How will we find 3 units?

S: Multiply 3 and 7 to get 21. → We could add 7 + 7 + 7. → We could put 3 of the 1 fifths together. That would be 21.

T: What is $\frac{3}{5}$ of 35?

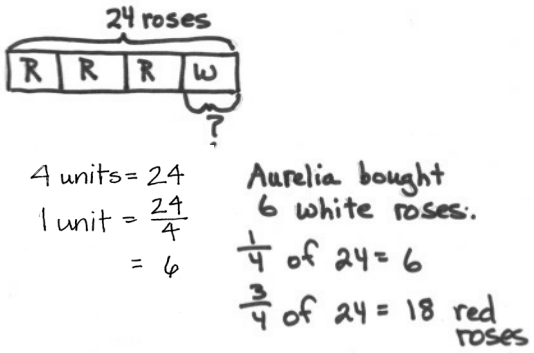
S: 21.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Students with fine motor deficits may find drawing tape diagrams difficult. Graph paper may provide some support. Online sources, such as the Thinking Blocks website, may also be helpful.

Problem 2

Aurelia buys 2 dozen roses. Of these roses, $\frac{3}{4}$ are red, and the rest are white. How many white roses did she buy?



T: What do you know about this problem? Turn and share with your partner.

S: I know the whole is 2 dozen, which is 24. → $\frac{3}{4}$ are red roses, and $\frac{1}{4}$ are white roses. The total is 24 roses. → The information in the problem is about red roses, but the question is about the other part—the white roses.

T: Discuss with your partner how you'll solve this problem.

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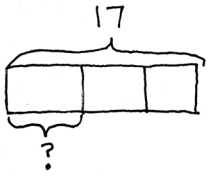
S: We can first find the total red roses and then subtract from 24 to get the white roses. → Since I know $\frac{1}{4}$ of the whole is white roses, I can find $\frac{1}{4}$ of 24 to find the white roses. That's faster.

T: Work with a partner to draw a tape diagram and solve.

S: (Work.)

T: Answer the question for this problem.

S: She bought 6 white roses.



3 units = 17

1 unit = $\frac{17}{3}$

= $5\frac{2}{3}$

$3\overline{)17}$ $5\frac{2}{3}$, -15, 2

Rosie used $5\frac{2}{3}$ yards of fabric.

Problem 3

Rosie had 17 yards of fabric. She used one-third of it to make a quilt. How many yards of fabric did Rosie use for the quilt?

T: What can you draw? Turn and share with your partner.

T: Compare this problem to the others we've done today.

S: The answer is not a whole number. → The quotient is not a whole number. → We were still looking for fractional parts, but the answer isn't a whole number.

T: We can draw a bar that shows 17 and divide it into thirds. How do we find the value of one unit?

S: Divide 17 by 3.

T: How much fabric is one-third of 17 yards?

S: $\frac{17}{3}$ yards. → $5\frac{2}{3}$ yards.

T How would you find 2 thirds of 17?

S: Double $5\frac{2}{3}$. → Multiply $5\frac{2}{3}$ times 2. → Subtract $5\frac{2}{3}$ from 17.

Repeat this sequence with $\frac{2}{5}$ of 11, if necessary.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

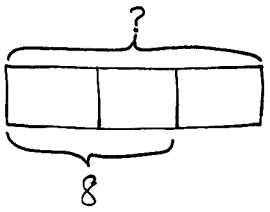
The added complexity of finding a fraction of a quantity that is not a multiple of the denominator may require a return to concrete materials for some students. Allow them access to materials that can be folded and cut to model Problem 3 physically. Five whole squares can be distributed into each unit of 1 third. Then, the remaining whole squares can be cut into thirds and distributed among the units of thirds. Be sure to make the connection to the fraction form of the division sentence and the written recording of the division algorithm.

Problem 4

$\frac{2}{3}$ of a number is 8. What is the number?

T: How is this problem different from the ones we just solved?

S: In the first problem, we knew the total and wanted to find a part of it. In this one, we know how much 2 thirds is but not the whole. → Last time, they told us the whole and asked us about a part. This time, they told us about a part and asked us to find the whole.



2 units = 8

1 unit = $\frac{8}{2}$

= 4

3 units = 3 × 4

= 12

The number is 12.

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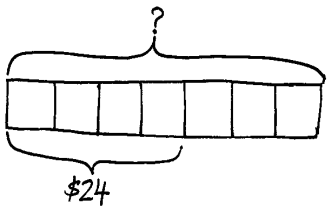
T: Draw a bar to represent the whole. What kind of units will we need to divide the whole into?
S: (Draw.) Thirds.
T: (Draw the bar divided into thirds.) What else do we know? Turn and tell your partner.
S: We know that 2 thirds is the same as 8, so it means we can label 2 of the units with a bracket and 8. → The units are thirds. We know about 2 of them. They are equal to 8 together. We don't know what the whole bar is worth, so we have to put a question mark there.
T: (Draw to show the labeling.) Label your bars.
S: (Label the bars.)
T: How can knowing what 2 units are worth help us find the whole?
S: Since we know that 2 units = 8, we can divide to find that 1 unit is equal to 4.
T: (Write 1 unit = 8 ÷ 2 = 4.) Let's record 4 inside each unit. (Show the recording.)
S: (Record the 4 inside each unit.)
T: Can we find the whole now?
S: Yes. We can add 4 + 4 + 4 = 12. → We can multiply 3 times 4, which is equal to 12.
T: (Write 3 units = 3 × 4 = 12.) Answer the question for this problem.
S: The number is 12.
T: Let's think about it and check to see if it makes sense. (Write $\frac{2}{3}$ of 12 = 8.) Work independently on your personal white board, and solve to find what 2 thirds of 12 is.

Problem 5

Tiffany spent $\frac{4}{7}$ of her money on a teddy bear. If the teddy bear cost $24, how much money did she have at first?



4 units = 24
1 unit = $\frac{24}{4}$
= 6
7 units = 7 × 6
= 42

She had $42 at first.

T: Which problem that we've worked on today is most similar to this one?
S: This one is just like Problem 4. We have information about a part, and we have to find the whole.
T: What can you draw? Turn and share with your partner.
S: We can draw a bar for all of the money. We can show what the teddy bear costs. It costs $24, and it's $\frac{4}{7}$ of her total money. We can put a question mark over the whole bar.
T: Do we have enough information to find the value of 1 unit?
S: Yes.
T: How much is one unit?
S: 4 units = $24, so 1 unit = $6.
T: How will we find the amount of money she had at first?
S: Multiply $6 by 7.

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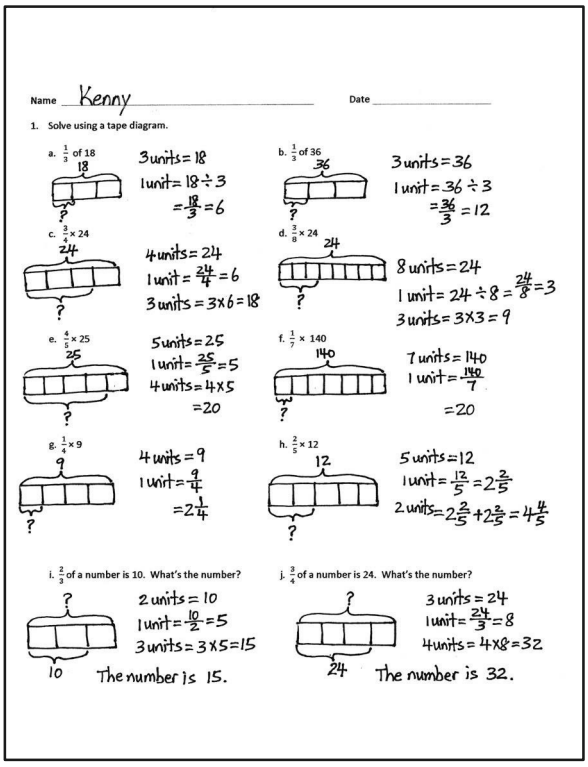
T: Say the multiplication sentence, starting with 7.
S: 7 × $6 = $42.
T: Answer the question in this problem.
S: Tiffany had $42 at first.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Kenny Date

1. Solve using a tape diagram.

a. $\frac{1}{3}$ of 18: 3 units = 18; 1 unit = 18 ÷ 3 = $\frac{18}{3}$ = 6

b. $\frac{1}{3}$ of 36: 3 units = 36; 1 unit = 36 ÷ 3 = $\frac{36}{3}$ = 12

c. $\frac{4}{7} \times 24$: 4 units = 24; 1 unit = $\frac{24}{4}$ = 6; 3 units = 3 × 6 = 18

d. $\frac{3}{8} \times 24$: 8 units = 24; 1 unit = 24 ÷ 8 = $\frac{24}{8}$ = 3; 3 units = 3 × 3 = 9

e. $\frac{4}{5} \times 25$: 5 units = 25; 1 unit = $\frac{25}{5}$ = 5; 4 units = 4 × 5 = 20

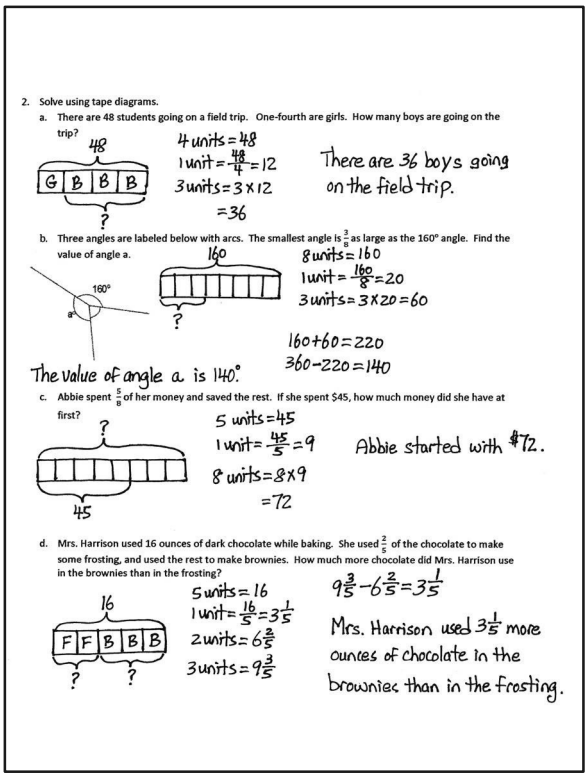
f. $\frac{1}{7} \times 140$: 7 units = 140; 1 unit = $\frac{140}{7}$ = 20

g. $\frac{1}{4} \times 9$: 4 units = 9; 1 unit = $\frac{9}{4}$ = $2\frac{1}{4}$

h. $\frac{2}{5} \times 12$: 5 units = 12; 1 unit = $\frac{12}{5}$ = $2\frac{2}{5}$; 2 units = $2\frac{2}{5} + 2\frac{2}{5}$ = $4\frac{4}{5}$

i. $\frac{2}{3}$ of a number is 10. What's the number? 2 units = 10; 1 unit = $\frac{10}{2}$ = 5; 3 units = 3 × 5 = 15. The number is 15.

j. $\frac{3}{4}$ of a number is 24. What's the number? 3 units = 24; 1 unit = $\frac{24}{3}$ = 8; 4 units = 4 × 8 = 32. The number is 32.


2. Solve using tape diagrams.

a. There are 48 students going on a field trip. One-fourth are girls. How many boys are going on the trip? 4 units = 48; 1 unit = $\frac{48}{4}$ = 12; 3 units = 3 × 12 = 36. There are 36 boys going on the field trip.

b. Three angles are labeled below with arcs. The smallest angle is $\frac{1}{8}$ as large as the 160° angle. Find the value of angle a. 8 units = 160; 1 unit = $\frac{160}{8}$ = 20; 3 units = 3 × 20 = 60; 160 + 60 = 220; 360 − 220 = 140. The value of angle a is 140°.

c. Abbie spent $\frac{5}{8}$ of her money and saved the rest. If she spent $45, how much money did she have at first? 5 units = 45; 1 unit = $\frac{45}{5}$ = 9; 8 units = 8 × 9 = 72. Abbie started with $72.

d. Mrs. Harrison used 16 ounces of dark chocolate while baking. She used $\frac{2}{5}$ of the chocolate to make some frosting, and used the rest to make brownies. How much more chocolate did Mrs. Harrison use in the brownies than in the frosting? 5 units = 16; 1 unit = $\frac{16}{5}$ = $3\frac{1}{5}$; 2 units = $6\frac{2}{5}$; 3 units = $9\frac{3}{5}$; $9\frac{3}{5} - 6\frac{2}{5} = 3\frac{1}{5}$. Mrs. Harrison used $3\frac{1}{5}$ more ounces of chocolate in the brownies than in the frosting.

Student Debrief (10 minutes)

Lesson Objective: Multiply any whole number by a fraction using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern relationships did you notice between Problems 1(a) and 1(b)? (The whole of 36 is twice as much as the whole of 18. 1 third of 36 is twice as much as 1 third of 18. 12 is twice as much as 6.)
- What pattern did you notice between Problems 1(c) and 1(d)? (The wholes are the same. The fraction of 3 eighths is half of 3 fourths. That is why the answer of 9 is also half of 18.)
- Look at Problems 1(e) and 1(f). We know that 4 fifths and 1 seventh aren't equal, so how did we get the same answer?

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- Compare Problems 1(c) and 1(j). How are they similar, and how are they different? (The questions involve the same numbers, but in Problem 1(c), 3 fourths is the unknown quantity, and in Problem 1(j), it is the known quantity. In Problem 1(c), the whole is known, but in Problem 1(j), the whole is unknown.)
- How did you solve for Problem 2(b)? Explain your strategy or solution to a partner.
- There are a couple of different methods to solve Problem 2(c). Find someone who used a different approach from yours, and explain your thinking.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. Solve using a tape diagram.

a. $\frac{1}{3}$ of 18

b. $\frac{1}{3}$ of 36

c. $\frac{3}{4} \times 24$

d. $\frac{3}{8} \times 24$

e. $\frac{4}{5} \times 25$

f. $\frac{1}{7} \times 140$

g. $\frac{1}{4} \times 9$

h. $\frac{2}{5} \times 12$

i. $\frac{2}{3}$ of a number is 10. What's the number?

j. $\frac{3}{4}$ of a number is 24. What's the number?

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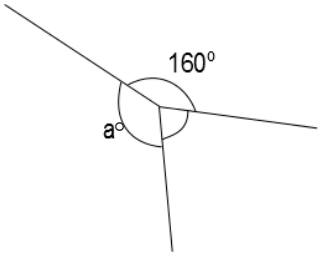
2. Solve using tape diagrams.

   a. There are 48 students going on a field trip. One-fourth are girls. How many boys are going on the trip?

   b. Three angles are labeled below with arcs. The smallest angle is $\frac{3}{8}$ as large as the 160° angle. Find the value of angle a.



   c. Abbie spent $\frac{5}{8}$ of her money and saved the rest. If she spent $45, how much money did she have at first?

   d. Mrs. Harrison used 16 ounces of dark chocolate while baking. She used $\frac{2}{5}$ of the chocolate to make some frosting and used the rest to make brownies. How much more chocolate did Mrs. Harrison use in the brownies than in the frosting?

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Name _____ Date _____

Solve using a tape diagram.

a. $\frac{3}{5}$ of 30

b. $\frac{3}{5}$ of a number is 30. What's the number?

c. Mrs. Johnson baked 2 dozen cookies. Two-thirds of the cookies were oatmeal. How many oatmeal cookies did Mrs. Johnson bake?

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Name _____ Date _____

1. Solve using a tape diagram.

a. $\frac{1}{4}$ of 24

b. $\frac{1}{4}$ of 48

c. $\frac{2}{3} \times 18$

d. $\frac{2}{6} \times 18$

e. $\frac{3}{7} \times 49$

f. $\frac{3}{10} \times 120$

g. $\frac{1}{3} \times 31$

h. $\frac{2}{5} \times 20$

i. $\frac{1}{4} \times 25$

j. $\frac{3}{4} \times 25$

k. $\frac{3}{4}$ of a number is 27. What's the number?

l. $\frac{2}{5}$ of a number is 14. What's the number?

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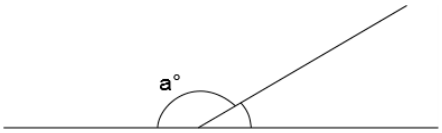
2. Solve using tape diagrams.

   a. A skating rink sold 66 tickets. Of these, $\frac{2}{3}$ were children's tickets, and the rest were adult tickets. What total number of adult tickets were sold?

   b. A straight angle is split into two smaller angles as shown. The smaller angle's measure is $\frac{1}{6}$ that of a straight angle. What is the value of angle a?



   c. Annabel and Eric made 17 ounces of pizza dough. They used $\frac{5}{8}$ of the dough to make a pizza and used the rest to make calzones. What is the difference between the amount of dough they used to make pizza and the amount of dough they used to make calzones?

   d. The New York Rangers hockey team won $\frac{3}{4}$ of their games last season. If they lost 21 games, how many games did they play in the entire season?

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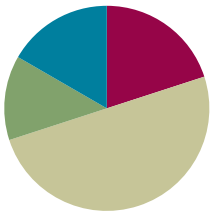
Lesson 8

Objective: Relate a fraction of a set to the repeated addition interpretation of fraction multiplication.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Convert Measures **4.MD.1** (5 minutes)
- Fractions as Whole Numbers **5.NF.3** (3 minutes)
- Multiply a Fraction Times a Whole Number **5.NF.4** (4 minutes)

Convert Measures (5 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Reference Sheet)

Note: This fluency activity prepares students for Lessons 9–12 content. Allow students to use the Grade 5 Mathematics Reference Sheet if they are confused, but encourage them to answer questions without referring to it.

T: (Write 1 ft = ____ in.) How many inches are in 1 foot?
S: 12 inches.
T: (Write 1 ft = 12 in. Below it, write 2 ft = ____ in.) 2 feet?
S: 24 inches.
T: (Write 2 ft = 24 in. Below it, write 3 ft = ____ in.) 3 feet?
S: 36 inches.
T: (Write 3 ft = 36 in. Below it, write 4 ft = ____ in.) 4 feet?
S: 48 inches.
T: (Write 4 ft = 48 in. Below it, write 10 ft = ____ in.) On your personal white board, write the equation.
S: (Write 10 ft = 120 in.)
T: (Write 10 ft × ____ = ____ in.) Write the multiplication equation you used to solve it.
S: (Write 10 ft × 12 = 120 in.)

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Continue with the following possible sequence: 1 pint = 2 cups, 2 pints = 4 cups, 3 pints = 6 cups, 9 pints = 18 cups, 1 yd = 3 ft, 2 yd = 6 ft, 3 yd = 9 ft, 7 yd = 21 ft, 1 gal = 4 qt, 2 gal = 8 qt, 3 gal = 12 qt, and 8 gal = 32 qt.

Fractions as Whole Numbers (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 5 and reviews denominators equivalent to hundredths. Instruct students to use their personal white boards for calculations that they cannot do mentally.

T: I'll say a fraction. You say it as a division problem. 4 halves.

S: $4 \div 2 = 2$.

Continue with the following possible sequence:

$\frac{6}{2}, \frac{12}{2}, \frac{52}{2}, \frac{40}{20}, \frac{60}{20}, \frac{120}{20}, \frac{740}{20}, \frac{10}{5}, \frac{15}{5}, \frac{45}{5}, \frac{75}{5}, \frac{100}{50}, \frac{150}{50}, \frac{400}{50}, \frac{700}{50}, \frac{8}{4}, \frac{12}{4}, \frac{20}{4}, \frac{72}{4}, \frac{50}{25}, \frac{75}{25}$, and $\frac{400}{25}$.

Multiply a Fraction Times a Whole Number (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 7 content.

T: (Project a tape diagram of 12 partitioned into 3 equal units. Shade in 1 unit.) What fraction of 12 is shaded?

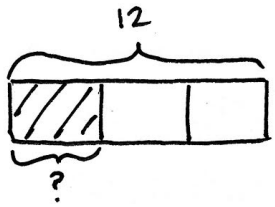
S: 1 third.

T: Read the tape diagram as a division equation.

S: $12 \div 3 = 4$.

T: (Write $12 \times$ ____ = 4.) On your personal white board, write the equation, filling in the missing fraction.

S: (Write $12 \times \frac{1}{3} = 4$.)

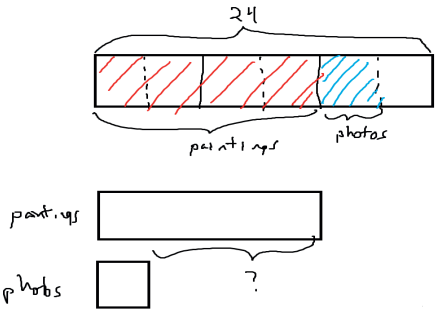


Continue with the following possible sequence: $28 \times \frac{1}{7}$, $\frac{1}{4} \times 24$, $\frac{3}{4} \times 24$, $\frac{1}{8} \times 56$, and $\frac{3}{8} \times 56$.

Application Problem (8 minutes)

Sasha organizes the art gallery in her town's community center. This month, she has 24 new pieces to add to the gallery.

Of the new pieces, $\frac{1}{6}$ of them are photographs, and $\frac{2}{3}$ of them are paintings. How many more paintings are there than photos?



6 units = 24
1 unit = $\frac{24}{6} = 4$

paintings = 4×4
= 16

photos = 1×4
= 4

$16 - 4 = ?$
$12 = ?$

There are 12 more paintings than photos.

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Note: This Application Problem requires students to find two fractions of the same set—a recall of the concepts from Lessons 6–7 in preparation for today's lesson.

Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1

$\frac{2}{3} \times 6 =$ ____

T: (Write 2×6 on the board.) Read this expression out loud.

S: 2 times 6.

T: In what different ways can we interpret the meaning of this expression? Discuss with your partner.

S: We can think of it as 6 times as much as 2. → 6 + 6. → We could think of 6 copies of 2. → 2 + 2 + 2 + 2 + 2 + 2.

T: True. We can find 2 copies of 6, and we can also think about 2 added 6 times. What is the property that allows us to multiply the factors in any order?

S: Commutative property.

T: (Write $\frac{2}{3} \times 6$ on the board.) How can we interpret this expression? Turn and talk.

S: 2 thirds of 6. → 6 copies of 2 thirds. → 2 thirds added together 6 times.

T: This expression can be interpreted in different ways, just as the whole number expression. We can say it's $\frac{2}{3}$ of 6 or 6 groups of $\frac{2}{3}$. (Write $\frac{2}{3} \times 6$ and $6 \times \frac{2}{3}$ on the board as shown below.)

T: Use a tape diagram to find 2 thirds of 6. (Point to $\frac{2}{3} \times 6$.)

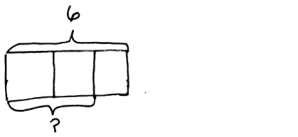
S: (Solve.)

$\frac{2}{3} \times 6 = \frac{2}{3}$ of 6 $\qquad$ $6 \times \frac{2}{3}$

T: Let me record our thinking. In the diagram, we see that 3 units is 6. (Write 3 units = 6.) We divide 6 by 3 to find 1 unit. (Write $\frac{6}{3}$.) So, 2 units is 2 times 6 divided by 3. (Write $2 \times \frac{6}{3}$ and the rest of the thinking as shown to the right.)

T: Now, let's think of it as 6 groups (or copies) of $\frac{2}{3}$ like you did in Grade 4. Solve it using repeated addition on your board.

S: (Solve.)



3 units = 6
1 unit = $\frac{6}{3}$
2 units = $2 \times \frac{6}{3}$
$= \frac{6}{3} + \frac{6}{3}$
$= \frac{6+6}{3}$
$= \frac{2 \times 6}{3}$
$= \frac{12}{3}$

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$
$= \frac{2+2+2+2+2+2}{3}$
$= \frac{6 \times 2}{3}$
$= \frac{12}{3}$

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T: (Write $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ on the board.)

T: What multiplication expression gave us 12?

S: 6 × 2.

T: (Write on the board.) What unit are we counting?

S: Thirds.

T: Let me write what I hear you saying. (Write (6 × 2) thirds on the board.) Now, let me write it another way. (Write = $\frac{6 \times 2}{3}$.) 6 times 2 thirds.

T: (Point to both $2 \times \frac{6}{3}$ and $\frac{6 \times 2}{3}$.) In both ways of thinking, what is the product? Why is it the same?

S: It's 12 thirds because 2 × 6 thirds is the same as 6 × 2 thirds. → It's the commutative property again. It doesn't matter what order we multiply; it's the same product.

T: How many wholes is 12 thirds? How much is 12 divided by 3?

S: 4.

T: Let's use something else we learned in Grade 4 to rename this fraction using larger units before we multiply. (Point to $\frac{2 \times 6}{3}$.) Look for a factor that is shared by the numerator and denominator. Turn and talk.

S: Two and 3 only have a common factor of 1, but 3 and 6 have a common factor of 3. → I know the numerator of 6 can be divided by 3 to get 2, and the denominator of 3 can be divided by 3 to get 1.

T: We can rename this fraction just like in Grade 4 by dividing both the numerator and denominator by 3. Watch me. 6 divided by 3 is 2. (Cross out 6, and write 2 above 6.) 3 divided by 3 is 1. (Cross out 3, and write 1 below 3.)

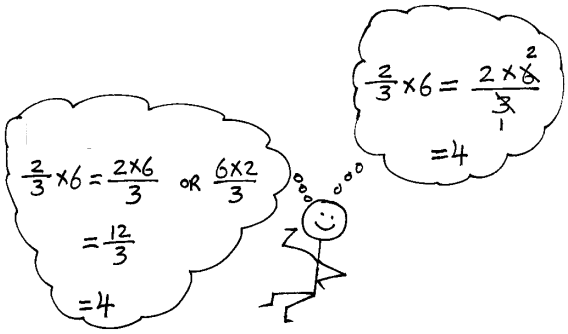
T: What does the numerator show now?

S: 2 × 2.

T: What's the denominator?

S: 1.

T: (Write $\frac{2 \times 2}{1} = \frac{4}{1}$.) This fraction was 12 thirds; now, it is 4 wholes. Did we change the amount of the fraction by naming it using larger units? How do you know?



S: It is the same amount. Thirds are smaller than wholes, so it requires 12 thirds to show the same amount as 4 wholes. → It is the same. The unit got larger, so the number we needed to show the amount got smaller. → There are 3 thirds in 1 whole, so 12 thirds makes 4 wholes. It is the same. → When we divide the numerator and denominator by the same number, it's like dividing by 1, and dividing by 1 doesn't change the number's value.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If students have difficulty remembering that dividing by a common factor allows a fraction to be renamed, consider a return to the Grade 4 notation for finding equivalent fractions as follows:

$\frac{2}{3} \times 9 = \frac{2 \times 9}{3} = \frac{2 \times 3 \times 3}{3}$.

The decomposition in the numerator makes the common factor of 3 apparent. Students may also be reminded that multiplying by $\frac{3}{3}$ is the same as multiplying by 1.

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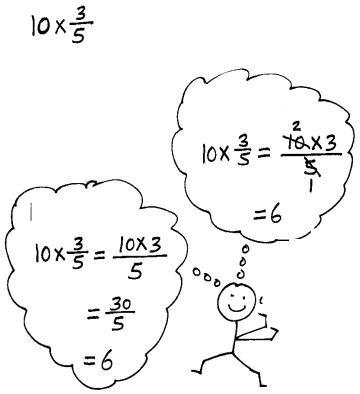
Problem 2

$\frac{3}{5} \times 10 =$ ____



T: (Write $\frac{3}{5} \times 10$ on board.) Finding $\frac{3}{5}$ of 10 is the same as finding the product of 10 copies of $\frac{3}{5}$. I can rewrite this expression in unit form as (10 × 3) fifths or a fraction. (Write $\frac{10 \times 3}{5}$.) 10 times 3 fifths. Multiply in your head, and say the product.

S: 30 fifths.

T: $\frac{30}{5}$ is equivalent to how many wholes?

S: 6 wholes.

T: So, if $10 \times \frac{3}{5}$ is equal to 6, is it also true that 3 fifths of 10 is 6? How do you know?

S: Yes, it is true. 1 fifth of 10 is 2, so 3 fifths would be 6. → The commutative property says we can multiply in any order. This is true of fractional numbers, too. So, the product would be the same. → 3 fifths is a little more than half, so 3 fifths of 10 should be a little more than 5. 6 is a little more than 5.

T: Now, let's work this problem again, but this time, let's find a common factor and rename it before we multiply. (Follow the sequence from Problem 1.)

S: (Work.)

T: Did dividing the numerator and denominator by the same common factor change the quantity? Why or why not?

S: (Share.)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

While the focus of today's lesson is the transition to a more abstract understanding of fraction of a set, do not hastily drop pictorial representations. Tape diagrams are powerful tools that help students make connections to the abstract. Throughout the lesson, continue to ask, "Can you draw something?"

These drawings also provide formative assessment opportunities for teachers and allow a glimpse into the thinking of students in real time.

Problem 3

$\frac{7}{6} \times 24 =$ ____

$\frac{7}{6} \times 27 =$ ____

T: Before we solve, what do you notice that is different this time?

S: The fraction of the set that we are finding is more than a whole this time. All of the others were fractions less than 1.

T: Let's estimate the size of our product. Turn and talk.

S: This is like the one from the Problem Set yesterday. We need more than a whole set, so the answer will be more than 24. → We need 1 sixth more than a whole set of 24, so the answer will be a little more than 24.

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T: (Write $\frac{24 \times 7}{6}$ on the board.) 24 times 7 sixths. Can you multiply 24 times 7 in your head?

S: You could, but it's a lot to think about to do it mentally.

T: Because this one is harder to calculate mentally, let's use the renaming strategies we've seen to solve this problem. Turn and share how we can begin.

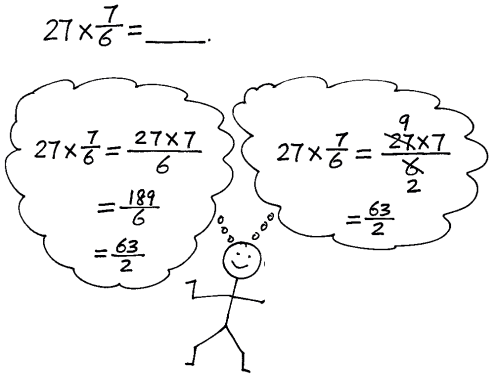
S: We can divide the numerator and denominator by the same common factor.

Continue with the sequence from Problem 2, having students name the common factor and rename as shown previously. Then, proceed to $\frac{7}{6} \times 27 =$ ____.

T: Compare this problem to the last one.

S: The whole is a little more than last time. → The fraction we are looking for is the same, but the whole is larger. → We probably need to rename this one before we multiply like the last one, because 7 × 27 is harder to do mentally.

T: Let's rename first. Name a factor that 27 and 6 share.



S: 3.

T: Let's divide the numerator and denominator by this common factor. 27 divided by 3 is 9. (Cross out 27, and write 9 above 27.) 6 divided by 3 is 2. (Cross out 6, and write 2 below 6.) We've renamed this fraction. What's the new name?

S: $\frac{9 \times 7}{2}$. (9 times 7 divided by 2.)

T: Has this made it easier for us to solve this mentally? Why?

S: Yes, the numbers are easier to multiply now. → The numerator is a basic fact now, and I know 9 × 7.

T: Have we changed the amount that is represented by this fraction? Turn and talk.

S: No. It's the same amount. We just renamed it using a larger unit. → We renamed it just like any other fraction by looking for a common factor. This doesn't change the amount.

T: Say the product as a fraction greater than one.

S: 63 halves. (Write = $\frac{63}{2}$.)

T: We could express $\frac{63}{2}$ as a mixed number, but we don't have to.

T: (Point to $\frac{27 \times 7}{6}$.) To compare, let's multiply without renaming and see if we get the same product.

T: What's the fraction?

S: $\frac{189}{6}$.

T: (Write = $\frac{189}{6}$.) Rewrite that as a fraction greater than 1 using the largest units that you can. What do you notice?

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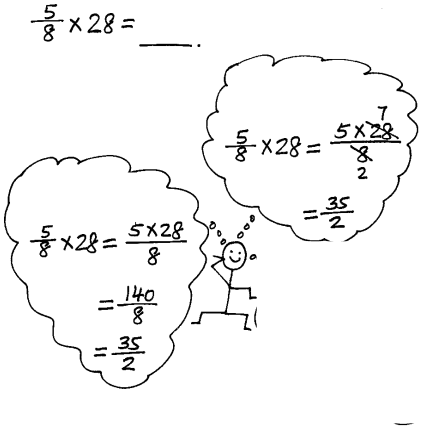
S: (Work to find $\frac{63}{2}$.) We get the same answer, but it was harder to find it. → 189 is a large number, so it's harder for me to find the common factor with 6. → I can't do it in my head. I needed to use paper and pencil to simplify.

T: So, sometimes, it makes our work easier and more efficient to rename with larger units, or simplify, first and then multiply.



Repeat this sequence with $\frac{5}{8} \times 28 =$ ____.

Problem 4

$\frac{2}{3}$ hour = ____ minutes

T: We are looking for part of an hour. Which part?

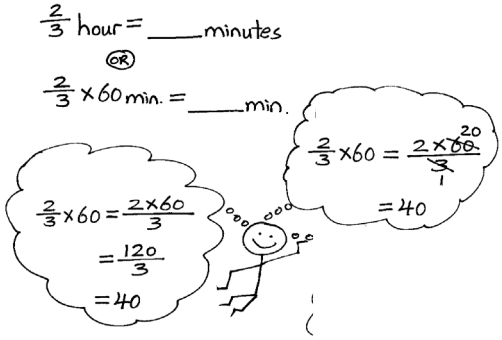
S: 2 thirds of an hour.

T: Will 2 thirds of an hour be more than 60 minutes or less? Why?

S: It should be less because it isn't a whole hour. → A whole hour, 60 minutes, would be 3 thirds. We only want 2 thirds, so it should be less than 60 minutes.

T: Turn and talk with your partner about how you might find 2 thirds of an hour.

S: I know the whole is 60 minutes, and the fraction I want is $\frac{2}{3}$. → We have to find what's $\frac{2}{3}$ of 60.



T: (Write $\frac{2}{3} \times 60$ min = ____ min.) Solve this problem independently. You may use any method you prefer.

S: (Solve.)

T: (Select students to share solutions with the class.)

Repeat this sequence with $\frac{3}{4}$ of a foot.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Today's Problem Set is lengthy. Students may benefit from additional guidance. Consider solving one problem from each section as a class before instructing students to solve the remainder of the problems independently.

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Student Debrief (10 minutes)

Lesson Objective: Relate a fraction of a set to the repeated addition interpretation of fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

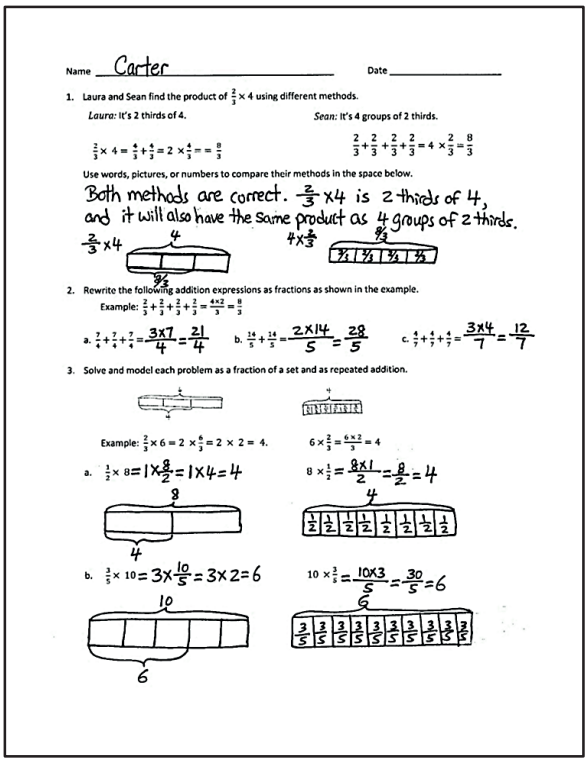
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Share and explain your solution for Problem 1 with a partner.
- What do you notice about Problems 2(a) and 2(c)? (Problem 2(a) is 3 groups of $\frac{7}{4}$, which is equal to $3 \times \frac{7}{4} = \frac{21}{4}$, and 2(c) is 3 groups of $\frac{4}{7}$, which is equal to $3 \times \frac{4}{7} = \frac{12}{7}$.)
- What do you notice about the solutions in Problems 3 and 4? (All of the products are whole numbers.)
- We learned to solve fraction of a set problems using the repeated addition strategy and multiplication and simplifying strategies today. Which one do you think is the most efficient way to solve a problem? Does it depend on the problems?
- Why is it important to learn more than one strategy to solve a problem?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



Name Carter Date

1. Laura and Sean find the product of $\frac{2}{3} \times 4$ using different methods.

Laura: It's 2 thirds of 4. *Sean:* It's 4 groups of 2 thirds.

$\frac{2}{3} \times 4 = \frac{4}{3} + \frac{4}{3} = 2 \times \frac{4}{3} = \frac{8}{3}$ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 4 \times \frac{2}{3} = \frac{8}{3}$

Use words, pictures, or numbers to compare their methods in the space below.

Both methods are correct. $\frac{2}{3} \times 4$ is 2 thirds of 4, and it will also have the same product as 4 groups of 2 thirds.

2. Rewrite the following addition expressions as fractions as shown in the example.

Example: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$

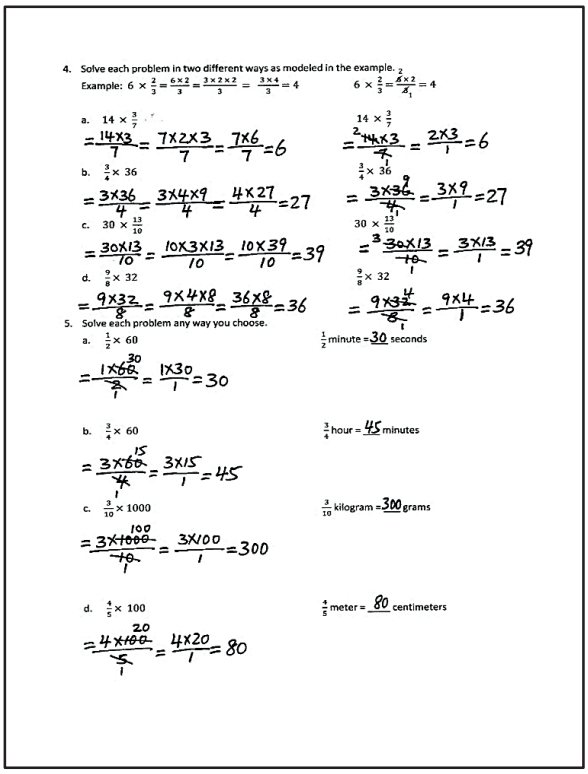
a. $\frac{7}{4} + \frac{7}{4} + \frac{7}{4} = \frac{3 \times 7}{4} = \frac{21}{4}$ b. $\frac{14}{5} + \frac{14}{5} = \frac{2 \times 14}{5} = \frac{28}{5}$ c. $\frac{4}{7} + \frac{4}{7} + \frac{4}{7} = \frac{3 \times 4}{7} = \frac{12}{7}$

3. Solve and model each problem as a fraction of a set and as repeated addition.

Example: $\frac{2}{3} \times 6 = 2 \times \frac{6}{3} = 2 \times 2 = 4$. $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = 4$

a. $\frac{1}{2} \times 8 = 1 \times \frac{8}{2} = 1 \times 4 = 4$ $8 \times \frac{1}{2} = \frac{8 \times 1}{2} = \frac{8}{2} = 4$

b. $\frac{3}{5} \times 10 = 3 \times \frac{10}{5} = 3 \times 2 = 6$ $10 \times \frac{3}{5} = \frac{10 \times 3}{5} = \frac{30}{5} = 6$



4. Solve each problem in two different ways as modeled in the example.

Example: $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = \frac{3 \times 2 \times 2}{3} = \frac{3 \times 4}{3} = 4$ $6 \times \frac{2}{3} = \frac{4 \times 2}{1} = 4$

a. $14 \times \frac{3}{7}$ $= \frac{14 \times 3}{7} = \frac{7 \times 2 \times 3}{7} = \frac{7 \times 6}{7} = 6$ $= \frac{2 \times 7 \times 3}{7} = \frac{2 \times 3}{1} = 6$

b. $\frac{3}{4} \times 36$ $= \frac{3 \times 36}{4} = \frac{3 \times 4 \times 9}{4} = \frac{4 \times 27}{4} = 27$ $= \frac{3 \times 36}{4} = \frac{3 \times 9}{1} = 27$

c. $30 \times \frac{13}{10}$ $= \frac{30 \times 13}{10} = \frac{10 \times 3 \times 13}{10} = \frac{10 \times 39}{10} = 39$ $= \frac{3 \times 30 \times 13}{10} = \frac{3 \times 13}{1} = 39$

d. $\frac{9}{8} \times 32$ $= \frac{9 \times 32}{8} = \frac{9 \times 4 \times 8}{8} = \frac{36 \times 8}{8} = 36$ $= \frac{9 \times 32}{8} = \frac{9 \times 4}{1} = 36$

5. Solve each problem any way you choose.

a. $\frac{1}{2} \times 60$ $= \frac{1 \times 60}{2} = \frac{1 \times 30}{1} = 30$ $\frac{1}{2}$ minute = 30 seconds

b. $\frac{3}{4} \times 60$ $= \frac{3 \times 60}{4} = \frac{3 \times 15}{1} = 45$ $\frac{3}{4}$ hour = 45 minutes

c. $\frac{3}{10} \times 1000$ $= \frac{3 \times 1000}{10} = \frac{3 \times 100}{1} = 300$ $\frac{3}{10}$ kilogram = 300 grams

d. $\frac{4}{5} \times 100$ $= \frac{4 \times 100}{5} = \frac{4 \times 20}{1} = 80$ $\frac{4}{5}$ meter = 80 centimeters

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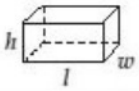
Grade 5 Mathematics Reference Sheet

FORMULAS



Right Rectangular Prism

Volume = lwh
Volume = Bh

CONVERSIONS

1 centimeter = 10 millimeters
1 meter = 100 centimeters = 1,000 millimeters
1 kilometer = 1,000 meters

1 gram = 1,000 milligrams
1 kilogram = 1,000 grams

1 pound = 16 ounces
1 ton = 2,000 pounds

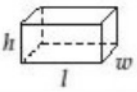
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 liter = 1,000 milliliters
1 kiloliter = 1,000 liters

1 mile = 5,280 feet
1 mile = 1,760 yards

Grade 5 Mathematics Reference Sheet

FORMULAS



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Name _____ Date _____

1. Laura and Sean find the product of $\frac{2}{3} \times 4$ using different methods.

Laura: It's 2 thirds of 4.

$\frac{2}{3} \times 4 = \frac{4}{3} + \frac{4}{3} = 2 \times \frac{4}{3} = \frac{8}{3}$

Sean: It's 4 groups of 2 thirds.

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 4 \times \frac{2}{3} = \frac{8}{3}$

Use words, pictures, or numbers to compare their methods in the space below.

2. Rewrite the following addition expressions as fractions as shown in the example.

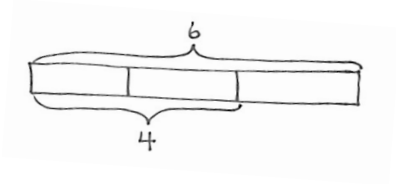
Example: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$

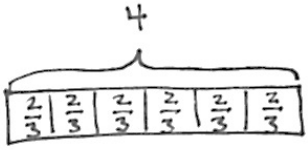
a. $\frac{7}{4} + \frac{7}{4} + \frac{7}{4} =$

b. $\frac{14}{5} + \frac{14}{5} =$

c. $\frac{4}{7} + \frac{4}{7} + \frac{4}{7} =$

3. Solve and model each problem as a fraction of a set and as repeated addition.





Example: $\frac{2}{3} \times 6 = 2 \times \frac{6}{3} = 2 \times 2 = 4$ $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = 4$

a. $\frac{1}{2} \times 8$ $8 \times \frac{1}{2}$

b. $\frac{3}{5} \times 10$ $10 \times \frac{3}{5}$

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4. Solve each problem in two different ways as modeled in the example.

Example: $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = \frac{3 \times 2 \times 2}{3} = \frac{3 \times 4}{3} = 4$

$6 \times \frac{2}{3} = \frac{\cancel{6}^{2} \times 2}{\cancel{3}_{1}} = 4$

a. $14 \times \frac{3}{7}$

$14 \times \frac{3}{7}$

b. $\frac{3}{4} \times 36$

$\frac{3}{4} \times 36$

c. $30 \times \frac{13}{10}$

$30 \times \frac{13}{10}$

d. $\frac{9}{8} \times 32$

$\frac{9}{8} \times 32$

5. Solve each problem any way you choose.

a. $\frac{1}{2} \times 60$

$\frac{1}{2}$ minute = _____ seconds

b. $\frac{3}{4} \times 60$

$\frac{3}{4}$ hour = _____ minutes

c. $\frac{3}{10} \times 1{,}000$

$\frac{3}{10}$ kilogram = _____ grams

d. $\frac{4}{5} \times 100$

$\frac{4}{5}$ meter = _____ centimeters

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Solve each problem in two different ways as modeled in the example.

Example: $\frac{2}{3} \times 6 = \frac{2 \times 6}{3} = \frac{12}{3} = 4$ $\frac{2}{3} \times 6 = \frac{2 \times \cancel{6}^{2}}{\cancel{3}_{1}} = 4$

a. $\frac{2}{3} \times 15$ $\frac{2}{3} \times 15$

b. $\frac{5}{4} \times 12$ $\frac{5}{4} \times 12$

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Name _____ Date _____

1. Rewrite the following expressions as shown in the example.

Example: $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{4\times 2}{3}=\frac{8}{3}$

a. $\frac{5}{3}+\frac{5}{3}+\frac{5}{3}$ b. $\frac{13}{5}+\frac{13}{5}$ c. $\frac{9}{4}+\frac{9}{4}+\frac{9}{4}$

2. Solve each problem in two different ways as modeled in the example.

Example: $\frac{2}{3}\times 6=\frac{2\times 6}{3}=\frac{12}{3}=4$ $\frac{2}{3}\times 6=\frac{2\times \cancel{6}^{2}}{\cancel{3}_{1}}=4$

a. $\frac{3}{4}\times 16$ $\frac{3}{4}\times 16$

b. $\frac{4}{3}\times 12$ $\frac{4}{3}\times 12$

c. $40\times\frac{11}{10}$ $40\times\frac{11}{10}$

d. $\frac{7}{6}\times 36$ $\frac{7}{6}\times 36$

e. $24\times\frac{5}{8}$ $24\times\frac{5}{8}$

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f. $18 \times \frac{5}{12}$ $18 \times \frac{5}{12}$

g. $\frac{10}{9} \times 21$ $\frac{10}{9} \times 21$

3. Solve each problem any way you choose.

a. $\frac{1}{3} \times 60$ $\frac{1}{3}$ minute = _____ seconds

b. $\frac{4}{5} \times 60$ $\frac{4}{5}$ hour = _____ minutes

c. $\frac{7}{10} \times 1000$ $\frac{7}{10}$ kilogram = _____ grams

d. $\frac{3}{5} \times 100$ $\frac{3}{5}$ meter = _____ centimeters

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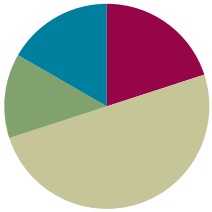
Lesson 9

Objective: Find a fraction of a measurement, and solve word problems.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Multiply Whole Numbers by Fractions with Tape Diagrams **5.NF.4** (4 minutes)
- Convert Measures **4.MD.1** (4 minutes)
- Multiply a Fraction and a Whole Number **5.NF.4** (4 minutes)

Multiply Whole Numbers by Fractions with Tape Diagrams (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 7 content.

T: (Project a tape diagram of 8 partitioned into 2 equal units. Shade in 1 unit.) What fraction of 8 is shaded?

S: 1 half.

T: Read the tape diagram as a division equation.

S: $8 \div 2 = 4$.

T: (Write $8 \times ____ = 4$.) On your personal white board, write the equation, filling in the missing fraction.

S: (Write $8 \times \frac{1}{2} = 4$.)

Continue with the following possible sequence: $35 \times \frac{1}{7}$, $\frac{1}{4} \times 16$, $\frac{3}{4} \times 16$, $\frac{1}{8} \times 48$, and $\frac{5}{8} \times 48$.

Convert Measures (4 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity prepares students for Lessons 9–12. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

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T: (Write 1 pt = ____ c.) How many cups are in 1 pint?
S: 2 cups.
T: (Write 1 pt = 2 c. Below it, write 2 pt = ____ c.) 2 pints?
S: 4 cups.
T: (Write 2 pt = 4 c. Below it, write 3 pt = ____ c.) 3 pints?
S: 6 cups.
T: (Write 3 pt = 6 c. Below it, write 7 pt = ____ c.) On your personal white board, write the equation.
S: (Write 7 pt = 14 c.)
T: Write the multiplication equation you used to solve it.
S: (Write 7 × 2 c = 14 c.)

Continue with the following possible sequence: 1 ft = 12 in, 2 ft = 24 in, 4 ft = 48 in, 1 yd = 3 ft, 2 yd = 6 ft, 3 yd = 9 ft, 9 yd = 27 ft, 1 gal = 4 qt, 2 gal = 8 qt, 3 gal = 12 qt, and 6 gal = 24 qt.

Multiply a Fraction and a Whole Number (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8 content.

T: (Write $\frac{1}{2} \times 4 =$ ____.) On your personal white board, write the equation as a repeated addition sentence and solve.
S: (Write $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2} = 2$.)
T: (Write $\frac{1}{2} \times 4 = \frac{__ \times __}{2}$.) On your personal white board, fill in the missing values to make a true number sentence.
S: (Write $\frac{1}{2} \times 4 = \frac{1 \times 4}{2}$.)
T: (Write $\frac{1}{2} \times 4 = \frac{1 \times 4}{2} = __ = ____$.) Fill in the missing numbers.
S: (Write $\frac{1}{2} \times 4 = \frac{1 \times 4}{2} = \frac{4}{2} = 2$.)
T: (Write $\frac{1}{2} \times 4 = \frac{1 \times 4}{2} =$ ____.) Find a common factor to simplify, and then multiply.
S: (Write $\frac{1}{2} \times 4 = \frac{1 \times \cancel{4}^{2}}{\cancel{2}_{1}} = \frac{2}{1} = 2$.)

Continue with the following possible sequence: $6 \times \frac{1}{3}$, $6 \times \frac{2}{3}$, $\frac{3}{4} \times 8$, and $9 \times \frac{2}{3}$.

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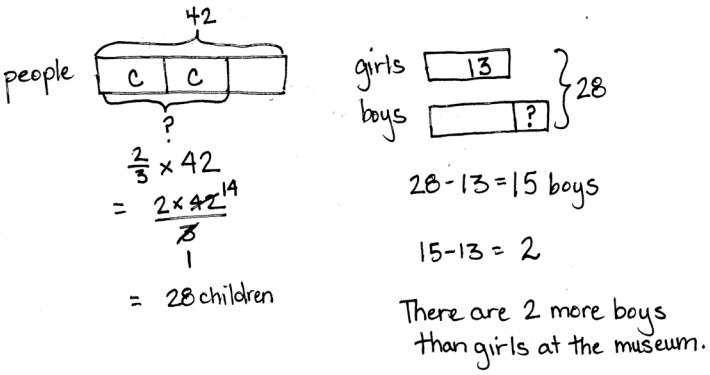
Application Problem (8 minutes)

There are 42 people at a museum. Two-thirds of them are children. How many children are at the museum?

Extension: If 13 of the children are girls, how many more boys than girls are at the museum?

Note: Today's Application Problem is a multi-step problem. Students must find a fraction of a set and then use that information to answer the question. The numbers are large enough to encourage simplifying strategies as taught in Lesson 8 without being overly burdensome for students who prefer to multiply, and then simplify, or students who still prefer to draw their solution using a tape diagram.

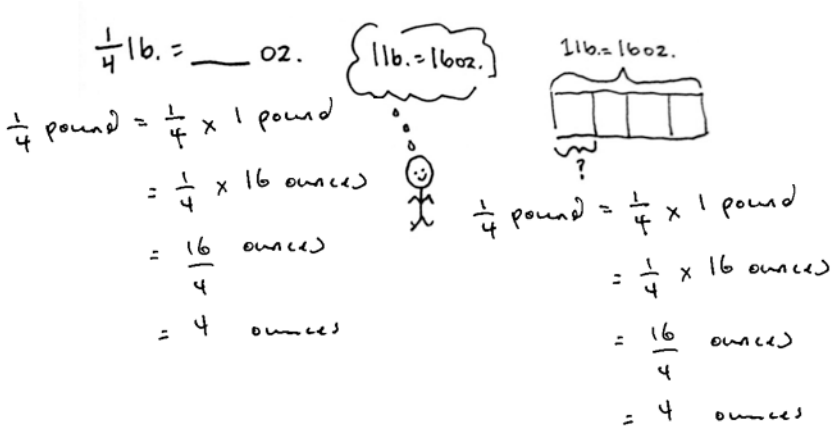


Concept Development (30 minutes)

Materials: (T) Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet, posted)
(S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Problem 1

$\frac{1}{4}$ lb = _____ oz



T: (Post Problem 1 on the board.) Which is a larger unit, pounds or ounces?

S: Pounds.

T: So, we are expressing a fraction of a larger unit as the smaller unit. We want to find $\frac{1}{4}$ of 1 pound. (Write $\frac{1}{4} \times 1$ lb.) We know that 1 pound is the same as how many ounces?

S: 16 ounces.

T: Let's rename the pound in our expression as ounces. Write it on your personal white board.

S: (Write $\frac{1}{4} \times 16$ ounces.)

T: (Write $\frac{1}{4} \times 1$ pound $= \frac{1}{4} \times 16$ ounces.) How do you know this is true?

S: It's true because we just renamed the pound as the same amount in ounces. → One pound is the same amount as 16 ounces.

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T: How will we find how many ounces are in a fourth of a pound? Turn and talk.

S: We can find $\frac{1}{4}$ of 16. → We can multiply $\frac{1}{4} \times 16$. → It's a fraction of a set. We'll just multiply 16 by a fourth. → We can draw a tape diagram and find one-fourth of 16.

T: Choose one with your partner and solve.

S: (Work.)

T: How many ounces are equal to one-fourth of a pound?

S: 4 ounces. (Write $\frac{1}{4}$ lb = 4 oz.)

T: So, each fourth of a pound in our tape diagram is equal to 4 ounces. How many ounces in two-fourths of a pound?

S: 8 ounces.

T: Three-fourths of a pound?

S: 12 ounces.

Problem 2

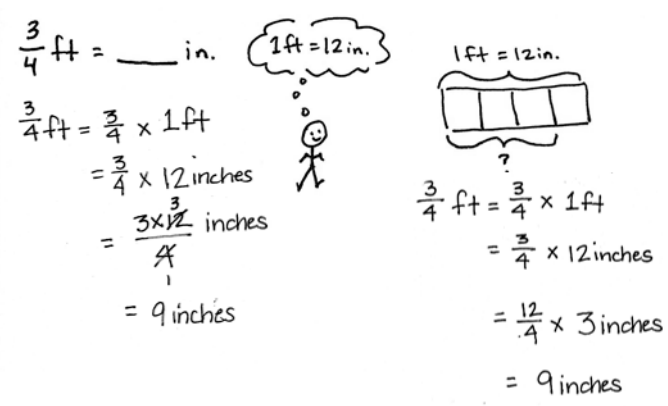
$\frac{3}{4}$ ft = _____ in

T: Compare this problem to the first one. Turn and talk.

S: We're still renaming a fraction of a larger unit as a smaller unit. → This time, we're changing feet to inches, so we must think about 12 instead of 16. → We were only finding 1 unit last time; this time, we must find 3 units.

T: (Write $\frac{3}{4} \times 1$ foot.) We know that 1 foot is the same as how many inches?

S: 12 inches.



T: Let's rename the foot in our expression as inches. Write it on your personal white board.

S: (Write $\frac{3}{4} \times 12$ inches.)

T: (Write $\frac{3}{4} \times 1$ ft = $\frac{3}{4} \times 12$ inches.) Is this true? How do you know?

S: This is just like last time. We didn't change the amount that we have in the expression. We just renamed the 1 foot using 12 inches. → Twelve inches and one foot are exactly the same length.

T: Before we solve this, let's estimate our answer. We are finding part of 1 foot. Will our answer be more than 6 inches or less than 6 inches? How do you know? Turn and talk.

S: Six inches is half a foot. We are looking for 3 fourths of a foot. Three-fourths is greater than one-half, so our answer will be more than 6. → It will be more than 6 inches. 6 is only half, and 3 fourths is almost a whole foot.

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T: Work with a neighbor to solve this problem. One of you can use multiplication to solve, and the other can use a tape diagram to solve. Check your neighbor's work when you're finished.

S: (Work and share.)

T: Reread the problem, and fill in the blank.

S: $\frac{3}{4}$ feet = 9 inches.

T: How can 3 fourths be equal to 9? Turn and talk.

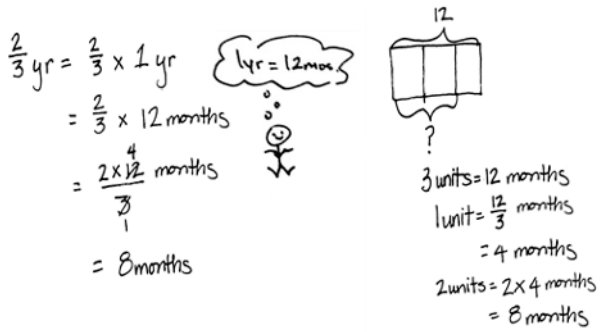
S: Because the units are different, the numbers will be different but show the same amount. → Feet are larger than inches, so it needs more inches than feet to show the same amount. → If you measured 3 fourths of a foot with a ruler and then measured 9 inches with a ruler, they would be exactly the same length. → If you measure the same length using feet and then using inches, you will always have more inches than feet because inches are smaller.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students to make conversions between fractions of gallons to pints or cups or fractions of a day to minutes or seconds.

Problem 3

Mr. Corsetti spends $\frac{2}{3}$ of every year in Florida. How many months does he spend in Florida each year?



T: Work independently. You may use either a tape diagram or a multiplication sentence to solve.

T: Use your work to answer the question.

S: Mr. Corsetti spends 8 months in Florida each year.

Repeat this sequence with $\frac{2}{3}$ yard = ____ feet and $\frac{2}{5}$ hour = ____ minutes.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

Lesson Objective: Find a fraction of a measurement, and solve word problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

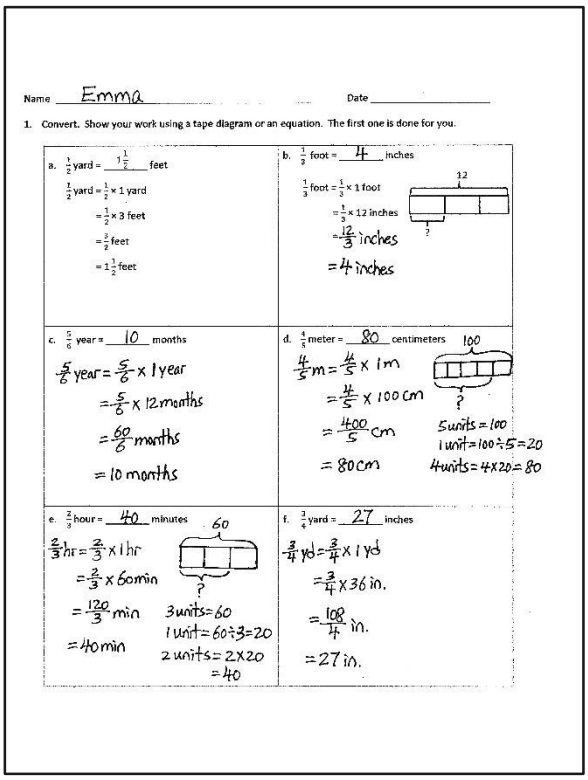
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Share and explain your solution for Problem 3 with your partner.
- In Problem 3, could you tell, without calculating, whether Mr. Paul bought more cashews or walnuts? How did you know?
- How did you solve Problem 3(c)? Is there more than one way to solve this problem? (Yes, there is more than one way to solve this problem, i.e., finding $\frac{7}{8}$ of 16 and $\frac{3}{4}$ of 16, and then subtracting, versus subtracting $\frac{7}{8} - \frac{3}{4}$, and then finding the fraction of 16.) Share your strategy with a partner.
- How did you solve Problem 3(d)? Share and explain your strategy with a partner.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



Name Emma Date

1. Convert. Show your work using a tape diagram or an equation. The first one is done for you.

a. $\frac{1}{2}$ yard = $1\frac{1}{2}$ feet

$\frac{1}{2}$ yard = $\frac{1}{2} \times 1$ yard

$= \frac{1}{2} \times 3$ feet

$= \frac{3}{2}$ feet

$= 1\frac{1}{2}$ feet

b. $\frac{1}{3}$ foot = 4 inches

$\frac{1}{3}$ foot = $\frac{1}{3} \times 1$ foot

$= \frac{1}{3} \times 12$ inches

$= \frac{12}{3}$ inches

$= 4$ inches

c. $\frac{5}{6}$ year = 10 months

$\frac{5}{6}$ year = $\frac{5}{6} \times 1$ year

$= \frac{5}{6} \times 12$ months

$= \frac{60}{6}$ months

$= 10$ months

d. $\frac{4}{5}$ meter = 80 centimeters

$\frac{4}{5}$ m = $\frac{4}{5} \times 1$ m

$= \frac{4}{5} \times 100$ cm

$= \frac{400}{5}$ cm

$= 80$ cm

5 units = 100

1 unit = $100 \div 5 = 20$

4 units = $4 \times 20 = 80$

e. $\frac{2}{3}$ hour = 40 minutes

$\frac{2}{3}$ hr = $\frac{2}{3} \times 1$ hr

$= \frac{2}{3} \times 60$ min

$= \frac{120}{3}$ min

$= 40$ min

3 units = 60

1 unit = $60 \div 3 = 20$

2 units = 2×20

$= 40$

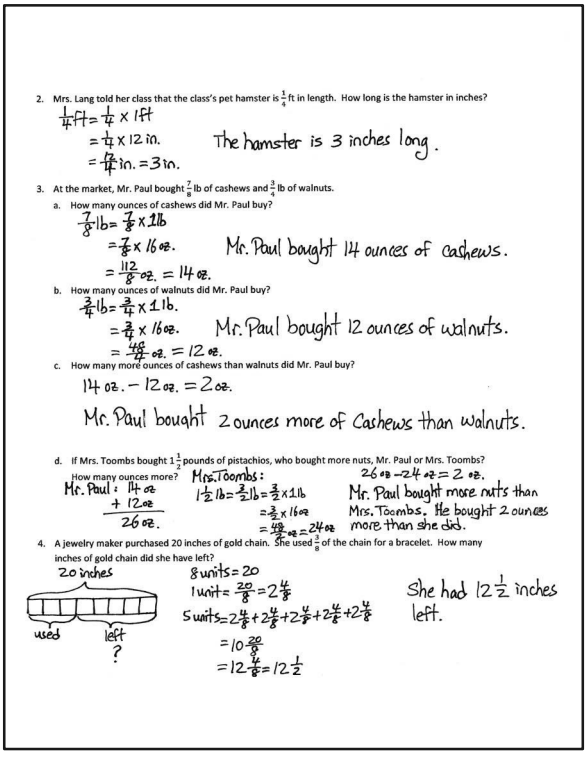
f. $\frac{3}{4}$ yard = 27 inches

$\frac{3}{4}$ yd = $\frac{3}{4} \times 1$ yd

$= \frac{3}{4} \times 36$ in.

$= \frac{108}{4}$ in.

$= 27$ in.



2. Mrs. Lang told her class that the class's pet hamster is $\frac{1}{4}$ ft in length. How long is the hamster in inches?

$\frac{1}{4}$ ft = $\frac{1}{4} \times 1$ ft

$= \frac{1}{4} \times 12$ in.

$= \frac{12}{4}$ in. $= 3$ in.

The hamster is 3 inches long.

3. At the market, Mr. Paul bought $\frac{7}{8}$ lb of cashews and $\frac{3}{4}$ lb of walnuts.

a. How many ounces of cashews did Mr. Paul buy?

$\frac{7}{8}$ lb = $\frac{7}{8} \times 1$ lb

$= \frac{7}{8} \times 16$ oz.

$= \frac{112}{8}$ oz. $= 14$ oz.

Mr. Paul bought 14 ounces of cashews.

b. How many ounces of walnuts did Mr. Paul buy?

$\frac{3}{4}$ lb = $\frac{3}{4} \times 1$ lb.

$= \frac{3}{4} \times 16$ oz.

$= \frac{48}{4}$ oz. $= 12$ oz.

Mr. Paul bought 12 ounces of walnuts.

c. How many more ounces of cashews than walnuts did Mr. Paul buy?

14 oz. − 12 oz. = 2 oz.

Mr. Paul bought 2 ounces more of cashews than walnuts.

d. If Mrs. Toombs bought $1\frac{1}{2}$ pounds of pistachios, who bought more nuts, Mr. Paul or Mrs. Toombs? How many ounces more?

Mr. Paul: 14 oz + 12 oz = 26 oz.

Mrs. Toombs: $1\frac{1}{2}$ lb = $\frac{3}{2}$ lb = $\frac{3}{2} \times 1$ lb

$= \frac{3}{2} \times 16$ oz

$= \frac{48}{2}$ oz. $= 24$ oz.

26 oz − 24 oz = 2 oz.

Mr. Paul bought more nuts than Mrs. Toombs. He bought 2 ounces more than she did.

4. A jewelry maker purchased 20 inches of gold chain. She used $\frac{5}{8}$ of the chain for a bracelet. How many inches of gold chain did she have left?

20 inches

used left ?

8 units = 20

1 unit = $\frac{20}{8} = 2\frac{4}{8}$

5 units = $2\frac{4}{8} + 2\frac{4}{8} + 2\frac{4}{8} + 2\frac{4}{8} + 2\frac{4}{8}$

$= 10\frac{20}{8}$

$= 12\frac{4}{8} = 12\frac{1}{2}$

She had $12\frac{1}{2}$ inches left.

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Name _____ Date _____

1. Convert. Show your work using a tape diagram or an equation. The first one is done for you.

a. $\frac{1}{2}$ yard = $1\frac{1}{2}$ feet

$\frac{1}{2}$ yard = $\frac{1}{2}$ × 1 yard

= $\frac{1}{2}$ × 3 feet

= $\frac{3}{2}$ feet

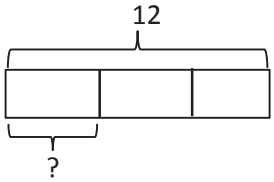
= $1\frac{1}{2}$ feet

b. $\frac{1}{3}$ foot = _____ inches

$\frac{1}{3}$ foot = $\frac{1}{3}$ × 1 foot

= $\frac{1}{3}$ × 12 inches

=



c. $\frac{5}{6}$ year = _____ months

d. $\frac{4}{5}$ meter = _____ centimeters

e. $\frac{2}{3}$ hour = _____ minutes

f. $\frac{3}{4}$ yard = _____ inches

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2. Mrs. Lang told her class that the class's pet hamster is $\frac{1}{4}$ ft in length. How long is the hamster in inches?

3. At the market, Mr. Paul bought $\frac{7}{8}$ lb of cashews and $\frac{3}{4}$ lb of walnuts.

   a. How many ounces of cashews did Mr. Paul buy?

   b. How many ounces of walnuts did Mr. Paul buy?

   c. How many more ounces of cashews than walnuts did Mr. Paul buy?

   d. If Mrs. Toombs bought $1\frac{1}{2}$ pounds of pistachios, who bought more nuts, Mr. Paul or Mrs. Toombs? How many ounces more?

4. A jewelry maker purchased 20 inches of gold chain. She used $\frac{3}{8}$ of the chain for a bracelet. How many inches of gold chain did she have left?

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Name _____ Date _____

1. Express 36 minutes as a fraction of an hour: 36 minutes = _____ hour

2. Solve.

   a. $\frac{2}{3}$ feet = _____ inches b. $\frac{2}{5}$ m = _____ cm c. $\frac{5}{6}$ year = _____ months

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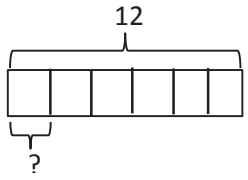
Name _____ Date _____

1. Convert. Show your work using a tape diagram or an equation. The first one is done for you.

a. $\frac{1}{4}$ yard = ___9___ inches $\frac{1}{4}$ yard = $\frac{1}{4} \times 1$ yard $= \frac{1}{4} \times 36$ inches $= \frac{36}{4}$ inches = 9 inches	b. $\frac{1}{6}$ foot = _____ inches $\frac{1}{6}$ foot = $\frac{1}{6} \times 1$ foot $= \frac{1}{6} \times 12$ inches = 
c. $\frac{3}{4}$ year = _____ months	d. $\frac{3}{5}$ meter = _____ centimeters
e. $\frac{5}{12}$ hour = _____ minutes	f. $\frac{2}{3}$ yard = _____ inches

2. Michelle measured the length of her forearm. It was $\frac{3}{4}$ of a foot. How long is her forearm in inches?

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3. At the market, Ms. Winn bought $\frac{3}{4}$ lb of grapes and $\frac{5}{8}$ lb of cherries.

   a. How many ounces of grapes did Ms. Winn buy?

   b. How many ounces of cherries did Ms. Winn buy?

   c. How many more ounces of grapes than cherries did Ms. Winn buy?

   d. If Mr. Phillips bought $1\frac{3}{4}$ pounds of raspberries, who bought more fruit, Ms. Winn or Mr. Phillips? How many ounces more?

4. A gardener has 10 pounds of soil. He used $\frac{5}{8}$ of the soil for his garden. How many pounds of soil did he use in the garden? How many pounds did he have left?

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Topic D

Fraction Expressions and Word Problems

5.OA.1, 5.OA.2, 5.NF.4a, 5.NF.6

Focus Standards:	5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
	5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 +7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
	5.NF.4a	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product of *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
	5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Instructional Days:	3	
Coherence -Links from:	G4–M2	Unit Conversions and Problem Solving with Metric Measurement
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions

Interpreting numerical expressions opens Topic D as students learn to evaluate expressions with parentheses, such as $3 \times (\frac{2}{3} - \frac{1}{5})$ or $\frac{2}{3} \times (7 + 9)$ (**5.OA.1**). They then learn to interpret numerical expressions, such as *3 times the difference between* $\frac{2}{3}$ *and* $\frac{1}{5}$ or *two thirds the sum of 7 and 9* (**5.OA.2**). Students generate word problems that lead to the same calculation (**5.NF.4a**), such as "Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank $\frac{2}{3}$ of the mixture. How much did Jack drink?" Solving word problems (**5.NF.6**) allows students to apply new knowledge of fraction multiplication in context, and tape diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

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A Teaching Sequence Toward Mastery of Fraction Expressions and Word Problems	
Objective 1:	**Compare and evaluate expressions with parentheses. (Lesson 10)**
Objective 2:	**Solve and create fraction word problems involving addition, subtraction, and multiplication. (Lessons 11–12)**

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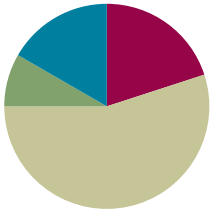
Lesson 10

Objective: Compare and evaluate expressions with parentheses.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Convert Measures from Small to Large Units **4.MD.1** (5 minutes)
- Multiply a Fraction and a Whole Number **5.NF.4** (3 minutes)
- Find the Unit Conversion **5.MD.2** (4 minutes)

Convert Measures from Small to Large Units (5 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity reviews Lesson 9 and prepares students for Lessons 10–12 content. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

T: (Write 12 in = __ ft.) How many feet are in 12 inches?
S: 1 foot.
T: (Write 12 in = 1 ft. Below it, write 24 in = __ ft.) 24 inches?
S: 2 feet.
T: (Write 24 in = 2 ft. Below it, write 36 in = __ ft.) 36 inches?
S: 3 feet.
T: (Write 36 in = 3 ft. Below it, write 48 in = __ ft.) 48 inches?
S: 4 feet.
T: (Write 48 in = 4 ft. Below it, 120 in = __ ft.) On your personal white board, write the equation.
S: (Write 120 in = 10 ft.)
T: (Write 120 in ÷ __ = __ ft.) Write the division equation you used to solve it.
S: (Write 120 in ÷ 12 = 10 ft.)

Continue with the following possible sequence: 2 c = 1 pt, 4 c = 2 pt, 6 c = 3 pt, 16 c = 8 pt, 3 ft = 1 yd, 6 ft = 2 yd, 9 ft = 3 yd, 27 ft = 9 yd, 4 qt = 1 gal, 8 qt = 2 gal, 12 qt = 3 gal, and 24 qt = 6 gal.

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Multiply a Fraction and a Whole Number (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

T: ($\frac{1}{2} \times 6 = \frac{__ \times __}{2}$.) On your personal white board, fill in the multiplication expression for the numerator.

S: (Write $\frac{1}{2} \times 6 = \frac{1 \times 6}{2}$.)

T: (Write $\frac{1}{2} \times 6 = \frac{1 \times 6}{2} = \text{—} = ___$.) Fill in the missing numbers.

S: (Write $\frac{1}{2} \times 6 = \frac{1 \times 6}{2} = \frac{6}{2} = 3$.)

T: (Write $\frac{1}{2} \times 6 = \frac{1 \times 6}{2} = ___$.) Find a common factor to simplify. Then, multiply.

S: (Write $\frac{1}{2} \times 6 = \frac{1 \times \overset{3}{\cancel{6}}}{\underset{1}{\cancel{2}}} = \frac{3}{1} = 3$.)

Continue with the following possible suggestions: $6 \times \frac{1}{3}$, $9 \times \frac{2}{3}$, $\frac{3}{4} \times 12$, and $12 \times \frac{5}{6}$.

Find the Unit Conversion (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 9.

T: 1 foot is equal to how many inches?

S: 12 inches.

T: (Write 1 ft = 12 inches to label the tape diagram. Below it, write $\frac{2}{3}$ ft = $\frac{2}{3} \times 1$ ft.) Rewrite the expression on the right of the equation on your personal white board, substituting 12 inches for 1 foot.

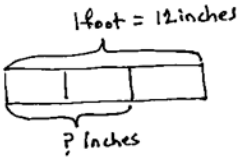
S: (Write $\frac{2}{3} \times 12$ inches.)

T: How many inches in each third? Represent the division using a fraction.

S: $\frac{12}{3}$.

T: How many inches in 2 thirds of a foot? Keep the division as a fraction.

S: (Write $\frac{12}{3} \times 2$ inches = 8 inches.)



$\frac{2}{3}$ foot = $\frac{2}{3} \times 1$ foot

$= \frac{2}{3} \times 12$ inches

$= \frac{12}{3} \times 2$ inches (Say "12 ÷ 3 × 2")

$= 4 \times 2$ inches

$= 8$ inches

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T: Let's read it. 12 divided by 3 times 2.

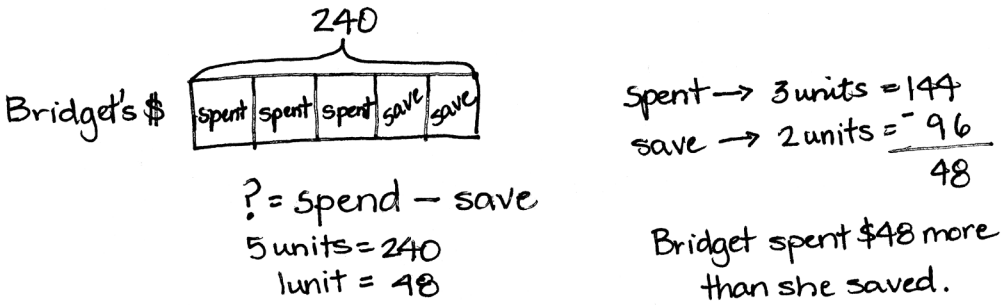
T: How many inches are equal to $\frac{2}{3}$ foot?

S: 8 inches.

Continue with the following possible sequence: $\frac{1}{4}$ lb = __ oz, $\frac{5}{6}$ yr = __ months, $\frac{7}{8}$ lb = __ oz, and $\frac{3}{4}$ hr = __ min.

Application Problem (5 minutes)

Bridget has $240. She spent $\frac{3}{5}$ of her money and saved the rest. How much more money did she spend than save?



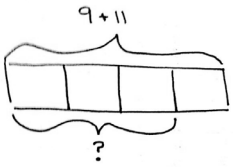
Note: This Application Problem provides a quick review of fraction of a set, which students have been working on in Topic C, and provides a bridge to the return to this work in Lesson 11. It is also a multi-step problem.

Concept Development (33 minutes)

Materials: (S) Personal white board

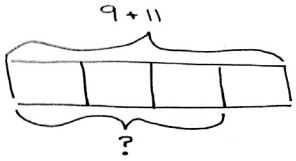
Problem 1: Write an expression to match a tape diagram. Then, evaluate.

a.



T: (Post the first tape diagram.) Read the expression that names the whole.

S: 9 + 11.



"3 fourths of the sum of 9 and 11"

$\frac{3}{4} \times (9+11)$

$= \frac{3}{4} \times 20$

$= \frac{3 \times 20^{5}}{4_{1}}$

$= \frac{15}{1}$

$= 15$

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T: What do we call the answer to an addition expression?

S: A sum.

T: So, our whole is the sum of 9 and 11. (Write the sum of 9 and 11 next to the tape diagram.) How many units is the sum being divided into?

S: Four.

T: What is the name of that fractional unit?

S: Fourths.

T: How many fourths are we trying to find?

S: 3 fourths.

T: So, this tape diagram is showing 3 fourths of the sum of 9 and 11. (Write 3 fourths next to the sum of 9 and 11.) Work with a partner to write a numerical expression to match these words.

S: $(9 + 11) \times \frac{3}{4}$. → $\frac{3}{4} \times (9 + 11)$. → $\frac{9 + 11}{4} \times 3$.

T: I noticed that many of you put parentheses around 9 + 11. Explain to a neighbor why that is necessary.

S: The parentheses tell us to add 9 and 11 first and then multiply. → If the parentheses weren't there, we would have to multiply first. We want to find the sum first and then multiply. → We can find the sum of 9 and 11 first and then divide the sum by 4 to find the value of one unit.

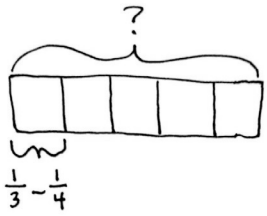
T: Work with a partner to evaluate or simplify this expression.

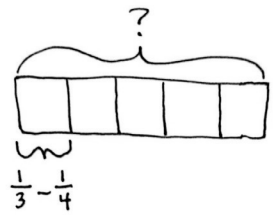
S: (Work to find 15.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If students have difficulty understanding the value of a unit as a subtraction sentence, write a whole number on one side of a small piece of construction paper and the equivalent subtraction expression on the other side. Place the paper on the model with the whole number facing up, and ask what must be done to find the whole. (Most students understand the process of multiplying to find the whole.) Write the multiplication expression using the paper, whole number side up. Then, flip the paper over, and show the parallel expression using the subtraction sentence.

b.





T: (Post the second tape diagram.) Look at this model. How is it different from the previous example?

S: This time, we don't know the whole. → In this diagram, the whole is being divided into fifths, not fourths. → Here, we know what 1 fifth is. We know it is the difference of $\frac{1}{3}$ and $\frac{1}{4}$. → We have to multiply the difference of $\frac{1}{3}$ and $\frac{1}{4}$ by 5 to find the whole.

T: Read the subtraction expression that tells the value of one unit (or 1 fifth) in the model. (Point to $\frac{1}{3} - \frac{1}{4}$.)

"5 times the difference of $\frac{1}{3}$ and $\frac{1}{4}$"

$5 \times \left(\frac{1}{3} - \frac{1}{4}\right)$

$= 5 \times \left(\frac{4}{12} - \frac{3}{12}\right)$

$= 5 \times \frac{1}{12}$

$= \frac{5 \times 1}{12}$

$= \frac{5}{12}$

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S: One-third minus one-fourth.

T: What is the name for the answer to a subtraction problem?

S: A difference.

T: This unit is the difference of one-third and one-fourth. (Write the difference of $\frac{1}{3}$ and $\frac{1}{4}$ next to the tape diagram.) How many of these $\left(\frac{1}{3}-\frac{1}{4}\right)$ units does our model show?

S: 5 units of $\frac{1}{3}-\frac{1}{4}$.

T: Work with a partner to write a numerical expression to match these words.

S: $5\times\left(\frac{1}{3}-\frac{1}{4}\right)$ or $\left(\frac{1}{3}-\frac{1}{4}\right)\times 5$.

T: Do we need parentheses for this expression?

S: Yes. We need to subtract first before multiplying.

T: Evaluate this expression independently. Then, compare your work with a neighbor.

Problem 2: Write and evaluate an expression from word form.

T: (Write the product of 4 and 2, divided by 3 on the board.) Read the expression.

S: The product of 4 and 2, divided by 3.

T: Work with a partner to write a matching numerical expression.

S: $(4\times 2)\div 3$. → $\frac{4\times 2}{3}$. → $4\times 2\div 3$.

T: Were the parentheses necessary here? Why or why not?

S: No. Because the product came first, and we can do multiplication and division left to right. We didn't need them. → I wrote it as a fraction. I didn't use parentheses because I knew before I could divide by 3 that I needed to find the product in the numerator.

T: Work independently to evaluate your expression. Express your answer as both a fraction greater than one and a mixed number. Check your work with a neighbor when you're finished.

S: (Work to find $\frac{8}{3}$ and $2\frac{2}{3}$. Then, check.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

It may be necessary to prompt students to use fraction notation for the division portion of the expression. Pointing out the format of the division sign—*dot over dot*—may serve as a good reminder. Reminding students of problems from the beginning of Module 4 may also be helpful (e.g., $2\div 3=\frac{2}{3}$).

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Problem 3: Evaluate and compare equivalent expressions.

a. $2 \div 3 \times 4$

b. 4 thirds doubled

c. $2 \div (3 \times 4)$

d. $\frac{2}{3} \times 4$

e. 4 copies of the sum of one-third and one-third

f. $(2 \div 3) \times 4$

T: Evaluate these expressions with your partner. Continue working until I call time. Be prepared to share.

S: (Work.)

T: Share your work with someone else's partner. What do you notice?

S: The answer is 8 thirds every time, except (c). → All of the expressions are equivalent, except (c). These are just different ways of expressing $\frac{8}{3}$.

T: What was different about (c)?

S: Since the expression had parentheses, we had to multiply first and then divide. It was equal to 2 twelfths. → It's tricky. In expression C, all of the numbers, digits, and operations are the same as the other expressions, but the order of the numbers and parentheses resulted in a different value.

T: Work with a partner to find another way of expressing $\frac{3}{5} \times 6$.

S: (Work. Possible expressions include: $3 \times (6 \div 5)$. → $3 \times 6 \div 5$. → $3 \times \frac{6}{5}$. → 6 times the value of 3 divided by 5.)

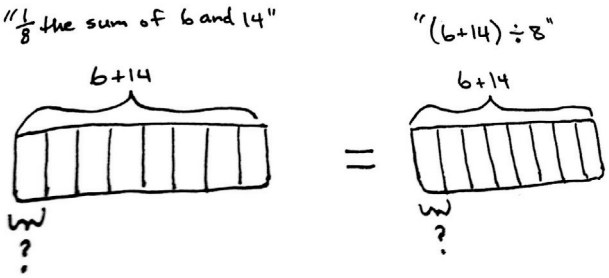
Invite students to share their expressions on the board and to discuss.

Problem 4: Compare expressions in word and numerical forms.

a. $\frac{1}{8}$ the sum of 6 and 14 ◯ $(6 + 14) \div 8$

b. $4 \times \frac{8}{3}$ ◯ 4 times the quotient of 3 and 8

c. Subtract 2 from $\frac{1}{2}$ of 9 ◯ $(11 \div 2) - 2$



T: Let's use <, >, or = to compare expressions. (Write $\frac{1}{8}$ the sum of 6 and 14 and $(6 + 14) \div 8$ on the board.) Draw a tape diagram for each expression, and compare them.

S: (Write $\frac{1}{8}$ the sum of 6 and 14 = $(6 + 14) \div 8$.)

T: What do you notice about the diagrams?

S: They are drawn exactly the same way. → We don't even need to evaluate the expressions to compare them. You can see that they will simplify to the same quantity. → I knew it would be the same before I drew it because finding 1 eighth of something and dividing by 8 are the same thing.

T: Look at the next pair of expressions. Work with your partner to compare them without calculating.

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S: (Work and write $4 \times \frac{8}{3} >$ 4 times the quotient of 3 and 8.)

T: How did you compare these expressions without calculating?

S: They both multiply something by 4. Since 8 thirds is greater than 3 eighths, the expression on the left is larger. → Since both expressions multiply with a factor of 4, the fraction that shows the smaller amount results in a product that is also less.

T: Compare the final pair of expressions independently without calculating. Be prepared to share your thoughts.

S: (Work and write subtract 2 from $\frac{1}{2}$ of 9 < (11 ÷ 2) – 2.)

T: How did you know which expression was greater? Turn and talk.

S: 11 divided by 2 is 11 halves, and 11 halves is greater than 9 halves. → Half of 9 is less than half of 11, and since we're subtracting 2 from both of them, the expression on the right is greater.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

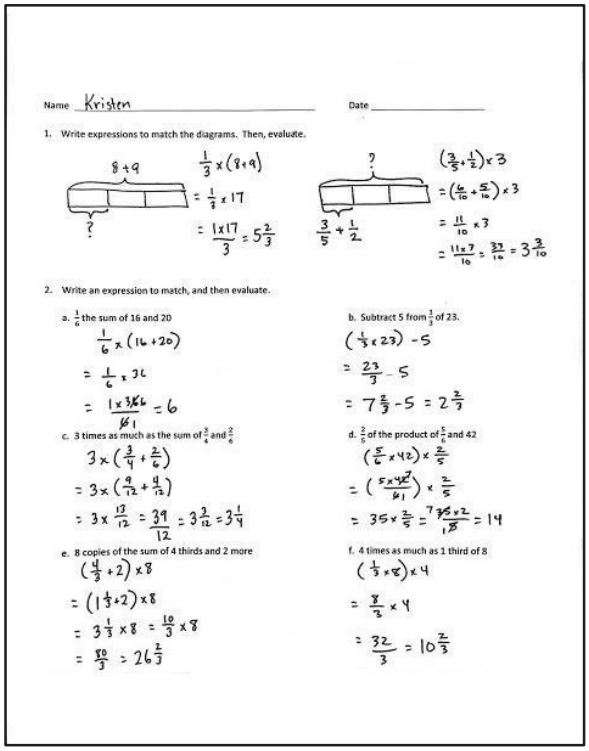
Lesson Objective: Compare and evaluate expressions with parentheses.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What relationships did you notice between the two tape diagrams in Problem 1?
- Share and explain your solution for Problem 3 with your partner.
- What were your strategies of comparing Problem 4? Explain it to your partner.


Name Kristen Date

1. Write expressions to match the diagrams. Then, evaluate.

8+9 ? $\frac{3}{5}+\frac{1}{2}$

2. Write an expression to match, and then evaluate.

a. $\frac{1}{6}$ the sum of 16 and 20

b. Subtract 5 from $\frac{1}{3}$ of 23.

c. 3 times as much as the sum of $\frac{3}{4}$ and $\frac{2}{6}$

d. $\frac{5}{6}$ of the product of $\frac{5}{6}$ and 42

e. 8 copies of the sum of 4 thirds and 2 more

f. 4 times as much as 1 third of 8

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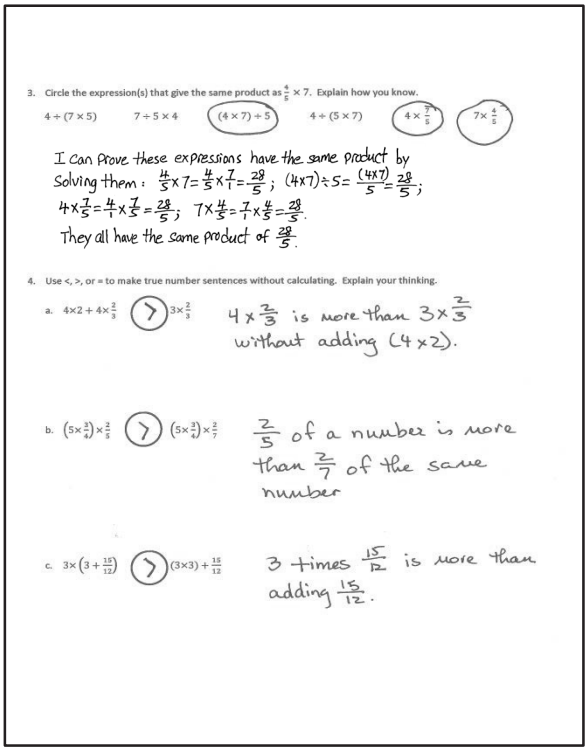
- How does the use of parentheses affect the answer in Problems 4(b) and 4(c)?
- Were you able to compare the expressions in Problem 4(c) without calculating? What made it more difficult than (a) and (b)?
- Explain to your partner how you created the line plot in Problem 5(d). Compare your line plot to your partner's.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



3. Circle the expression(s) that give the same product as $\frac{4}{5} \times 7$. Explain how you know.

$4 \div (7 \times 5)$ $\quad 7 \div 5 \times 4$ $\quad (4 \times 7) \div 5$ $\quad 4 \div (5 \times 7)$ $\quad 4 \times \frac{7}{5}$ $\quad 7 \times \frac{4}{5}$

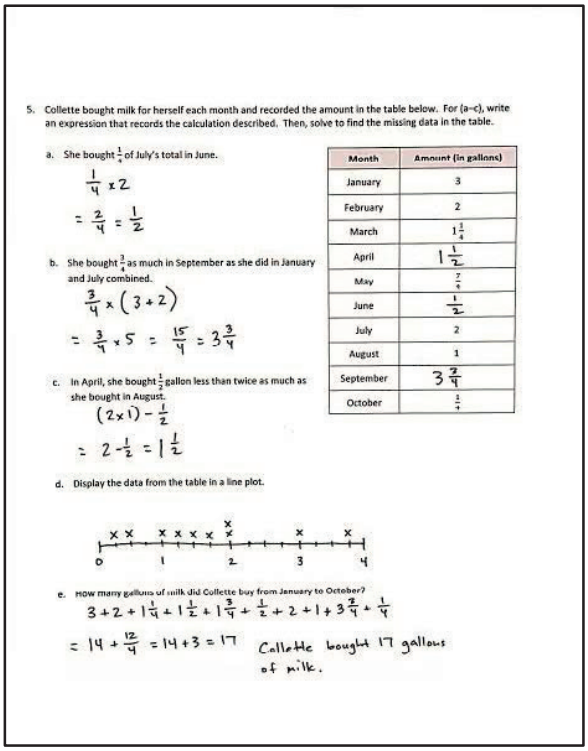
I can prove these expressions have the same product by solving them: $\frac{4}{5} \times 7 = \frac{4}{5} \times \frac{7}{1} = \frac{28}{5}$; $(4 \times 7) \div 5 = \frac{(4 \times 7)}{5} = \frac{28}{5}$; $4 \times \frac{7}{5} = \frac{4}{1} \times \frac{7}{5} = \frac{28}{5}$; $7 \times \frac{4}{5} = \frac{7}{1} \times \frac{4}{5} = \frac{28}{5}$. They all have the same product of $\frac{28}{5}$.

4. Use <, >, or = to make true number sentences without calculating. Explain your thinking.

a. $4 \times 2 + 4 \times \frac{2}{3}$ (>) $3 \times \frac{2}{3}$ $\quad 4 \times \frac{2}{3}$ is more than $3 \times \frac{2}{3}$ without adding (4×2).

b. $(5 \times \frac{3}{7}) \times \frac{2}{5}$ (>) $(5 \times \frac{2}{7}) \times \frac{2}{7}$ $\quad \frac{2}{5}$ of a number is more than $\frac{2}{7}$ of the same number

c. $3 \times (3 + \frac{10}{12})$ (>) $(3 \times 3) + \frac{10}{12}$ $\quad 3$ times $\frac{15}{12}$ is more than adding $\frac{15}{12}$.



5. Collette bought milk for herself each month and recorded the amount in the table below. For (a–c), write an expression that records the calculation described. Then, solve to find the missing data in the table.

a. She bought $\frac{1}{4}$ of July's total in June.

$\frac{1}{4} \times 2$

$= \frac{2}{4} = \frac{1}{2}$

b. She bought $\frac{3}{4}$ as much in September as she did in January and July combined.

$\frac{3}{4} \times (3 + 2)$

$= \frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$

c. In April, she bought $\frac{1}{2}$ gallon less than twice as much as she bought in August.

$(2 \times 1) - \frac{1}{2}$

$= 2 - \frac{1}{2} = 1\frac{1}{2}$

Month	Amount (in gallons)
January	3
February	2
March	$1\frac{1}{4}$
April	$1\frac{1}{2}$
May	$\frac{7}{4}$
June	$\frac{1}{2}$
July	2
August	1
September	$3\frac{3}{4}$
October	$\frac{3}{4}$

d. Display the data from the table in a line plot.

e. How many gallons of milk did Collette buy from January to October?

$3 + 2 + 1\frac{1}{4} + 1\frac{1}{2} + 1\frac{3}{4} + \frac{1}{2} + 2 + 1 + 3\frac{3}{4} + \frac{1}{4}$

$= 14 + \frac{12}{4} = 14 + 3 = 17$ Collette bought 17 gallons of milk.

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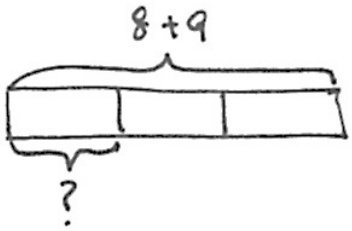
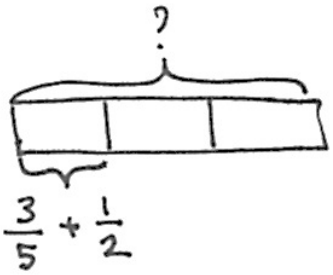
Name _____ Date _____

1. Write expressions to match the diagrams. Then, evaluate.





2. Write an expression to match, and then evaluate.

a. $\frac{1}{6}$ the sum of 16 and 20

b. Subtract 5 from $\frac{1}{3}$ of 23.

c. 3 times as much as the sum of $\frac{3}{4}$ and $\frac{2}{6}$

d. $\frac{2}{5}$ of the product of $\frac{5}{6}$ and 42

e. 8 copies of the sum of 4 thirds and 2 more

f. 4 times as much as 1 third of 8

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3. Circle the expression(s) that give the same product as $\frac{4}{5} \times 7$. Explain how you know.

$4 \div (7 \times 5)$ $7 \div 5 \times 4$ $(4 \times 7) \div 5$ $4 \div (5 \times 7)$ $4 \times \frac{7}{5}$ $7 \times \frac{4}{5}$

4. Use <, >, or = to make true number sentences without calculating. Explain your thinking.

a. $4 \times 2 + 4 \times \frac{2}{3}$ ◯ $3 \times \frac{2}{3}$

b. $\left(5 \times \frac{3}{4}\right) \times \frac{2}{5}$ ◯ $\left(5 \times \frac{3}{4}\right) \times \frac{2}{7}$

c. $3 \times \left(3 + \frac{15}{12}\right)$ ◯ $(3 \times 3) + \frac{15}{12}$

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5. Collette bought milk for herself each month and recorded the amount in the table below. For (a)–(c), write an expression that records the calculation described. Then, solve to find the missing data in the table.

a. She bought $\frac{1}{4}$ of July's total in June.

b. She bought $\frac{3}{4}$ as much in September as she did in January and July combined.

c. In April, she bought $\frac{1}{2}$ gallon less than twice as much as she bought in August.

Month	Amount (in gallons)
January	3
February	2
March	$1\frac{1}{4}$
April	
May	$\frac{7}{4}$
June	
July	2
August	1
September	
October	$\frac{1}{4}$

d. Display the data from the table in a line plot.

e. How many gallons of milk did Collette buy from January to October?

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Name _____ Date _____

1. Rewrite these expressions using words.

   a. $\frac{3}{4} \times \left(2\frac{2}{5} - \frac{5}{6}\right)$

   b. $2\frac{1}{4} + \frac{8}{3}$

2. Write an expression, and then solve.

   Three less than one-fourth of the product of eight thirds and nine

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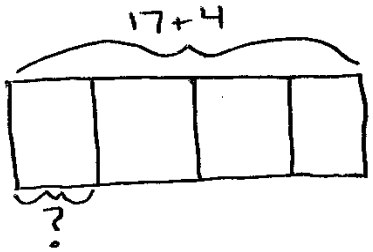
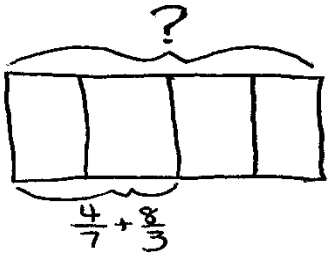
Name _____ Date _____

1. Write expressions to match the diagrams. Then, evaluate.





2. Circle the expression(s) that give the same product as $6 \times \frac{3}{8}$. Explain how you know.

$8 \div (3 \times 6)$ $3 \div 8 \times 6$ $(6 \times 3) \div 8$ $(8 \div 6) \times 3$ $6 \times \frac{8}{3}$ $\frac{3}{8} \times 6$

3. Write an expression to match, and then evaluate.

a. $\frac{1}{8}$ the sum of 23 and 17

b. Subtract 4 from $\frac{1}{6}$ of 42.

c. 7 times as much as the sum of $\frac{1}{3}$ and $\frac{4}{5}$

d. $\frac{2}{3}$ of the product of $\frac{3}{8}$ and 16

e. 7 copies of the sum of 8 fifths and 4

f. 15 times as much as 1 fifth of 12

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4. Use <, >, or = to make true number sentences without calculating. Explain your thinking.

a. $\frac{2}{3} \times (9 + 12)$ ◯ $15 \times \frac{2}{3}$

b. $\left(3 \times \frac{5}{4}\right) \times \frac{3}{5}$ ◯ $\left(3 \times \frac{5}{4}\right) \times \frac{3}{8}$

b. $6 \times \left(2 + \frac{32}{16}\right)$ ◯ $(6 \times 2) + \frac{32}{16}$

5. Fantine bought flour for her bakery each month and recorded the amount in the table to the right. For (a)–(c), write an expression that records the calculation described. Then, solve to find the missing data in the table.

a. She bought $\frac{3}{4}$ of January's total in August.

b. She bought $\frac{7}{8}$ as much in April as she did in October and July combined.

Month	Amount (in pounds)
January	3
February	2
March	$1\frac{1}{4}$
April	
May	$\frac{9}{8}$
June	
July	$1\frac{1}{4}$
August	
September	$\frac{11}{4}$
October	$\frac{3}{4}$

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c. In June, she bought $\frac{1}{8}$ pound less than three times as much as she bought in May.

d. Display the data from the table in a line plot.

e. How many pounds of flour did Fantine buy from January to October?

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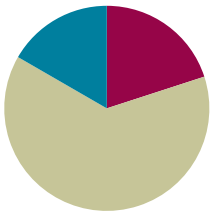
Lesson 11

Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(38 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Convert Measures **4.MD.1** (5 minutes)
- Multiply Whole Numbers by Fractions Using Two Methods **5.NF.4** (3 minutes)
- Write the Expression to Match the Diagram **5.NF.4** (4 minutes)

Convert Measures (5 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity reviews Lessons 9–10 and prepares students for Lessons 11–12 content. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

T: (Write 2 c = __ pt.) How many pints are in 2 cups?
S: 1 pint.
T: (Write 2 c = 1 pt. Below it, write 4 c = __ pt.) 4 cups?
S: 2 pints.
T: (Write 4 c = 2 pt. Below it, write 6 c = __ pt.) 6 cups?
S: 3 pints.
T: (Write 6 c = 3 pt. Below it, write 20 c = __ pt.) On your personal white board, write the equation.
S: (Write 20 c = 10 pt.)
T: (Write 20 c ÷ __ = __ pt.) Write the division equation you used to solve it.
S: (Write 20 c ÷ 2 = 10 pt.)

Continue with the following possible sequence: 12 in = 1 ft, 24 in = 2 ft, 48 in = 4 ft, 3 ft = 1 yd, 6 ft = 2 yd, 9 ft = 3 yd, 24 ft = 8 yd, 4 qt = 1 gal, 8 qt = 2 gal, 12 qt = 3 gal, and 36 qt = 9 gal.

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Multiply Whole Numbers by Fractions Using Two Methods (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

T: (Write $\frac{1}{2} \times 8 = \frac{__ \times __}{2}$.) On your personal white board, write the equation, and fill in the multiplication expression for the numerator.

S: (Write $\frac{1}{2} \times 8 = \frac{1 \times 8}{2}$.)

T: (Write $\frac{1}{2} \times 8 = \frac{1 \times 8}{2} = __ = __$.) Fill in the missing numbers.

S: (Write $\frac{1}{2} \times 8 = \frac{1 \times 8}{2} = \frac{8}{2} = 4$.)

T: (Write $\frac{1}{2} \times 8 = \frac{1 \times 8}{2}$.) Divide by a common factor and solve.

S: (Write $\frac{1}{2} \times 8 = \frac{1 \times \cancel{8}^{4}}{\cancel{2}_{1}} = 4$.)

T: Did you reach the same answer using both methods?

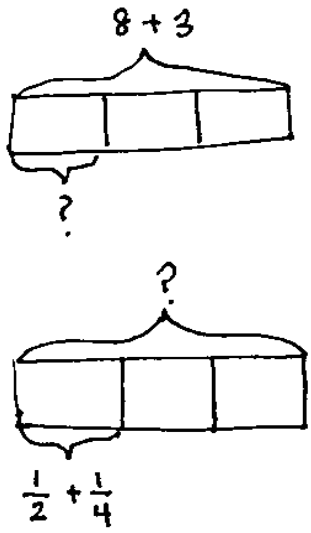
S: Yes.

Continue with the following possible sequence: $12 \times \frac{1}{4}$, $12 \times \frac{3}{4}$, $\frac{2}{3} \times 15$, and $18 \times \frac{5}{6}$.

Write the Expression to Match the Diagram (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.



T: (Project a tape diagram partitioned into 3 equal parts with 8 + 3 as the whole.) Say the value of the whole.

S: 11.

T: On your personal white board, write an expression to match the diagram.

S: (Write (8 + 3) ÷ 3.)

T: Solve the expression.

S: (Beneath (8 + 3) ÷ 3, write $\frac{11}{3}$. Beneath $\frac{11}{3}$, write $3\frac{2}{3}$.)

Repeat this sequence with the following suggested problem: (5 + 6) ÷ 3.

T: (Project a tape diagram partitioned into 3 equal parts. Beneath one of the units, write $\frac{1}{2} + \frac{1}{4}$.) On your personal white board, write an expression to match the diagram.

S: (Write $\left(\frac{1}{2} + \frac{1}{4}\right) \times 3$.)

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T: Solve the expression.

S: (Beneath $\frac{1}{2} + \frac{1}{4} \times 3$, write $\frac{3}{4} \times 3$. Beneath it, write $\frac{9}{4}$. Also, beneath that, write $2\frac{1}{4}$.)

Continue with the following possible suggested problem: $\left(\frac{1}{2} + \frac{2}{3}\right) \times 5$.

Concept Development (38 minutes)

Materials: (S) Problem Set

Note: Because today's lesson involves solving word problems, time allocated to the Application Problem has been allotted to the Concept Development.

Suggested Delivery of Instruction for Solving Lesson 11 Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the other students work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give students two minutes to finish their work on that question, sharing their work and thinking with a peer. All students should then write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

A general instructional note on today's problems: The problem solving in today's lesson requires that students combine their previous knowledge of adding and subtracting fractions with new knowledge of multiplying to find fractions of a set. The problems have been designed to encourage flexibility in terms of thinking by offering many avenues for solving each one. Be sure to conclude the work with plenty of time for students to present and compare approaches.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

When a task offers varied approaches for solving, an efficient way to have all students' work shared is to hold a Museum Walk. Using this approach, student work is displayed, and then students walk around from one piece to the next. This approach is similar to how one would view art in a gallery.

This method for sharing works best when a purpose for the observing is provided. For example, students might be asked to note similarities and differences in the drawing of a model or approach to a calculation. Students can indicate the similarities or differences by using color-coded sticky notes that are attached to the displays. Another way to react to the Museum Walk is to write about what they notice in journals.

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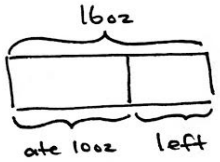
Problem 1

Kim and Courtney share a 16-ounce box of cereal. By the end of the week, Kim has eaten $\frac{3}{8}$ of the box, and Courtney has eaten $\frac{1}{4}$ of the box of cereal. What fraction of the box is left?

Method 1: Kim: $\frac{3}{8}$ of 16 oz $= \frac{3 \times \cancel{16}^{2}}{\cancel{8}_{1}}$ oz $= 6$ oz

Courtney: $\frac{1}{4}$ of 16 oz $= \frac{16}{4}$ oz $= 4$ oz

6 oz $+ 4$ oz $= 10$ oz



16 oz − 10 oz = 6 oz

$\frac{6}{16} = \frac{3}{8}$

$\frac{3}{8}$ of the box is left.

Method 2: $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

$\frac{5}{8}$ of 16 oz $= \frac{5 \times \cancel{16}^{2}}{\cancel{8}_{1}}$ oz $= 10$ oz

$1 - \frac{5}{8} = \frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

$\frac{3}{8}$ of the box is left.

To complete Problem 1, students must find fractions of a set and use skills learned in Module 3 to add or subtract fractions.

As exemplified, students may solve this multi-step word problem using different methods. Consider demonstrating these two methods of solving Problem 1 if both methods are not mentioned by students. Point out that the rest of today's Problem Set can be solved using multiple strategies as well.

If desired, this problem's complexity may be increased by changing the amount of the cereal in the box to 20 ounces and Courtney's fraction to $\frac{1}{3}$. This will produce a mixed number for both girls. Kim's portion becomes $7\frac{1}{2}$ ounces, and Courtney's becomes $6\frac{2}{3}$.

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Problem 2

Mathilde has 20 pints of green paint. She uses $\frac{2}{5}$ of it to paint a landscape and $\frac{3}{10}$ of it while painting a clover. She decides that, for her next painting, she will need 14 pints of green paint. How much more paint will she need to buy?

Method 1:

landscape: $\frac{2}{5}$ of 20 pt = $\frac{2 \times 20^{4}}{8_{1}}$ pt = 8 pt

clover: $\frac{3}{10}$ of 20 pt = $\frac{3 \times 20^{2}}{10_{1}}$ pt = 6 pt

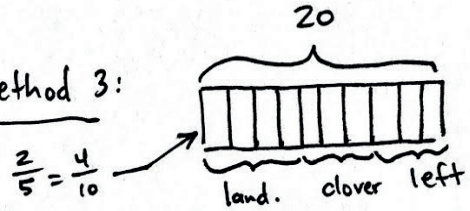
used: 8 pt + 6 pt = 14 pt

left over: 20 pt − 14 pt = 6 pt

needed: 14 pt − 6 pt = 8 pt

She needs 8 more pints of paint.

Method 3:



$\frac{2}{5} = \frac{4}{10}$

$\frac{3}{10}$ of 20 = $\frac{3 \times 20^{2}}{10_{1}}$ = 6

14 − 6 = 8

She needs 8 more pints.

Method 2:

$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

$\frac{7}{10}$ of 20 pt = $\frac{7 \times 20^{2}}{10_{1}}$ pt = 14 pt

20 − 14 = 6

14 − 6 = 8

She needs 8 more pints.

Complexity is increased here because students are called on to maintain a high level of organization as they keep track of the attribute of paint *used* and *not used*. Multiple approaches should be encouraged. For Methods 1 and 2, the *used* paint is the focus of the solution. Students may choose to find the fractions of the whole (fraction of a set) Mathilde has used on each painting; they may also first add the separate fractions before finding the fraction of the whole. Subtracting that portion from the 14 pints she'll need for her next project yields the answer to the question. Method 3 finds the leftover paint and simply subtracts it from the 14 pints needed for the next painting.

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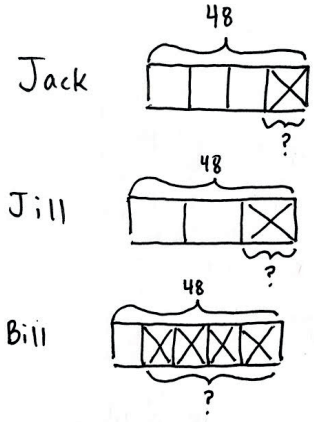
Problem 3

Jack, Jill, and Bill each carried a 48-ounce bucket full of water down the hill. By the time they reached the bottom, Jack's bucket was only $\frac{3}{4}$ full, Jill's was $\frac{2}{3}$ full, and Bill's was $\frac{1}{6}$ full. How much water did they spill altogether on their way down the hill?



Method 1:

Jack: $\frac{3}{4}$ of 48 oz = $\frac{3 \times \overset{12}{\cancel{48}}}{\cancel{4}_1}$ oz = 36 oz

Jill: $\frac{2}{3}$ of 48 oz = $\frac{2 \times \overset{16}{\cancel{48}}}{\cancel{3}_1}$ oz = 32 oz

Bill: $\frac{1}{6}$ of 48 oz = $\frac{1 \times \overset{8}{\cancel{48}}}{\cancel{6}_1}$ oz = 8 oz

48 oz − 36 oz = 12 oz

48 oz − 32 oz = 16 oz

48 oz − 8 oz = 40 oz

They spilled 68 oz.

12 oz
16 oz
+ 40 oz
68 oz

Method 2:

Jack: $\frac{3}{4}$ full → $\frac{1}{4}$ of 48 oz = $\frac{1 \times \overset{12}{\cancel{48}}}{\cancel{4}_1}$ oz = 12 oz

Jill: $\frac{2}{3}$ full → $\frac{1}{3}$ of 48 oz = $\frac{1 \times \overset{16}{\cancel{48}}}{\cancel{3}_1}$ oz = 16 oz

Bill: $\frac{1}{6}$ full → $\frac{5}{6}$ of 48 oz = $\frac{5 \times \overset{8}{\cancel{48}}}{\cancel{6}_1}$ oz = 40 oz

12 oz + 16 oz + 40 oz = 68 oz

They spilled 68 ounces.

This problem is very similar to Problem 2 in that students keep track of one attribute—spilled water or unspilled water. However, the inclusion of a third person, Bill, requires that students keep track of more information. In Method 1, a student may opt to find the fraction of water remaining in each bucket. This process requires students to then subtract those portions from the 48 ounces that each bucket held originally. In Method 2, students may decide to find what fraction of the water has been spilled by *counting on* to a whole (e.g., if 3 fourths remain in Jack's bucket, then only 1 fourth has been spilled). This is a more direct approach to the solution because subtraction from 48 is not necessary.

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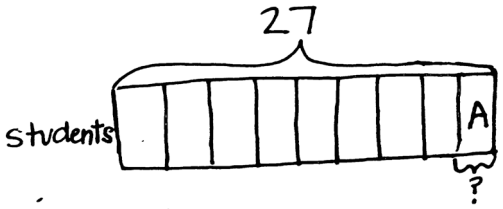
Problem 4

Mrs. Diaz makes 5 dozen cookies for her class. One-ninth of her 27 students are absent the day she brings the cookies. If she shares the cookies equally among the students who are present, how many cookies will each student get?



4) Method 1:

9 units = 27

1 unit = $\frac{27}{9}$

= 3

27 − 3 = 24

24 students present

5 × 12 = 60

$\frac{60}{24} \div \frac{12}{12} = \frac{5}{2} = 2\frac{1}{2}$

Each student gets $2\frac{1}{2}$ cookies.

Method 2: 5 dozen = 5 × 12 = 60

$\frac{1}{9} \times 27 = \frac{1 \times 27^{3}}{9_{1}} = 3$

3 students absent

27 − 3 = 24

60 ÷ 24 = $2\frac{1}{2}$

$2\frac{12}{24} = 2\frac{1}{2}$

24 | 60

−48

12

Each student gets $2\frac{1}{2}$ cookies.

Method 3: 5 dozen = 5 × 12 = 60

$\frac{1}{9}$ absent → $\frac{8}{9}$ present

$\frac{8}{9} \times 27 = \frac{8 \times 27^{3}}{9_{1}} = 24$

24 students present

60 ÷ 24 = $2\frac{12}{24} = 2\frac{1}{2}$

Each student gets $2\frac{1}{2}$ cookies.

This problem is straightforward, yet the division of the cookies at the end provides an opportunity to call out the division interpretation of a fraction. With quantities such as 60 and 24, students likely lean toward the long division algorithm, so using fraction notation to show the division may need to be discussed as an alternative. Using the fraction and renaming using larger units may be the more efficient approach given the quantities. The similarities and differences of these approaches certainly bear a moment's discussion. Additionally, the practicality of sharing cookies in twenty-fourths can result in a discussion of renaming $2\frac{12}{24}$ as $2\frac{1}{2}$.

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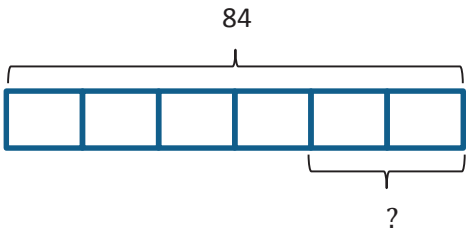
Problem 5

Create a story problem about a fish tank for the tape diagram below. Your story must include a fraction.



5) There are 84 fish in the fish tank. $\frac{4}{6}$ of the fish are goldfish, and the rest are guppies. How many guppies are in the fish tank?

5) There are 84 organisms in a freshwater aquarium. One half of the organisms are top-feeding fish. One sixth are crustaceans. The remaining organisms are apple snails. How many snails are in the tank?

In this problem, students are shown a tape diagram marking 84 as the whole and partitioned into 6 equal units (or sixths). The question mark should signal students to find $\frac{2}{6}$ of the whole.

Students are asked to create a word problem about a fish tank. Students should be encouraged to use their creativity while generating a word problem but remain mathematically sound. Two sample stories are included here, but this is a good opportunity to have students share their own word problems aloud.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Generally, early finishers for this lesson are students who use an abstract, more procedural approach to solving. These students might be asked to work the problems again using well-drawn tape diagrams that explain why their calculations are valid. These models could be displayed in hallways or placed in a class book.

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Student Debrief (10 minutes)

Lesson Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

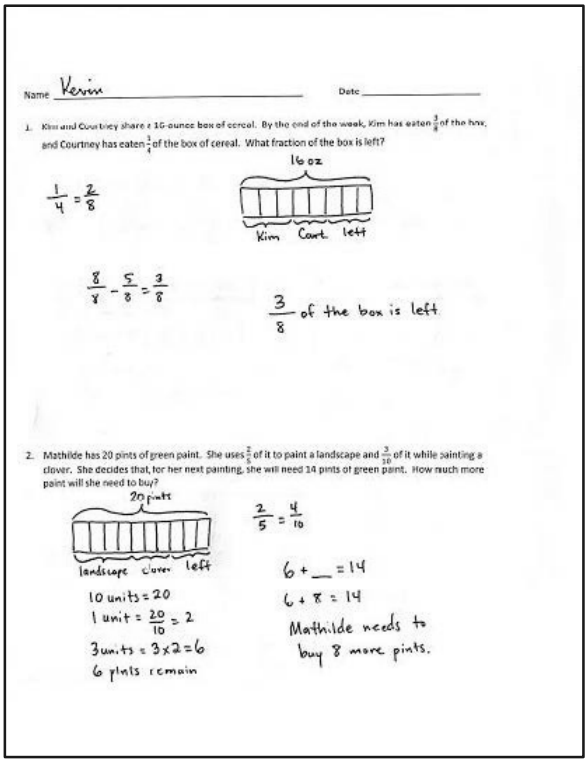
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How are the problems alike? How are they different?
- How many strategies can you use to solve the problems?
- How was your solution the same and different from those that were demonstrated?
- Did you see other solutions that surprised you or made you see the problems differently?
- How many different story problems can you create for Problem 5?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


Name Kevin Date

1. Kim and Courtney share a 16-ounce box of cereal. By the end of the week, Kim has eaten $\frac{3}{8}$ of the box, and Courtney has eaten $\frac{1}{4}$ of the box of cereal. What fraction of the box is left?

$\frac{1}{4} = \frac{2}{8}$

16 oz

Kim Court left

$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

$\frac{3}{8}$ of the box is left

2. Mathilde has 20 pints of green paint. She uses $\frac{2}{5}$ of it to paint a landscape and $\frac{3}{10}$ of it while painting a clover. She decides that, for her next painting, she will need 14 pints of green paint. How much more paint will she need to buy?

20 pints

landscape clover left

$\frac{2}{5} = \frac{4}{10}$

10 units = 20

1 unit = $\frac{20}{10}$ = 2

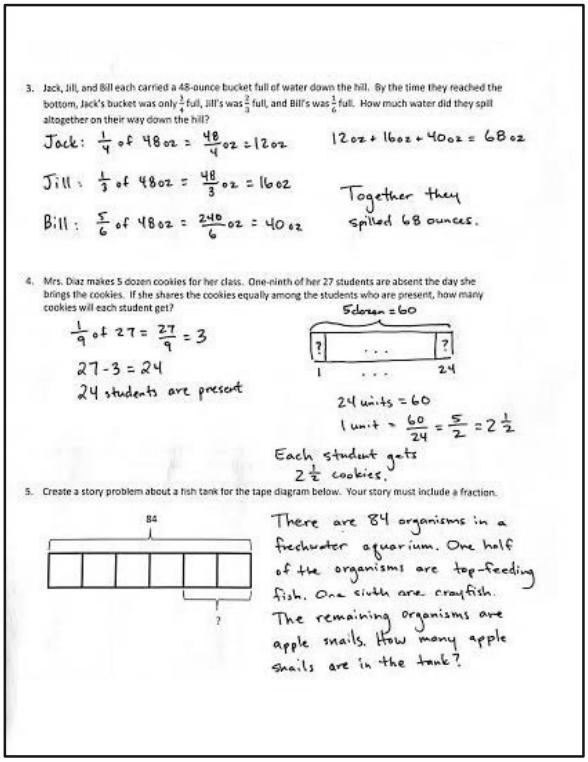
3 units = 3 x 2 = 6

6 pints remain

6 + ___ = 14

6 + 8 = 14

Mathilde needs to buy 8 more pints.


3. Jack, Jill, and Bill each carried a 48-ounce bucket full of water down the hill. By the time they reached the bottom, Jack's bucket was only $\frac{1}{4}$ full, Jill's was $\frac{1}{3}$ full, and Bill's was $\frac{5}{6}$ full. How much water did they spill altogether on their way down the hill?

Jack: $\frac{1}{4}$ of 48 oz = $\frac{48}{4}$ oz = 12 oz

Jill: $\frac{1}{3}$ of 48 oz = $\frac{48}{3}$ oz = 16 oz

Bill: $\frac{5}{6}$ of 48 oz = $\frac{240}{6}$ oz = 40 oz

12 oz + 16 oz + 40 oz = 68 oz

Together they spilled 68 ounces.

4. Mrs. Diaz makes 5 dozen cookies for her class. One-ninth of her 27 students are absent the day she brings the cookies. If she shares the cookies equally among the students who are present, how many cookies will each student get?

$\frac{1}{9}$ of 27 = $\frac{27}{9}$ = 3

27 - 3 = 24

24 students are present

5 dozen = 60

? ... ?

1 ... 24

24 units = 60

1 unit = $\frac{60}{24} = \frac{5}{2} = 2\frac{1}{2}$

Each student gets $2\frac{1}{2}$ cookies.

5. Create a story problem about a fish tank for the tape diagram below. Your story must include a fraction.

84

?

There are 84 organisms in a freshwater aquarium. One half of the organisms are top-feeding fish. One sixth are crayfish. The remaining organisms are apple snails. How many apple snails are in the tank?

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Name _____ Date _____

1. Kim and Courtney share a 16-ounce box of cereal. By the end of the week, Kim has eaten $\frac{3}{8}$ of the box, and Courtney has eaten $\frac{1}{4}$ of the box of cereal. What fraction of the box is left?

2. Mathilde has 20 pints of green paint. She uses $\frac{2}{5}$ of it to paint a landscape and $\frac{3}{10}$ of it while painting a clover. She decides that, for her next painting, she will need 14 pints of green paint. How much more paint will she need to buy?

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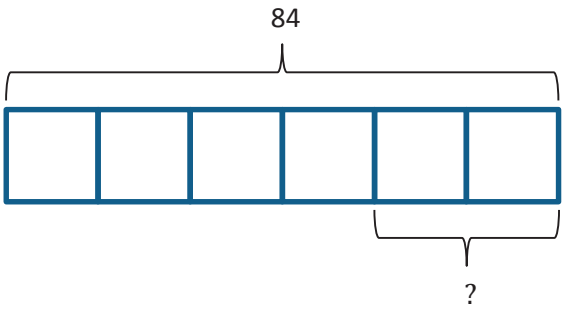
3. Jack, Jill, and Bill each carried a 48-ounce bucket full of water down the hill. By the time they reached the bottom, Jack's bucket was only $\frac{3}{4}$ full, Jill's was $\frac{2}{3}$ full, and Bill's was $\frac{1}{6}$ full. How much water did they spill altogether on their way down the hill?

4. Mrs. Diaz makes 5 dozen cookies for her class. One-ninth of her 27 students are absent the day she brings the cookies. If she shares the cookies equally among the students who are present, how many cookies will each student get?

5. Create a story problem about a fish tank for the tape diagram below. Your story must include a fraction.



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Name _____ Date _____

Use a tape diagram to solve.

$\frac{2}{3}$ of 5

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Name _____ Date _____

1. Jenny's mom says she has an hour before it's bedtime. Jenny spends $\frac{1}{3}$ of the hour texting a friend and $\frac{1}{4}$ of the time brushing her teeth and putting on her pajamas. She spends the rest of the time reading her book. How many minutes did Jenny read?

2. A-Plus Auto Body is painting designs on a customer's car. They had 18 pints of blue paint on hand. They used $\frac{1}{2}$ of it for the flames and $\frac{1}{3}$ of it for the sparks. They need $7\frac{3}{4}$ pints of blue paint to paint the next design. How many more pints of blue paint will they need to buy?

3. Giovanna, Frances, and their dad each carried a 10-pound bag of soil into the backyard. After putting soil in the first flower bed, Giovanna's bag was $\frac{5}{8}$ full, Frances's bag was $\frac{2}{5}$ full, and their dad's was $\frac{3}{4}$ full. How many pounds of soil did they put in the first flower bed altogether?

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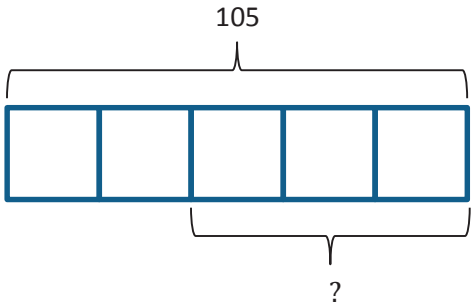
4. Mr. Chan made 252 cookies for the Annual Fifth Grade Class Bake Sale. They sold $\frac{3}{4}$ of them, and $\frac{3}{9}$ of the remaining cookies were given to PTA. members. Mr. Chan allowed the 12 student helpers to divide the cookies that were left equally. How many cookies will each student get?

5. Using the tape diagram below, create a story problem about a farm. Your story must include a fraction.



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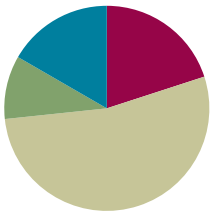
Lesson 12

Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Convert Measures **4.MD.1** (4 minutes)
- Multiply a Fraction and a Whole Number **5.NF.3** (4 minutes)
- Write the Expression to Match the Diagram **5.NF.4** (4 minutes)

Convert Measures (4 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity reviews Lessons 9–11 and prepares students for Lesson 12 content. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

T: (Write 1 ft = __ in.) How many inches are in 1 foot?
S: 12 inches.
T: (Write 1 ft = 12 in. Below it, write 2 ft = __ in.) 2 feet?
S: 24 inches.
T: (Write 2 ft = 24 in. Below it, write 4 ft = __ in.) 4 feet?
S: 48 inches.
T: Write the multiplication equation you used to solve it.
S: (Write 4 ft × 12 = 48 in.)

Continue with the following possible sequence: 1 pint = 2 cups, 2 pints = 4 cups, 7 pints = 14 cups, 1 yard = 3 ft, 2 yards = 6 ft, 6 yards = 18 ft, 1 gal = 4 qt, 2 gal = 8 qt, and 9 gal = 36 qt.

T: (Write 2 c = __ pt.) How many pints are in 2 cups?

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S: 1 pint.

T: (Write 2 c = 1 pt. Below it, write 4 c = __ pt.) 4 cups?

S: 2 pints.

T: (Write 4 c = 2 pt. Below it, write 10 c = __ pt.) 10 cups?

S: 5 pints.

T: Write the division equation you used to solve it.

S: (Write 10 c ÷ 2 = 5 pt.)

Continue with the following possible sequence: 12 in = 1 ft, 36 in = 3 ft, 3 ft = 1 yd, 12 ft = 4 yd, 4 qt = 1 gal, and 28 qt = 7 gal.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

For English language learners and struggling students, provide the conversion reference sheet included in Lesson 8.

Multiply a Fraction and a Whole Number (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 9–11.

T: (Write 9 ÷ 3 =__.) Say the division sentence.

S: 9 ÷ 3 = 3.

T: (Write $\frac{1}{3} \times 9 =$__.) Say the multiplication sentence.

S: $\frac{1}{3} \times 9 = 3$.

T: (Write $\frac{2}{3} \times 9 =$__.) On your personal white board, write the multiplication sentence.

S: (Write $\frac{2}{3} \times 9 = 6$.)

T: (Write $9 \times \frac{2}{3} =$__.) On your personal white board, write the multiplication sentence.

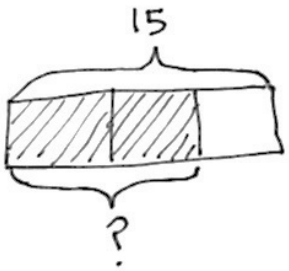
S: (Write $9 \times \frac{2}{3} = 6$.)

Continue with the following possible sequence: $12 \div 6$, $\frac{1}{6} \times 12$, $\frac{5}{6} \times 12$, $12 \times \frac{5}{6}$, $\frac{1}{8} \times 24$, $24 \times \frac{1}{8}$, $24 \times \frac{3}{8}$, $\frac{2}{3} \times 12$, and $12 \times \frac{3}{4}$.

Write the Expression to Match the Diagram (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 10–11.



T: (Project a tape diagram partitioned into 3 equal units with 15 as the whole and 2 units shaded.) Say the value of the whole.

S: 15.

T: On your personal white board, write an expression to match the diagram using a fraction.

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S: (Write $\frac{2}{3} \times 15$ or $15 \times \frac{2}{3}$.)

T: To solve, we can write 15 divided by 3 to find the value of one unit, times 2. (Write $\frac{15}{3} \times 2$ while saying the words.)

T: Find the value of the expression.

S: (Write $\frac{15}{3} \times 2 = 10$.)

Continue this process with the following suggested problems: $\frac{3}{5} \times 45$, $\frac{3}{4} \times 32$, $\frac{5}{6} \times 54$, and $\frac{7}{8} \times 64$.

Application Problem (6 minutes)

Complete the table.

$\frac{2}{3}$ yard	_____ foot(feet)
4 pounds	_____ ounce(s)
8 tons	_____ pound(s)
$\frac{3}{4}$ gallon	_____ quart(s)
$\frac{5}{12}$ year	_____ month(s)
$\frac{4}{5}$ hour	_____ minute(s)

Note: The chart requires students to work within many customary systems reviewing the work of Lesson 9. Students may need a conversion chart (Lesson 8) to scaffold this problem.

$\frac{2}{3}$ yard = 2 feet

4 pounds = 56 ounces

8 tons = 16,000 pounds

$\frac{3}{4}$ gallons = 3 quarts

$\frac{5}{12}$ year = 5 months

$\frac{4}{5}$ hour = 48 minutes

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Concept Development (32 minutes)

Materials: (S) Problem Set

Suggested Delivery of Instruction for Solving Lesson 12 Word Problems

1. **Model the problem.**

   Have two pairs of student who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

   - Can you draw something?
   - What can you draw?
   - What conclusions can you reach from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. **Calculate to solve and write a statement.**

Give students two minutes to finish their work on that question, sharing their work and thinking with a peer. All students should then write their equations and statements of the answer.

3. **Assess the solution for reasonableness.**

Give students one to two minutes to assess and explain the reasonableness of their solution.

A general instructional note on today's problems: Today's problems are more complex than those found in Lesson 11. All are multi-step problems. Students should be strongly encouraged to draw before attempting to solve. As in Lesson 11, multiple approaches to solving all of the problems are possible. Students should be given time to share and compare thinking during the Debrief.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The complexity of the language involved in today's problems may pose significant challenges to English language learners or students with learning differences that affect language processing. Consider pairing these students with students in the class who are adept at drawing clear models. These visuals and the peer interaction they generate can be invaluable bridges to making sense of the written word.

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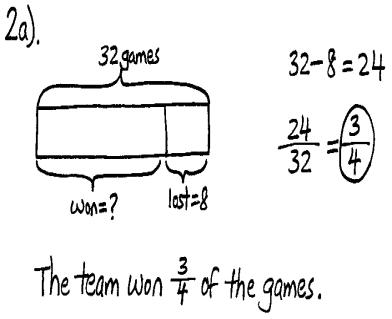
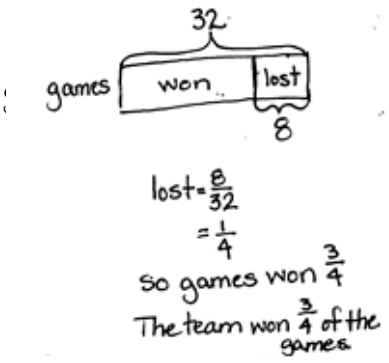
Problem 1

A baseball team played 32 games and lost 8. Katy was the catcher in $\frac{5}{8}$ of the winning games and $\frac{1}{4}$ of the losing games.

a. What fraction of the games did the team win?

b. In how many games did Katy play catcher?





While Part A is relatively straightforward, there are still various approaches for solving. Students may find the difference between the number of games played and lost to find the number of games won (24), expressing this difference as a fraction ($\frac{24}{32}$). Alternatively, they may conclude that the losing games are $\frac{1}{4}$ of the total and deduce that winning games must constitute $\frac{3}{4}$. Watch for students distracted by the fractions $\frac{5}{8}$ and $\frac{1}{4}$ written in the stem, and try to involve them in the solution to Part (a). Complexity increases as students must use the fraction of a set strategy twice, carefully matching each fraction with the appropriate number of games and finally combining the number of games that Katy played to find the total.

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Problem 2

In Mrs. Elliott's garden, $\frac{1}{8}$ of the flowers are red, $\frac{1}{4}$ of them are purple, and $\frac{1}{5}$ of the remaining flowers are pink. If there are 128 flowers, how many flowers are pink?

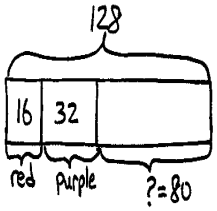
Method 1:

Red: $\frac{1}{8}$ of 128 $= \frac{1 \times 128}{8} = 16$

Purple: $\frac{1}{4}$ of 128 $= \frac{1 \times 128}{4} = 32$

$128 - 16 - 32 = 80$

Pink: $\frac{1}{5}$ of 80 $= \frac{1 \times 80}{5} = 16$



Method 2:

$\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

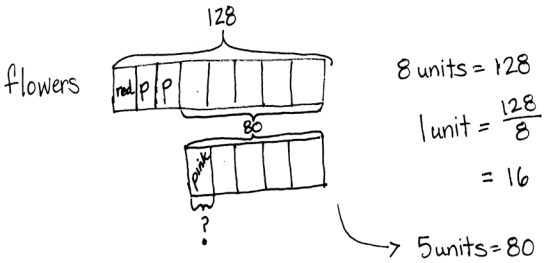
$\frac{3}{8}$ of 128 $= \frac{3 \times 128}{8} = \frac{3 \times 16}{1} = 48$

$128 - 48 = 80$

$\frac{1}{5}$ of 80 $= \frac{1 \times 80}{5} = 16$

16 flowers are pink.

Method 3:



8 units = 128

1 unit = $\frac{128}{8}$

= 16

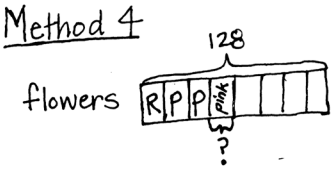
5 units = 80

1 unit = $\frac{80}{5}$

= 16

16 flowers are pink

Method 4



8 units = 128

1 unit = $\frac{128}{8}$

= 16

$\frac{1}{5}$ of rest = 16

16 flowers are pink

The complexity for this problem increases as students are asked to find the number of pink flowers in the garden. This portion of the flowers refers to 1 fifth of the *remaining* flowers (i.e., 1 fifth of those that are not red or purple), *not* 1 fifth of the total. Some students may realize (as in Method 4) that 1 fifth of the remainder is simply equal to 1 unit or 16 flowers. Multiple methods of drawing and solving are possible. Some of the possibilities are pictured above.

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Problem 3

Lillian and Darlene plan to get their homework finished within one hour. Darlene completes her math homework in $\frac{3}{5}$ hour. Lillian completes her math homework with $\frac{5}{6}$ hour remaining. Who completes her homework faster, and by how many minutes?

Bonus: Give the answer as a fraction of an hour.

Method 1:

4) Darlene: $\frac{3}{5}$ of an hour = $\frac{3}{5} \times 60 = \frac{3 \times 60^{12}}{5_1} = 36$ min

Lillian: $\frac{5}{6}$ of an hour remaining → $\frac{1}{6}$ of an hour

$\frac{1}{6} \times 60 = \frac{1 \times 60^{10}}{6_1} = 10$ min

36 min − 10 min = 26 min

Lillian completed the homework faster. She did it 26 minutes faster.

BONUS: 26 min as a fraction of an hour

→ $\frac{26}{60} = \frac{13}{30}$ of an hour faster

Method 2:

$\frac{3}{5} \times 60 = \frac{60}{5} \times 3 = 12 \times 3 = 36$ min

$\frac{1}{6} \times 60 = \frac{60}{6} = 10$ min

Lillian completed the homework 26 minutes faster than Darlene.

The way in which Lillian's time is expressed establishes some complexity in this problem. Students must recognize that she only took $\frac{1}{6}$ hour to complete the assignment. Many students may quickly recognize that Lillian worked faster as $\frac{1}{6} < \frac{3}{5}$. However, students must proceed further to find exactly how many minutes faster. The bonus requires them to give the fraction of an hour. Simplification of this fraction should not be required but may be discussed.

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Problem 4

Create and solve a story problem about a baker and some flour whose solution is given by the expression $\frac{1}{4} \times (3 + 5)$.

The pastry chef mixes $\frac{1}{4}$ cup of sugar into each cup of flour. He uses 3 cups of wheat flour and 5 cups of rice flour. How many cups of sugar does he use in all?

$\frac{1}{4} \times (3+5)$
$= \frac{3+5}{4}$
$= \frac{8}{4}$
$= 2$ He uses 2 cups of sugar.

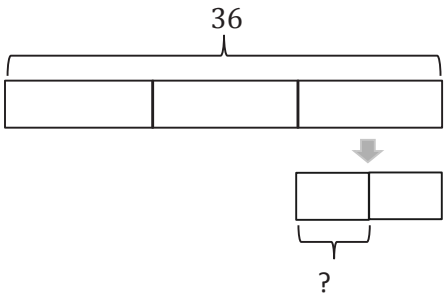
A baker combined 3 cups of sugar with 5 cups of flour together in a bowl. He used $\frac{1}{4}$ of the mixture to bake cookies. How many cups of the mixture did he use?

$\frac{1}{4} \times (3+5)$
$= \frac{1}{4} \times 8$
$= \frac{1 \times 8}{4}$
$= 2$ He used 2 cups of the mixture.

Working backward from expression to story may be challenging for some students. Since the expression given contains parentheses, the story created must first involve the addition or combining of 3 and 5. For students who require assistance, drawing a tape diagram first may help. Then, asking the simple prompt, "What would a baker add together or combine?" may be enough to help students begin. Evaluating $\frac{1}{4} \times (3 + 5)$ should pose no significant challenge to students. Note that the story of the chef interprets the expression as repeated addition of a fourth where the story of the baker interprets the expression as a fraction of a set.

Problem 5

Create and solve a story problem about a baker and 36 kilograms of an ingredient that is modeled by the following tape diagram. Include at least one fraction in your story.



A baker poured a bag of 36 kg of flour equally into 3 containers. He then used $\frac{1}{2}$ of one container of flour to make pretzels. How many kilograms of flour did he use to make the pretzels?

3 units = 36 kg
1 unit = $36 \div 3 = 12$ kg
$\frac{1}{2}$ of 12 = $\frac{1 \times 12}{2} = 6$ kg
He used 6 kg of flour to make pretzels.

Again, students are asked to both create and then solve a story problem, this time using a given tape diagram. The challenge here is that this tape diagram implies a two-step word problem. The whole, 36, is first partitioned into thirds, and then one of those thirds is divided in half. The story students create should reflect this two-part drawing. Students should be encouraged to share aloud, as well as discuss their stories and thought process for solving.

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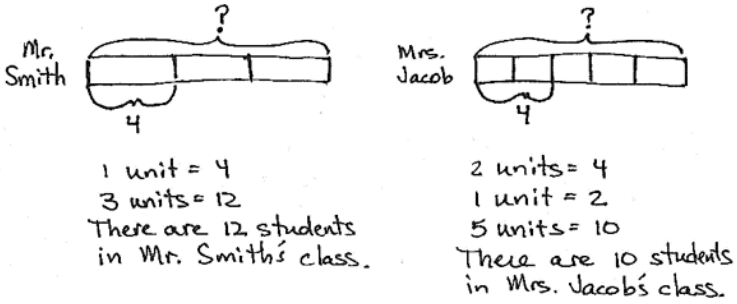
Problem 6

Of the students in Mr. Smith's fifth-grade class, $\frac{1}{3}$ were absent on Monday. Of the students in Mrs. Jacobs's class, $\frac{2}{5}$ were absent on Monday. If there were 4 students absent in each class on Monday, how many students are in each class?

For this problem, students need to find the whole. An interesting aspect of this problem is that fractional amounts of different wholes can be the same amount. In this case, two-fifths of 10 is the same as one-third of 12. This should be discussed with students.

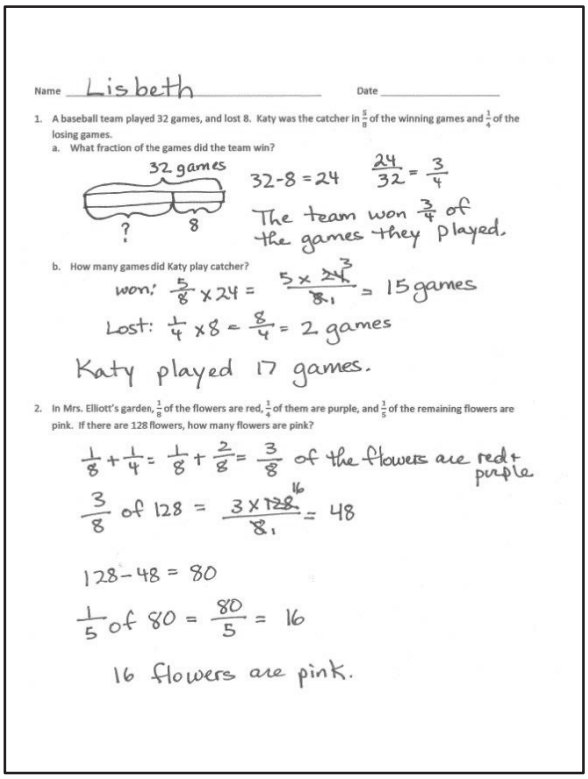


Student Debrief (10 minutes)

Lesson Objective: Solve and create fraction word problems involving addition, subtraction, and multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


Name Lisbeth Date

1. A baseball team played 32 games, and lost 8. Katy was the catcher in $\frac{5}{8}$ of the winning games and $\frac{1}{4}$ of the losing games.
   a. What fraction of the games did the team win?

32 games; ? ; 8

32 − 8 = 24 $\frac{24}{32} = \frac{3}{4}$

The team won $\frac{3}{4}$ of the games they played.

   b. How many games did Katy play catcher?

won: $\frac{5}{8} \times 24 = \frac{5 \times 24^{3}}{8_{1}} = 15$ games

Lost: $\frac{1}{4} \times 8 = \frac{8}{4} = 2$ games

Katy played 17 games.

2. In Mrs. Elliott's garden, $\frac{1}{8}$ of the flowers are red, $\frac{1}{4}$ of them are purple, and $\frac{3}{5}$ of the remaining flowers are pink. If there are 128 flowers, how many flowers are pink?

$\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$ of the flowers are red + purple

$\frac{3}{8}$ of 128 = $\frac{3 \times 128^{16}}{8_{1}} = 48$

128 − 48 = 80

$\frac{1}{5}$ of 80 = $\frac{80}{5} = 16$

16 flowers are pink.

Any combination of the questions below may be used to lead the discussion.

- How are the problems similar? How are they different?
- How many strategies can you use to solve the problems?
- How was your solution the same and different from those that were demonstrated?
- Did you see other solutions that surprised you or made you see the problems differently?
- How many different story problems can you create for Problems 5 and 6?

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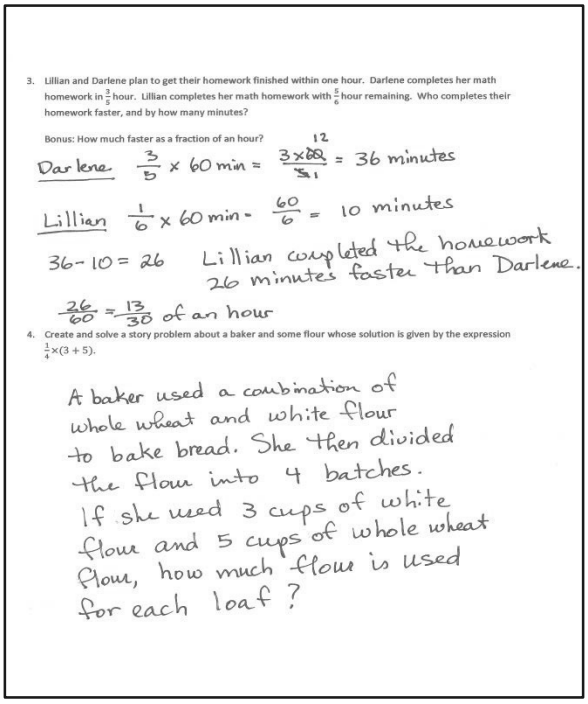
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



3. Lillian and Darlene plan to get their homework finished within one hour. Darlene completes her math homework in $\frac{3}{5}$ hour. Lillian completes her math homework with $\frac{5}{6}$ hour remaining. Who completes their homework faster, and by how many minutes?

Bonus: How much faster as a fraction of an hour?

Darlene $\frac{3}{5} \times 60 \text{ min} = \frac{3 \times 60}{5} = 36$ minutes

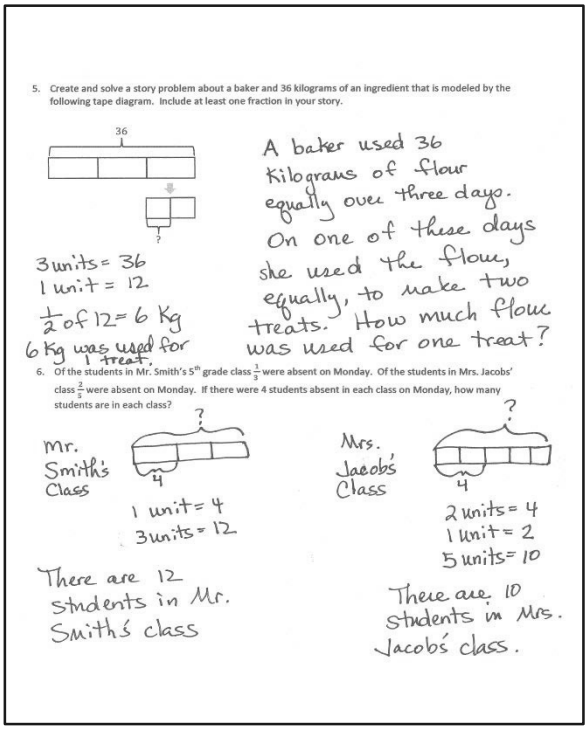
Lillian $\frac{1}{6} \times 60 \text{ min} = \frac{60}{6} = 10$ minutes

$36 - 10 = 26$ Lillian completed the homework 26 minutes faster than Darlene.

$\frac{26}{60} = \frac{13}{30}$ of an hour

4. Create and solve a story problem about a baker and some flour whose solution is given by the expression $\frac{1}{4} \times (3 + 5)$.

A baker used a combination of whole wheat and white flour to bake bread. She then divided the flour into 4 batches. If she used 3 cups of white flour and 5 cups of whole wheat flour, how much flour is used for each loaf?



5. Create and solve a story problem about a baker and 36 kilograms of an ingredient that is modeled by the following tape diagram. Include at least one fraction in your story.

3 units = 36
1 unit = 12
$\frac{1}{2}$ of 12 = 6 kg
6 kg was used for 1 treat.

A baker used 36 kilograms of flour equally over three days. On one of these days she used the flour, equally, to make two treats. How much flour was used for one treat?

6. Of the students in Mr. Smith's 5th grade class $\frac{1}{3}$ were absent on Monday. Of the students in Mrs. Jacobs' class $\frac{2}{5}$ were absent on Monday. If there were 4 students absent in each class on Monday, how many students are in each class?

Mr. Smith's Class

1 unit = 4
3 units = 12

There are 12 students in Mr. Smith's class

Mrs. Jacobs' Class

2 units = 4
1 unit = 2
5 units = 10

There are 10 students in Mrs. Jacobs' class.

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Name _____ Date _____

1. A baseball team played 32 games and lost 8. Katy was the catcher in $\frac{5}{8}$ of the winning games and $\frac{1}{4}$ of the losing games.

   a. What fraction of the games did the team win?

   b. In how many games did Katy play catcher?

2. In Mrs. Elliott's garden, $\frac{1}{8}$ of the flowers are red, $\frac{1}{4}$ of them are purple, and $\frac{1}{5}$ of the remaining flowers are pink. If there are 128 flowers, how many flowers are pink?

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3. Lillian and Darlene plan to get their homework finished within one hour. Darlene completes her math homework in $\frac{3}{5}$ hour. Lillian completes her math homework with $\frac{5}{6}$ hour remaining. Who completes her homework faster, and by how many minutes?

   Bonus: Give the answer as a fraction of an hour.

4. Create and solve a story problem about a baker and some flour whose solution is given by the expression $\frac{1}{4} \times (3 + 5)$.

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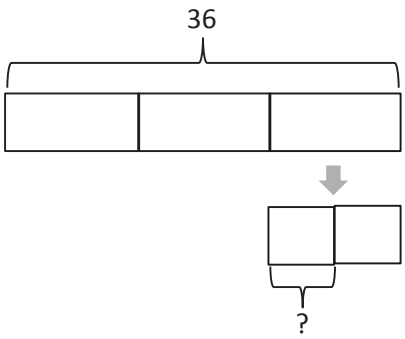
5. Create and solve a story problem about a baker and 36 kilograms of an ingredient that is modeled by the following tape diagram. Include at least one fraction in your story.



6. Of the students in Mr. Smith's fifth-grade class, $\frac{1}{3}$ were absent on Monday. Of the students in Mrs. Jacobs' class, $\frac{2}{5}$ were absent on Monday. If there were 4 students absent in each class on Monday, how many students are in each class?

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Name _____ Date _____

In a classroom, $\frac{1}{6}$ of the students are wearing blue shirts, and $\frac{2}{3}$ are wearing white shirts. There are 36 students in the class. How many students are wearing a shirt other than blue or white?

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Name _____ Date _____

1. Terrence finished a word search in $\frac{3}{4}$ the time it took Frank. Charlotte finished the word search in $\frac{2}{3}$ the time it took Terrence. Frank finished the word search in 32 minutes. How long did it take Charlotte to finish the word search?

2. Ms. Phillips ordered 56 pizzas for a school fundraiser. Of the pizzas ordered, $\frac{2}{7}$ of them were pepperoni, 19 were cheese, and the rest were veggie pizzas. What fraction of the pizzas was veggie?

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3. In an auditorium, $\frac{1}{6}$ of the students are fifth graders, $\frac{1}{3}$ are fourth graders, and $\frac{1}{4}$ of the remaining students are second graders. If there are 96 students in the auditorium, how many second graders are there?

4. At a track meet, Jacob and Daniel compete in the 220 m hurdles. Daniel finishes in $\frac{3}{4}$ of a minute. Jacob finishes with $\frac{5}{12}$ of a minute remaining. Who ran the race in the faster time?

Bonus: Express the difference in their times as a fraction of a minute.

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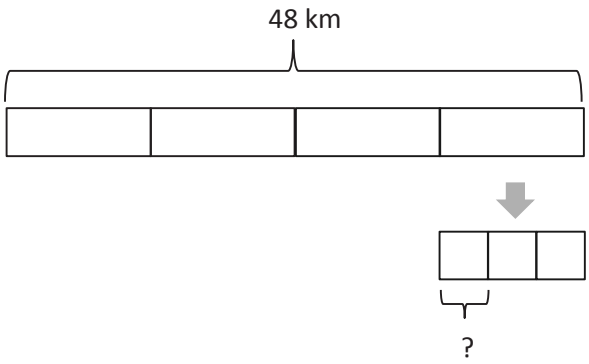
5. Create and solve a story problem about a runner who is training for a race. Include at least one fraction in your story.



6. Create and solve a story problem about two friends and their weekly allowance whose solution is given by the expression $\frac{1}{5} \times (12 + 8)$.

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Name _____ Date _____

1. Multiply or divide. Draw a model to explain your thinking.

a. $\frac{1}{2} \times 6$

b. $\frac{1}{2} \times 7$

c. $\frac{3}{4} \times 12$

d. $\frac{2}{5} \times 30$

e. $\frac{1}{3}$ of 2 feet = _____ inches

f. $\frac{1}{6}$ of 3 yards = _____feet

g. $\left(3 + \frac{1}{2}\right) \times 14$

h. $4\frac{2}{3} \times 13$

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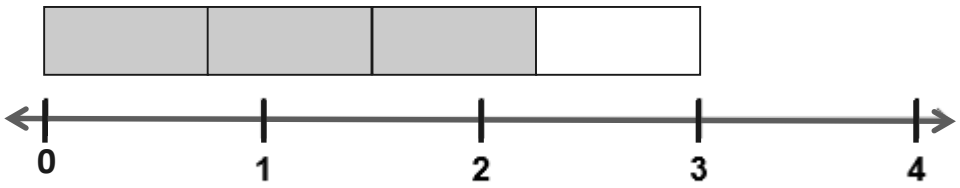
2. If the whole bar is 3 units long, what is the length of the shaded part of the bar? Write a multiplication equation for the diagram, and then solve.



3. Circle the expression(s) that are equal to $\frac{3}{5} \times 6$. Explain why the others are *not* equal using words, pictures, or numbers.

   a. $3 \times (6 \div 5)$

   b. $3 \div (5 \times 6)$

   c. $(3 \times 6) \div 5$

   d. $3 \times \frac{6}{5}$

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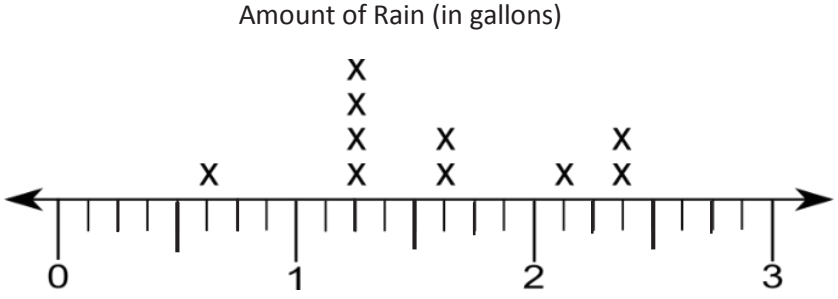
4. Write the following as expressions.

   a. One-third the sum of 6 and 3.

   b. Four times the quotient of 3 and 4.

   c. One-fourth the difference between $\frac{2}{3}$ and $\frac{1}{2}$.

5. Mr. Schaum used 10 buckets to collect rainfall in various locations on his property. The following line plot shows the amount of rain collected in each bucket in gallons. Write an expression that includes multiplication to show how to find the total amount of water collected in gallons. Then, solve your expression.



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6. Mrs. Williams uses the following recipe for crispy rice treats. She decides to make $\frac{2}{3}$ of the recipe.

2 cups melted butter
24 oz marshmallows
13 cups rice crispy cereal

a. How much of each ingredient will she need? Write an expression that includes multiplication. Solve by multiplying.

b. How many fluid ounces of butter will she use? (Use your measurement conversion chart, if you wish.)

c. When the crispy rice treats have cooled, Mrs. Williams cuts them into 30 equal pieces. She gives two-fifths of the treats to her son and takes the rest to school. How many treats will Mrs. Williams take to school? Use any method to solve.

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Mid-Module Assessment Task Standards Addressed — Topics A–D

Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 +7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)*

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

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Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **5.NF.4a** **5.MD.1**	The student draws valid models and/or arrives at the correct product for three or fewer items.	The student draws valid models and/or arrives at the correct product for four or five items.	The student draws valid models and/or arrives at the correct product for six or seven items.	The student correctly answers all eight items and draws valid models: a. 3 b. $3\frac{1}{2}$ c. 9 d. 12 e. 8 inches f. $1\frac{1}{2}$ feet g. 49 h. $60\frac{2}{3}$
2 **5.NF.4a** **5.NF.3**	The student shows no evidence of being able to express the length of the shaded area.	The student approximates the length of the shaded bar but does not write a multiplication equation.	The student is able to write the correct multiplication equation for the diagram but incorrectly states the length of the shaded part of the bar.	The student correctly: ▪ Writes a multiplication equation: $\frac{3}{4} \times 3 = \frac{9}{4}$ or $2\frac{1}{4}$. ▪ Finds the length of the shaded part of the bar as $\frac{9}{4}$ or $2\frac{1}{4}$.

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A Progression Toward Mastery				
3 **5.OA.1**	The student is unable to identify any equal expressions.	The student correctly identifies one equal expression.	The student correctly identifies two equal expressions.	The student correctly: ▪ Identifies (a), (c), and (d) as equal to $\frac{3}{5} \times 6$. ▪ Explains why (b) is not equal.
4 **5.OA.2**	The student is unable to write expressions for (a), (b), or (c).	The student correctly writes one expression.	The student correctly writes two expressions.	The student correctly writes three expressions: a. $\frac{1}{3} \times (6 + 3)$ b. $4 \times (3 \div 4)$ or $4 \times \frac{3}{4}$ c. $\frac{1}{4} \times \left(\frac{2}{3} - \frac{1}{2}\right)$
5 **5.NF.4a** **5.NF.6** **5.MD.2**	The student is neither able to produce a multiplication expression that identifies the data from the line plot nor find the total gallons of water collected.	The student is either able to write a multiplication expression that accounts for all data points on the line plot or find the total gallons of water collected.	The student is able to write a multiplication expression that correctly accounts for all the data points on the line plot when finding the total gallons of water collected but makes a calculation error.	The student correctly: ▪ Accounts for all data points in the line plot in the multiplication expression. ▪ Finds the total gallons of water collected as $15\frac{6}{8}$ gallons or $15\frac{3}{4}$ gallons.
6 **5.NF.4a** **5.NF.6** **5.MD.1**	The student correctly calculates two correct answers.	The student is able to correctly calculate three correct answers.	The student is able to correctly calculate four correct answers.	The student correctly: a. Calculates $1\frac{1}{3}$ c butter, 16 oz of marshmallows, $8\frac{2}{3}$ c of cereal. b. Converts $1\frac{1}{3}$ c butter to $10\frac{2}{3}$ fluid ounces. c. Uses an equation or model and finds the number of treats taken to school as 18 treats.

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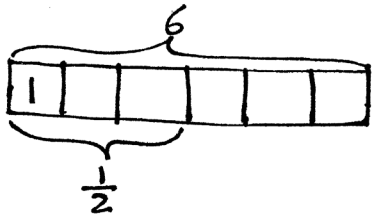
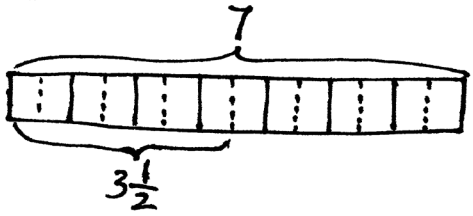
Name Hayley Date _____

1. Multiply or divide. Draw a model to explain your thinking.

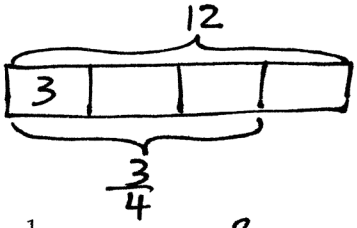
a. $\frac{1}{2} \times 6 = 6 \div 2 = 3$



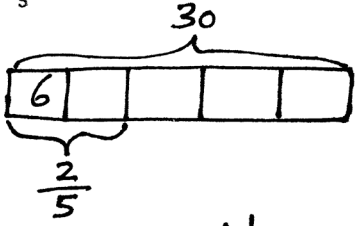
b. $\frac{1}{2} \times 7 = 7 \div 2 = 3\frac{1}{2}$



c. $\frac{3}{4} \times 12 = 9$

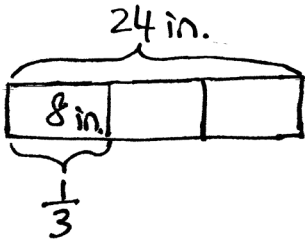


d. $\frac{2}{5} \times 30 = 12$

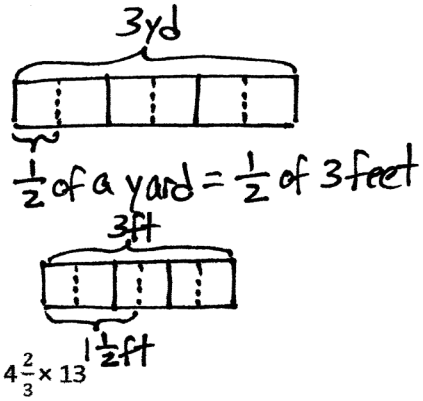


e. $\frac{1}{3}$ of 2 feet = 8 inches

2 x 12 inches = 24 inches



f. $\frac{1}{6}$ of 3 yards = $1\frac{1}{2}$ feet

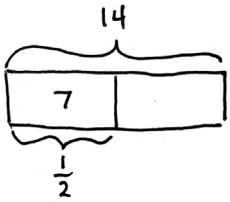


g. $\left(3 + \frac{1}{2}\right) \times 14$

$= (3 \times 14) + (\frac{1}{2} \times 14)$

$= 42 + 7$

$= 49$



h. $4\frac{2}{3} \times 13$

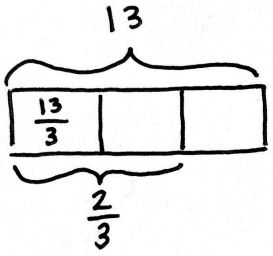
$= (4 \times 13) + (\frac{2}{3} \times 13)$

$= 52 + \frac{2 \times 13}{3}$

$= 52 + \frac{26}{3}$

$= 52 + 8\frac{2}{3}$

$= 60\frac{2}{3}$



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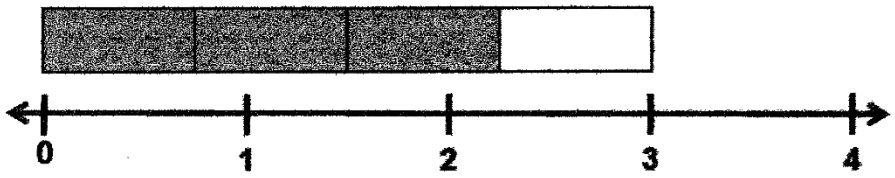
2. If the whole bar is 3 units long, what is the length of the shaded part of the bar? Write a multiplication equation for the diagram, and then solve.



$\frac{3}{4} \times 3 = \frac{3 \times 3}{4} = \frac{9}{4} = 2\frac{1}{4}$

The shaded part of the bar is $2\frac{1}{4}$ units long.

3. Circle the expression(s) that are equal to $\frac{3}{5} \times 6$. Explain why the others are *not* equal using words, pictures, or numbers.

a. $3 \times (6 \div 5)$

b. $3 \div (5 \times 6)$

$= 3 \div 30$

$= 0.1$

c. $(3 \times 6) \div 5$

d. $3 \times \frac{6}{5}$

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4. Write the following as expressions.
   a. One-third the sum of 6 and 3.

$\frac{1}{3} \times (6+3)$

   b. Four times the quotient of 3 and 4.

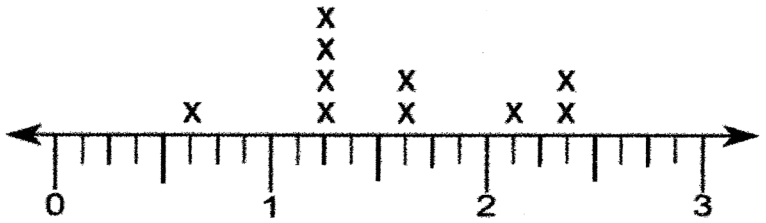
$4 \times (3 \div 4)$

   c. One-fourth the difference between $\frac{2}{3}$ and $\frac{1}{2}$.

$\frac{1}{4} \times (\frac{2}{3} - \frac{1}{2})$

5. Mr. Schaum used 10 buckets to collect rainfall in various locations on his property. The following line plot shows the amount of rain collected in each bucket in gallons. Write an expression that includes multiplication to show how to find the total amount of water collected in gallons. Then solve your expression.

Amount of Rain (in gallons)



$\frac{5}{8} + (4 \times 1\frac{2}{8}) + (2 \times 1\frac{5}{8}) + 2\frac{1}{8} + (2 \times 2\frac{3}{8})$

$= \frac{5}{8} + 4 + \frac{4 \times 2}{8} + 2 + \frac{2 \times 5}{8} + 2\frac{1}{8} + 4 + \frac{2 \times 3}{8}$

$= 12 + \frac{5}{8} + \frac{8}{8} + \frac{10}{8} + \frac{1}{8} + \frac{6}{8}$

$= 13 + \frac{22}{8}$

$= 15\frac{6}{8} = 15\frac{3}{4}$

Mr. Schaum collected $15\frac{3}{4}$ gallons of water.

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6. Mrs. Williams uses the following recipe for crispy rice treats. She decides to make $\frac{2}{3}$ of the recipe.

2 cups melted butter
24 oz marshmallows
13 cups rice crispy cereal

a. How much of each ingredient will she need? Write an expression that includes multiplication. Solve by multiplying.

Butter: $\frac{2}{3} \times 2 \text{ cups} = \frac{2 \times 2}{3} = \frac{4}{3} = 1\frac{1}{3}$ cups

Marshmallows: $\frac{2}{3} \times 24 \text{ oz.} = \frac{2 \times 24}{3} = \frac{48}{3} = 16$ oz.

Cereal: $\frac{2}{3} \times 13 \text{ cups} = \frac{2 \times 13}{3} = \frac{26}{3} = 8\frac{2}{3}$ cups

She will need $1\frac{1}{3}$ cups of butter, 16 ounces of marshmallows, and $8\frac{2}{3}$ cups of rice crispy cereal.

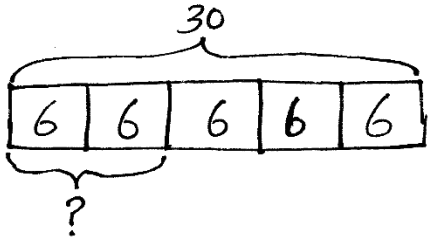
b. How many fluid ounces of butter will she use? (Use your measurement conversion chart if you wish.)

1 cup = 8 ounces

$1\frac{1}{3} \times 8 = (1 \times 8) + (\frac{1}{3} \times 8)$
$= 8 + \frac{8}{3}$
$= 8 + 2\frac{2}{3}$
$= 10\frac{2}{3}$

She will use $10\frac{2}{3}$ fluid ounces of butter.

c. When the crispy rice treats have cooled, Mrs. Williams cuts them into 30 equal pieces. She gives two-fifths of the treats to her son and takes the rest to school. How many treats will Mrs. Williams take to school? Use any method to solve.



$$\begin{array}{r} 30 \\ -\ 12 \\ \hline 18 \end{array}$$

Mrs. Williams will take 18 treats to school.

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Topic E

Multiplication of a Fraction by a Fraction

5.NBT.7, 5.NF.4a, 5.NF.6, 5.MD.1, 5.NF.4b

Focus Standards:	5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
	5.NF.4a	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product of $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*
	5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
	5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Instructional Days:	8	
Coherence -Links from:	G4–M6	Decimal Fractions
	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions
	G6–M4	Expressions and Equations

Topic E introduces students to multiplication of fractions by fractions—both in fraction and decimal form (**5.NF.4a**, **5.NBT.7**). The topic starts with multiplying a unit fraction by a unit fraction and progresses to multiplying two non-unit fractions. Students use area models, rectangular arrays, and tape diagrams to model the multiplication. These familiar models help students draw parallels between whole number and fraction multiplication, as well as solve word problems. This intensive work with fractions positions students to extend their previous work with decimal-by-whole number multiplication to decimal-by-decimal multiplication. Just as students used unit form to multiply fractional units by wholes in Module 2 (e.g., 3.5 × 2 = 35 tenths × 2 ones = 70 tenths), they connect fraction-by-fraction multiplication to multiply fractional units-by-fractional units (3.5 × 0.2 = 35 tenths × 2 tenths = 70 hundredths).

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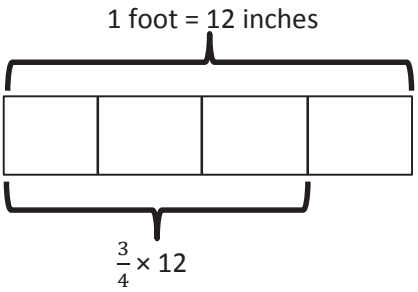
$\frac{3}{4}$ of a foot = $\frac{3}{4}$ × (1 foot)

= $\frac{3}{4}$ × (12 inches)

= 9 inches



Express $5\frac{3}{4}$ feet as inches.

$5\frac{3}{4}$ feet = (5 × 12) inches + ($\frac{3}{4}$ × 12) inches

= 60 + 9 inches

= 69 inches

Reasoning about decimal placement is an integral part of these lessons. Finding fractional parts of customary measurements and measurement conversion (**5.MD.1**) concludes Topic E. Students convert smaller units to fractions of a larger unit (e.g., 6 inches = $\frac{1}{2}$ foot). The inclusion of customary units provides a meaningful context for many common fractions (e.g., $\frac{1}{2}$ pint = 1 cup, $\frac{1}{3}$ yard = 1 foot, $\frac{1}{4}$ gallon = 1 quart). This topic, together with the fraction concepts and skills learned in Module 3, opens the door to a wide variety of application word problems (**5.NF.6**).

A Teaching Sequence Toward Mastery of Multiplication of a Fraction by a Fraction

Objective 1: Multiply unit fractions by unit fractions. (Lesson 13)

Objective 2: Multiply unit fractions by non-unit fractions. (Lesson 14)

Objective 3: Multiply non-unit fractions by non-unit fractions. (Lesson 15)

Objective 4: Solve word problems using tape diagrams and fraction-by-fraction multiplication. (Lesson 16)

Objective 5: Relate decimal and fraction multiplication. (Lessons 17–18)

Objective 6: Convert measures involving whole numbers, and solve multi-step word problems. (Lesson 19)

Objective 7: Convert mixed unit measurements, and solve multi-step word problems. (Lesson 20)

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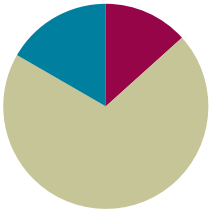
Lesson 13

Objective: Multiply unit fractions by unit fractions.

Suggested Lesson Structure



Fluency Practice	(8 minutes)
Concept Development	(42 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (8 minutes)

- Multiply a Fraction and a Whole Number **5.NF.3** (4 minutes)
- Convert Measures **4.MD.1** (4 minutes)

Multiply a Fraction and a Whole Number (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 9–11.

T: (Write $\frac{8}{4}$.) Say the division sentence with the answer.

S: $8 \div 4 = 2$.

T: (Write $\frac{1}{4} \times 8 =$ ___.) Say the multiplication sentence with the answer.

S: $\frac{1}{4} \times 8 = 2$.

T: (Write $\frac{3}{4} \times 8 =$ ___.) On your personal white board, write the multiplication sentence and solve.

S: (Write $\frac{3}{4} \times 8 = 6$.)

T: (Write $8 \times \frac{3}{4} =$ ___.) Try this problem.

S: (Write $8 \times \frac{3}{4} = 6$.)

Continue with the following possible sequence: $\frac{18}{6}$, $\frac{1}{6} \times 18$, $\frac{5}{6} \times 18$, $18 \times \frac{5}{6}$, $\frac{1}{8} \times 16$, $16 \times \frac{1}{8}$, $16 \times \frac{5}{8}$, $\frac{2}{3} \times 15$, and $20 \times \frac{3}{4}$.

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Convert Measures (4 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity reviews Lesson 12 and prepares students for the present lesson. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

Convert the following. Draw a tape diagram, if necessary.

a. $\frac{1}{3}$ yd = _____ ft = _____ in

b. $\frac{2}{3}$ yd = _____ ft = _____ in

c. $\frac{1}{3}$ hr = _____ min

d. $\frac{2}{3}$ hr = _____ min

e. $\frac{1}{4}$ yr = _____ mos

f. $\frac{3}{4}$ yr = _____ mos

a. $\frac{1}{3}$ yd = 1 ft = 12 in

b. $\frac{2}{3}$ yd = 2 ft = 24 in

c. $\frac{1}{3}$ hr = 20 min

d. $\frac{2}{3}$ hr = 40 min

e. $\frac{1}{4}$ yr = 3 mos

f. $\frac{3}{4}$ yr = 9 mos

Concept Development (42 minutes)

Materials: (S) Personal white board, 4" × 2" rectangular paper (several pieces per student), scissors

Note: Today's lesson is lengthy, so the time normally allotted for an Application Problem has been allocated to the Concept Development. The last problem in the sequence can be considered the Application Problem for today.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

While the lesson moves to the pictorial level of representation fairly quickly, be aware that many students may need the scaffold of the concrete model (paper folding and shading) to fully comprehend the concepts. Make these materials available, and model their use throughout the remainder of the module.

Problem 1

Jan has 4 pans of crispy rice treats. She sends $\frac{1}{2}$ of the pans to school with her children. How many pans of crispy rice treats does Jan send to school?

Note: To progress from finding a fraction of a whole number to a fraction of a fraction, the following sequence is then used: 2 pans, 1 pan, $\frac{1}{2}$ pan.

T: (Post Problem 1 on the board, and read it aloud with students.) Work with your partner to write a multiplication sentence that explains your thinking. Be prepared to share. (Allow students time to work.)

T: What fraction of the pans does Jan send to school?

S: One-half of them.

$\frac{1}{2}$ of 4 pans = 2 pans

$\frac{1}{2}$ of 2 pans = 1 pan

$\frac{1}{2}$ of 1 pan = $\frac{1}{2}$ pan

$\frac{1}{2}$ of $\frac{1}{2}$ pan = $\frac{1}{4}$ pan

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T: How many pans of crispy rice treats did Jan have at first?

S: 4 pans.

T: What is one-half of 4 pans?

S: 2 pans.

T: Show the multiplication sentence that you wrote to explain your thinking.

S: (Show $\frac{1}{2} \times 4$ pans = 2 pans or $4 \times \frac{1}{2} = 2$ pans.)

T: Say the answer in a complete sentence.

S: Jan sent 2 pans of crispy rice treats to school.

T: (Erase the 4 in the text of the problem, and replace it with a 2.) Imagine that Jan has 2 pans of treats. If she still sends half of the pans to school, how many pans will she send? Write a multiplication sentence to show how you know.

S: (Write $\frac{1}{2} \times 2$ pans = 1 pan.)

T: (Replace the 2 in the problem with a 1.) Now, imagine that she only has 1 pan. If she still sends half to school, how many pans will she send? Write the multiplication sentence.

S: (Write $\frac{1}{2} \times 1$ pan = $\frac{1}{2}$ pan.)

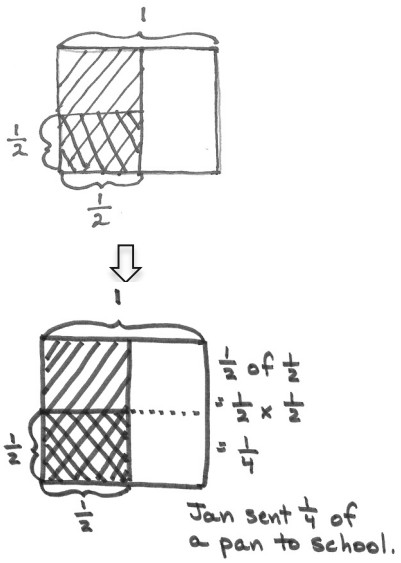
T: (Erase the 1 in the problem, and replace it with $\frac{1}{2}$. Read the problem aloud with students.) What if Jan only has half a pan and wants to send half of it to school? What is different about this problem?

S: There's only $\frac{1}{2}$ of a pan instead of a whole pan. → Jan is still sending half the treats to school, but now we'll find half of a half, not half of 1. → The amount we have is less than a whole.

T: Let's say that your piece of paper represents the pan of treats. Turn and talk to your partner about how you can use your rectangular paper to find out what fraction of the whole pan of treats Jan sent to school.

S: (May fold or shade the paper to show the problem.)

T: Many of you shaded half of your paper and then partitioned that half into 2 equal parts and shaded one of them, like so. (Model as seen to the right.)



MP.4 T: We now have two different size units shaded in our model. I can see the part that Jan sent to school, but I need to name this unit. To name the part she sent (point to the double-shaded unit), all of the units in the whole must be the same size as this one. Turn and talk to your partner about how we can split the rest of the pan so that all the units are the same as our double-shaded one. Use your paper to show your thinking.

S: We could cut the other half in half, too. That would make 4 units of the same size. → We could keep cutting across the rest of the whole. That would make the whole pan cut into 4 equal parts. → Half of a half is a fourth.

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T: Let me record that. (Partition the unshaded half using a dotted line.) Look at our model. What's the name for the smallest units we have drawn now?

S: Fourths.

T: She sent half of the treats she had, but what fraction *of the whole pan* of treats did Jan send to school?

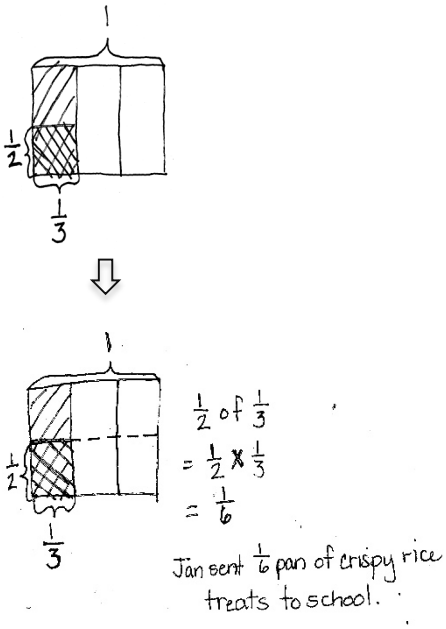
S: One-fourth of the whole pan.

T: Write a multiplication sentence that shows your thinking.

S: (Write $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.)

Problem 2

Jan has $\frac{1}{3}$ pan of crispy rice treats. She sends $\frac{1}{2}$ of the treats to school with her children. What fraction of a pan of crispy rice treats does Jan send to school?



T: (Erase $\frac{1}{2}$ in the text of Problem 1, and replace it with $\frac{1}{3}$.) Imagine that Jan only has a third of a pan, and she still wants to send half of the treats to school. Will she be sending a greater amount or a smaller amount of treats to school than she sent in our last problem? How do you know? Turn and discuss with your partner.

S: It will be a smaller part of a whole pan because she had half a pan before. Now, she only has 1 third of a pan. → 1 third is less than 1 half, so half of a third is less than half of a half. → 1 half is larger than 1 third, so she sent more in the last problem than this one.

T: We need to find $\frac{1}{2}$ of $\frac{1}{3}$ pan. (Write $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2} \times \frac{1}{3}$ on the board.) I'll draw a rectangular fraction model to represent this problem while you use your paper to model it. (Draw a rectangle on the board.) This rectangle shows 1 whole pan. (Label 1 above the rectangle.) Fold your paper, and then shade it to show how much of this one pan Jan has at first.

S: (Fold in thirds, and shade 1 third of the whole.)

T: (On the board, partition the rectangle vertically into 3 parts, shade in 1 of them, and label $\frac{1}{3}$ below it.) What fraction of the treats did Jan send to school?

S: One-half.

T: Jan sends $\frac{1}{2}$ of this part to school. (Point to 1 shaded portion.) How can I show $\frac{1}{2}$ of this part? Turn and talk to your partner, and show your thinking with your paper.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

There are students who notice the patterns within the algorithm quickly and want to use it to find the product. Be sure those students are questioned deeply and can articulate the reasoning and meaning of the product in relationship to the whole.

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S: We can draw a line to cut it in half. → We need to split it into 2 equal parts and shade only 1 of them.

T: I hear you saying that I should partition the one-third into 2 equal parts and then shade only 1. (Draw a horizontal line through the shaded third, and shade the bottom half again to create a double shade. Label the double-shaded area as $\frac{1}{2}$.)

T: Again, now I have two different size shaded units. What do I need to do with this horizontal line to be able to name the units? Turn and talk.

S: We could cut the other thirds in half, too. That would make 6 units the same size. → We could keep cutting across the rest of the whole. That would make the whole pan cut into 6 equal parts. → 1 third is the same as 2 sixths. Half of 2 sixths is 1 sixth.

T: Let me record that. (Partition the unshaded thirds using a dotted line.) What's the name for the units we have drawn now?

S: Sixths.

T: What fraction of the pan of treats did Jan send to school?

S: One-sixth of the whole pan.

T: One-half of one-third is one-sixth. (Write $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.)

Repeat a similar sequence with Problem 3, but have students draw a rectangular fraction model on their papers rather than folding their papers. Be sure that students articulate clearly the finding of a common unit, which lets them name the product.

Problem 3

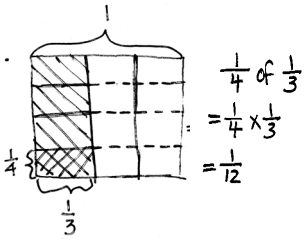
Jan has $\frac{1}{3}$ a pan of crispy rice treats. She sends $\frac{1}{4}$ of the treats to school with her children. What fraction of a pan of crispy rice treats does Jan send to school?

T: (Write $\frac{1}{4}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{4}$ on the board.) Let's compare finding 1 fourth of 1 third with finding 1 third of 1 fourth. What do you notice about these problems? Turn and talk.

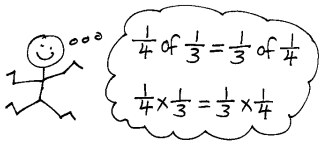
S: They both have 1 fourth and 1 third in them, but they're flip-flopped. → They have the same factors, but they are in a different order.

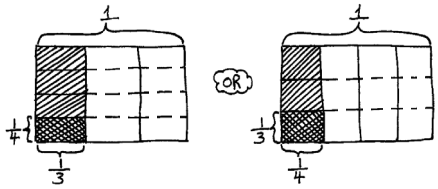
T: Will the order of the factors affect the size of the product? Talk to your partner.

S: No, it doesn't when we multiply whole numbers. → But is that true for fractions, too? → That means 1 fourth of 1 third is the same as a third of a fourth.



Jan sent $\frac{1}{12}$ pan of crispy rice treats to school.





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T: We just drew the rectangular fraction model for $\frac{1}{4}$ of $\frac{1}{3}$. Let's now draw it for $\frac{1}{3}$ of $\frac{1}{4}$ to find out if we will have the same answer. In $\frac{1}{3}$ of $\frac{1}{4}$, the amount we start with is 1 fourth of the pan. Draw a rectangle, shade $\frac{1}{4}$, and label it. (Draw a rectangular box, and cut it vertically into 4 equal parts, and label $\frac{1}{4}$. Point to the shaded part.) How do I take a third of this fourth?

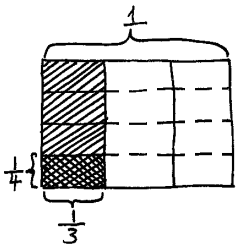
S: Cut the fourth into 3 parts.

T: How will we name this new unit?

S: To name this new unit, we need to cut the other fourths into 3 equal parts, too.

T: (Partition each unit into thirds, double-shade the portion showing $\frac{1}{3}$ of $\frac{1}{4}$, and label $\frac{1}{3}$.) How many of these units make our whole?

S: Twelve.



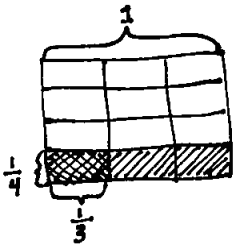
T: What is their name?

S: Twelfths.

T: What's $\frac{1}{3}$ of $\frac{1}{4}$?

S: 1 twelfth.

T: (Write $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.) These multiplication sentences have the same answer, but the shape of the twelfth is different. How do you know that 12 equal parts can be different shapes, yet the same fraction?



S: What matters is that they are 12 equal parts of the same whole. → The same applies to a square; there are a lot of ways to show a half, or 2 equal parts. The area has to be the same, not the shape.

T: True. What matters is that the parts have the same area. We can prove $\frac{1}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{3}$ with another drawing. Start with the same pan. Draw fourths horizontally, and shade 1 fourth. Now, let's double-shade 1 third of that fourth (extend the units with dotted lines). Is the exact same amount shaded in the two pans?

S: Yes!

T: So, we see in another way that $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{3}$ of $\frac{1}{4}$. Review how to prove that with our rectangles. Turn and talk.

S: We shade a fourth of a third, drawing the thirds vertically first. Then, we shaded a third of a fourth, drawing the fourths horizontally first. They were exactly the same part of the whole. → I can shade a fourth and then take a third of it, or I can shade a third and then take a fourth of it. I get $\frac{1}{12}$ either way.

T: What do we know about multiplication that supports the truth of the number sentence $\frac{1}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{3}$?

S: The commutative property works with fractions the same as whole numbers. The order of the factors doesn't change the product. → Taking a fourth of a third is like taking a smaller part of a larger unit, while taking a third of a fourth is like taking a larger part of a smaller unit. Either way, you're finding the same size share.

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T: We can express $\frac{1}{4}$ of $\frac{1}{3}$ as $\frac{1}{4}\times\frac{1}{3}$ or $\frac{1}{3}\times\frac{1}{4}$. (Write $\frac{1}{4}\times\frac{1}{3}=\frac{1}{3}\times\frac{1}{4}$.) They are equivalent expressions.

Problem 4

A sales lot is filled with vehicles for sale. $\frac{1}{3}$ of the vehicles are pickup trucks. $\frac{1}{3}$ of the trucks are white. What fraction of all the vehicles are white pickup trucks?

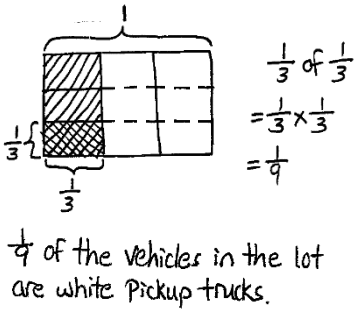
T: (Post Problem 4 on the board, and read it aloud with students.) Work with your partner to draw a rectangular fraction model and solve. Write a multiplication sentence to show your thinking. (Allow students time to work.)

T: What is a third of one-third?

S: $\frac{1}{9}$.

T: Say the answer to the question in a complete sentence.

S: One-ninth of the vehicles in the lot are white pickup trucks.



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

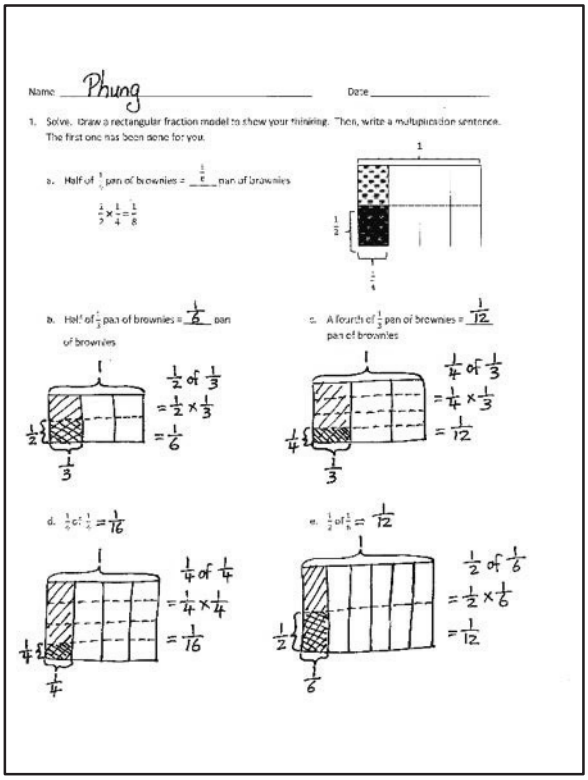
Lesson Objective: Multiply unit fractions by unit fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what is the relationship between Parts (a) and (d)? (Part (a) is double Part (d).) Between Parts (b) and (c)? (Part (b) is double Part (c).) Between Parts (b) and (e)? (Part (b) is double Part (e).)



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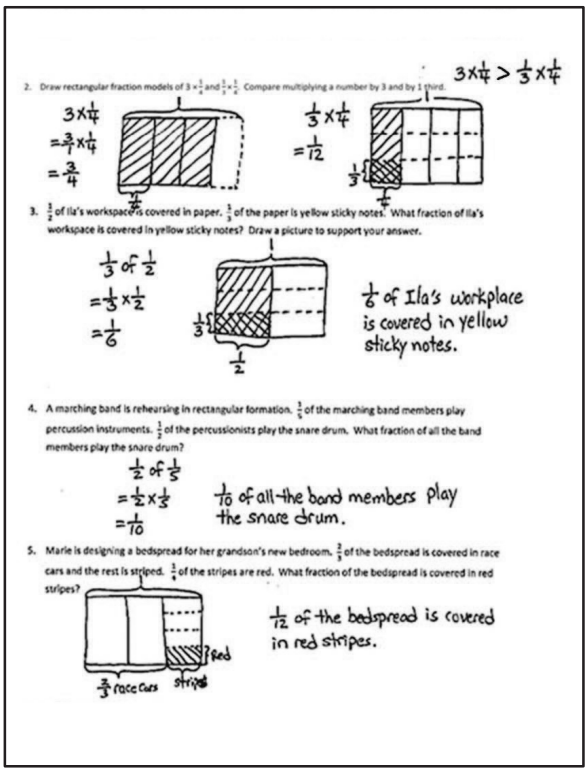
- Why is the product for Problem 1(d) smaller than 1(c)? Explain your reasoning to your partner.
- Share and compare your solution with a partner for Problem 2.
- Compare and contrast Problem 3 and Problem 1(b). Discuss with your partner.
- How is solving for the product of a fraction and a whole number the same as or different from solving a fraction of a fraction? Can you use some of the similar strategies? Explain your thinking to a partner.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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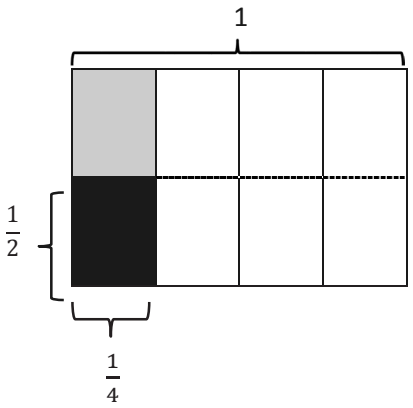
Name _____ Date _____

1. Solve. Draw a rectangular fraction model to show your thinking. Then, write a multiplication sentence. The first one has been done for you.

a. Half of $\frac{1}{4}$ pan of brownies = $\frac{1}{8}$ pan of brownies.

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$



b. Half of $\frac{1}{3}$ pan of brownies = _____ pan of brownies.

c. A fourth of $\frac{1}{3}$ pan of brownies = _____ pan of brownies.

d. $\frac{1}{4}$ of $\frac{1}{4}$

e. $\frac{1}{2}$ of $\frac{1}{6}$

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2. Draw rectangular fraction models of $3 \times \frac{1}{4}$ and $\frac{1}{3} \times \frac{1}{4}$. Compare multiplying a number by 3 and by 1 third.

3. $\frac{1}{2}$ of Ila's workspace is covered in paper. $\frac{1}{3}$ of the paper is covered in yellow sticky notes. What fraction of Ila's workspace is covered in yellow sticky notes? Draw a picture to support your answer.

4. A marching band is rehearsing in rectangular formation. $\frac{1}{5}$ of the marching band members play percussion instruments. $\frac{1}{2}$ of the percussionists play the snare drum. What fraction of all the band members play the snare drum?

5. Marie is designing a bedspread for her grandson's new bedroom. $\frac{2}{3}$ of the bedspread is covered in race cars, and the rest is striped. $\frac{1}{4}$ of the stripes are red. What fraction of the bedspread is covered in red stripes?

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model, and write a number sentence to show your thinking.

$\frac{1}{3} \times \frac{1}{3} =$

2. Ms. Sheppard cuts $\frac{1}{2}$ of a piece of construction paper. She uses $\frac{1}{6}$ of the piece to make a flower. What fraction of the sheet of paper does she use to make the flower?

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model to show your thinking.

a. Half of $\frac{1}{2}$ cake = _____ cake.

b. One-third of $\frac{1}{2}$ cake = _____ cake.

c. $\frac{1}{4}$ of $\frac{1}{2}$

d. $\frac{1}{2} \times \frac{1}{5}$

e. $\frac{1}{3} \times \frac{1}{3}$

f. $\frac{1}{4} \times \frac{1}{3}$

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2. Noah mows $\frac{1}{2}$ of his property and leaves the rest wild. He decides to use $\frac{1}{5}$ of the wild area for a vegetable garden. What fraction of the property is used for the garden? Draw a picture to support your answer.

3. Fawn plants $\frac{2}{3}$ of the garden with vegetables. Her son plants the remainder of the garden. He decides to use $\frac{1}{2}$ of his space to plant flowers, and in the rest, he plants herbs. What fraction of the entire garden is planted in flowers? Draw a picture to support your answer.

4. Diego eats $\frac{1}{5}$ of a loaf of bread each day. On Tuesday, Diego eats $\frac{1}{4}$ of the day's portion before lunch. What fraction of the whole loaf does Diego eat before lunch on Tuesday? Draw a rectangular fraction model to support your thinking.

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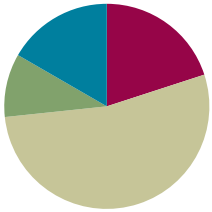
Lesson 14

Objective: Multiply unit fractions by non-unit fractions.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Sprint: Multiply a Fraction and a Whole Number **5.NF.3** (8 minutes)
- Fractions as Whole Numbers **5.NF.4** (4 minutes)

Sprint: Multiply a Fraction and a Whole Number (8 minutes)

Materials: (S) Multiply a Fraction and a Whole Number Sprint

Note: This Sprint reviews Lessons 9–12 content.

Fractions as Whole Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 5, as well as denominators that are easily converted to hundredths. Instruct students to use their personal white boards for calculations they cannot do mentally.

T: I'll say a fraction. You say it as a division equation, and give the quotient. 4 halves.

S: $4 \div 2 = 2$.

Continue with the following possible sequence: $\frac{6}{2}$, $\frac{14}{2}$, $\frac{54}{2}$, $\frac{40}{20}$, $\frac{80}{20}$, $\frac{180}{20}$, $\frac{960}{20}$, $\frac{10}{5}$, $\frac{15}{5}$, $\frac{35}{5}$, $\frac{85}{5}$, $\frac{100}{50}$, $\frac{150}{50}$, $\frac{300}{50}$, $\frac{900}{50}$, $\frac{8}{4}$, $\frac{12}{4}$, $\frac{24}{4}$, $\frac{96}{4}$, $\frac{50}{25}$, $\frac{75}{25}$, $\frac{800}{25}$.

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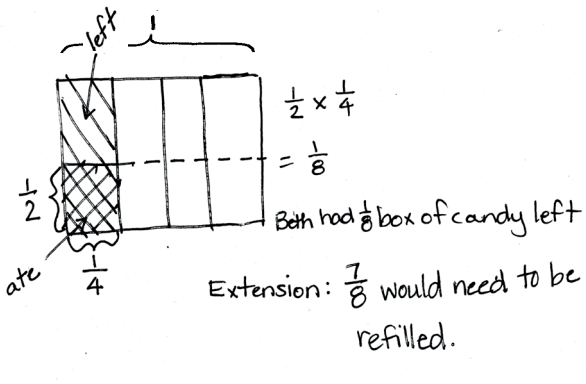
Application Problem (6 minutes)

Solve by drawing a rectangular fraction model and writing a multiplication sentence.

Beth had $\frac{1}{4}$ box of candy. She ate $\frac{1}{2}$ of the candy. What fraction of the whole box does she have left?

Extension: If Beth decides to refill the box, what fraction of the box would need to be refilled?

Note: This Application Problem stimulates prior knowledge of the multiplication of unit fractions by unit fractions in preparation for today's lesson.

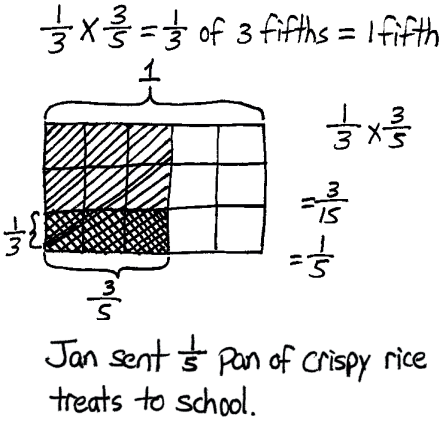


Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1

Jan had $\frac{3}{5}$ pan of crispy rice treats. She sent $\frac{1}{3}$ of the treats to school. What fraction of the whole pan did she send to school?



T: (Write Problem 1 on the board.) How is this problem different from the ones we solved yesterday? Turn and talk.

S: Yesterday, Jan always had 1 unit fraction of treats. She had 1 half or 1 third or 1 fourth. Today, she has 3 fifths. This one has a 3 in one of the numerators. → We only multiplied unit fractions yesterday.

T: In this problem, what are we finding $\frac{1}{3}$ of?

S: 3 fifths of a pan of treats.

T: Before we find $\frac{1}{3}$ of Jan's $\frac{3}{5}$, visualize this. If there are 3 bananas, how many would $\frac{1}{3}$ of the bananas be? Turn and talk.

S: Well, if you have 3 bananas, one-third of that is just 1 banana. → One-third of 3 of any unit is just one of those units. → 1 third of 3 is always 1. It doesn't matter what the unit is.

T: What is $\frac{1}{3}$ of 3 *pens*?

S: 1 pen.

T: What is $\frac{1}{3}$ of 3 *books*?

S: 1 book.

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T: (Write = $\frac{1}{3}$ of 3 fifths.) So then, what is $\frac{1}{3}$ of 3 *fifths*?

S: 1 fifth.

T: (Write = 1 fifth.) $\frac{1}{3}$ of 3 fifths equals 1 fifth. Let's draw a rectangular fraction model to prove your thinking. Draw a rectangular fraction model showing $\frac{3}{5}$.

S/T: (Draw, shade, and label the rectangular fraction model.)

T: If we want to show $\frac{1}{3}$ of $\frac{3}{5}$, what must we do to each of these 3 units? (Point to each of the shaded fifths.)

S: Split each one into thirds.

T: Yes, partition each of these units, these fifths, into 3 equal parts.

S/T: (Partition, shade, and label the rectangular fraction model.)

T: To name these parts, what must we do to the rest of the whole?

S: Partition the other fifths into 3 equal parts as well.

T: Show that using dotted lines. What new unit have we created?

S: Fifteenths.

T: How many fifteenths are in the whole?

S: 15.

T: How many fifteenths are double-shaded?

S: 3.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Consider allowing learners who grasp these multiplication concepts quickly to draw models and create story problems to accompany them. If there are technology resources available, allow these students to produce screencasts explaining fraction-by-fraction multiplication for absent or struggling classmates.

T: (Write $\frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$ next to the rectangular fraction model.) I thought we said that our answer was 1 fifth. So, how is it that our model shows 3 fifteenths? Turn and talk.

S: 3 fifteenths is another way to show 1 fifth. → I can see 5 equal groups in this model. They each have 3 fifteenths in them. Only 1 of those 5 is double-shaded, so it's really only 1 fifth shaded here, too. → The answer is 1 fifth. It's just chopped into fifteenths in the model.

T: Let's explore that a bit. Looking at your model, how many groups of 3 fifteenths do you see? Turn and talk.

MP.2

S: There are 5 groups of 3 fifteenths in the whole. → I see 1 group that's double-shaded. I see 2 more groups that are single-shaded, and then there are 2 groups that aren't shaded at all. That makes 5 groups of 3 fifteenths.

T: Out of the 5 groups that we see, how many are double-shaded?

S: 1 group.

T: 1 out of 5 groups is what fraction?

S: 1 fifth.

T: Does our rectangular fraction model support our thinking from before?

S: Yes. $\frac{1}{3}$ of 3 fifths equals 1 fifth.

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Problem 2

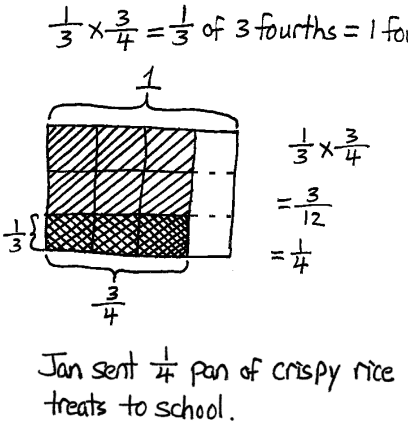
Jan had $\frac{3}{4}$ pan of crispy rice treats. She sent $\frac{1}{3}$ of the treats to school. What fraction of the whole pan did she send to school?

T: What are we finding $\frac{1}{3}$ of this time?

S: $\frac{1}{3}$ of 3 fourths.

T: (Write = $\frac{1}{3}$ of 3 fourths.) Based on what we learned in the previous problem, what do you think the answer will be for $\frac{1}{3}$ of 3 fourths? Whisper and tell a partner.

S: Just like $\frac{1}{3}$ of 3 apples is equal to 1 apple, and $\frac{1}{3}$ of 3 fifths is equal to 1 fifth. We know that $\frac{1}{3}$ of 3 fourths is equal to 1 fourth. → We are taking 1 third of 3 units again. The units are fourths this time, so the answer is 1 fourth.



T: Work with a neighbor to solve one-third of 3 fourths. One of you can draw the rectangular fraction model, while the other writes a matching number sentence.

S: (Work and share.)

T: In your area model, when you partitioned each of the fourths into 3 equal parts, what new unit did you create?

S: Twelfths.

T: How many twelfths represent 1 third of 3 fourths?

S: 3 twelfths.

T: Say 3 twelfths in its simplest form.

S: 1 fourth.

T: So, $\frac{1}{3}$ of 3 fourths is equal to what?

S: 1 fourth.

T: Look back at the two problems we just solved. If $\frac{1}{3}$ of 3 fifths is 1 fifth and $\frac{1}{3}$ of 3 fourths is 1 fourth, what then is $\frac{1}{3}$ of 3 eighths?

S: 1 eighth.

T: $\frac{1}{3}$ of 3 tenths?

S: 1 tenth.

T: $\frac{1}{3}$ of 3 hundredths?

S: 1 hundredth.

T: Based on what you've just learned, what is $\frac{1}{4}$ of 4 fifths?

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S: 1 fifth.

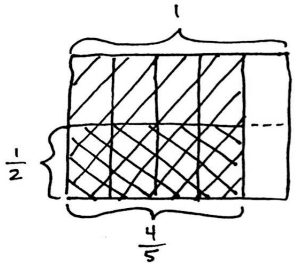
T: $\frac{1}{2}$ of 2 fifths?

S: 1 fifth.

T: $\frac{1}{4}$ of 4 sevenths?

S: 1 seventh.

Problem 3: $\frac{1}{2} \times \frac{4}{5}$



$\frac{1}{2}$ of 4 fifths = 2 fifths

$\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$

$= \frac{2}{5}$

T: We need to find 1 half of 4 fifths. If this were 1 half of 4 bananas, how many bananas would we have?

S: 2 bananas.

T: How can you use this thinking to help you find 1 half of 4 fifths? Turn and talk.

S: It's half of 4, so it must be 2. This time, it's 4 fifths, so half would be 2 fifths. → Half of 4 is always 2. It doesn't matter that it is fifths. The answer is 2 fifths.

T: It sounds like we agree that 1 half of 4 fifths is 2 fifths. Let's draw a rectangular fraction model to confirm our thinking. Work with your partner.

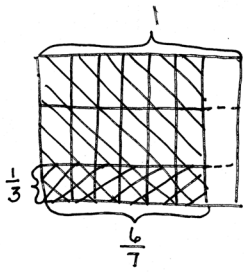
S: (Draw.)

T: I notice that our rectangular fraction model shows that the product is 4 tenths, but we said a moment ago that our product was 2 fifths. Did we make a mistake? Why or why not?

S: No. 4 tenths is just another name for 2 fifths. → I can see 5 groups of 2 tenths, but only 2 of them are double-shaded. Two out of 5 groups is another way to say 2 fifths.

Repeat this sequence with $\frac{1}{3} \times \frac{6}{7}$.

T: What patterns do you notice in our multiplication sentences? Turn and talk.



$\frac{1}{3}$ of 6 sevenths = 2 sevenths

$\frac{1}{3} \times \frac{6}{7} = \frac{6}{21}$

$= \frac{2}{7}$

S: I notice that the denominator in the product is the product of the two denominators in the factors before we simplified. → I notice that you can just multiply the numerators and then multiply the denominators to get the numerator and denominator in the final answer. → When you split the amount in the second factor into thirds, it's like tripling the units, so it's just like multiplying the first unit by 3. But the units become smaller, so you have the same amount that you started with.

T: As we are modeling the rest of our problems, let's see if this pattern continues.

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Problem 4

$\frac{3}{4}$ of Benjamin's garden is planted in vegetables. Carrots are planted in $\frac{1}{2}$ of his vegetable section of the garden. How much of Benjamin's garden is planted in carrots?

T: Write a multiplication expression to represent the amount of his garden planted in carrots.

S: $\frac{1}{2} \times \frac{3}{4}$. → $\frac{1}{2}$ of $\frac{3}{4}$.

T: I'll write this in unit form. (Write $\frac{1}{2}$ of 3 fourths on the board.) Compare this problem to the previous ones. Turn and talk.

S: This one seems trickier because all the others were easy to halve. They were all even numbers of units. → This is half of 3. I know that's 1 and 1 half, but the unit is fourths, and I don't know how to say $1\frac{1}{2}$ fourths.

T: Could we name 3 fourths of Benjamin's garden using another unit that makes it easier to halve? Turn and talk with your partner, and then write the amount in unit form.

S: We need a unit that lets us name 3 fourths with an even number of units. We could use 6 eighths. → 6 eighths is the same amount as 3 fourths, and 6 is a multiple of 2.

T: What is 1 half of 6?

S: 3.

T: So, what is 1 half of 6 eighths?

S: 3 eighths.

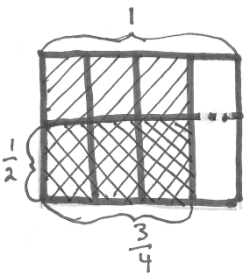
T: Let's draw our rectangular fraction model to confirm our thinking. (Allow students time to draw.)

T: Looking at our model, what was the new unit that we used to name the parts of the garden?

S: Eighths.

T: How much of Benjamin's garden is planted in carrots?

S: 3 eighths.



$\frac{1}{2}$ of 3 fourths = ? fourths

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

$\frac{3}{8}$ of Benjamin's garden is planted in carrots.

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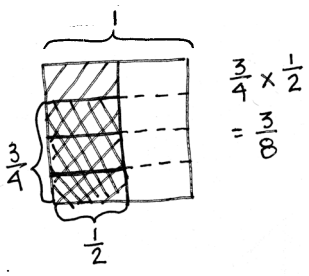
Problem 5: $\frac{3}{4}$ of $\frac{1}{2}$

T: (Post Problem 5 on the board.) Solve this by drawing a rectangular fraction model and writing a multiplication sentence. (Allow students time to work.)



T: Compare this model to the one we drew for Benjamin's garden. Turn and talk.

S: It's similar. The fractions are the same, but when you draw this one, you have to start with 1 half and then chop that into fourths. → The model for this problem looks like what we drew for Benjamin's garden, except it's been turned on its side. → When we wrote the multiplication sentence, the factors were switched around. → This time, we're finding 3 fourths of a half, not a half of 3 fourths. → If this were another garden, less of the garden is planted in vegetables overall. Last time, it was 3 fourths of the garden; this time, it would be only half. The fraction of the whole garden that is carrots is the same, but now, there is only 1 eighth of the garden planted in other vegetables. Last time, 3 eighths of the garden would have had other vegetables.

T: I hear you saying that $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{1}{2}$ are equivalent expressions. (Write $\frac{1}{2}\times\frac{3}{4}=\frac{3}{4}\times\frac{1}{2}$.) Can you give an equivalent expression for $\frac{1}{2}\times\frac{3}{5}$?

S: $\frac{1}{2}$ of $\frac{3}{5}$. → $\frac{3}{5}$ of $\frac{1}{2}$. → $\frac{3}{5}\times\frac{1}{2}$.

T: Show me another pair of equivalent expressions that involve fraction multiplication.

S: (Work and share.)

Problem 6

Mr. Becker, the gym teacher, uses $\frac{3}{5}$ of his kickballs in class. Half of the remaining balls are given to students for recess. What fraction of all the kickballs is given to students for recess?

T: (Post Problem 6, and read it aloud with students.) This time, let's solve using a tape diagram.

S/T: (Draw a tape diagram.)

T: What fraction of the balls does Mr. Becker use in class?

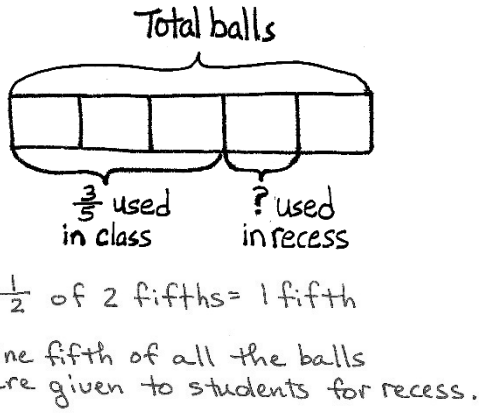
S: 3 fifths. (Partition the diagram into fifths, and label $\frac{3}{5}$ *used in class*.)

T: What fraction of the balls is remaining?

S: 2 fifths.

T: How many of those balls are given to students for recess?

S: One-half of them.



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T: What is one-half of 2?

S: 1.

T: What's one-half of 2 fifths?

S: 1 fifth.

T: Write a number sentence, and make a statement to answer the question.

S: $\frac{1}{2}$ of 2 fifths = 1 fifth. One-fifth of Mr. Becker's kickballs are given to students to use at recess.

Repeat this sequence using $\frac{1}{3} \times \frac{3}{5}$.

Problem Set (10 minutes)

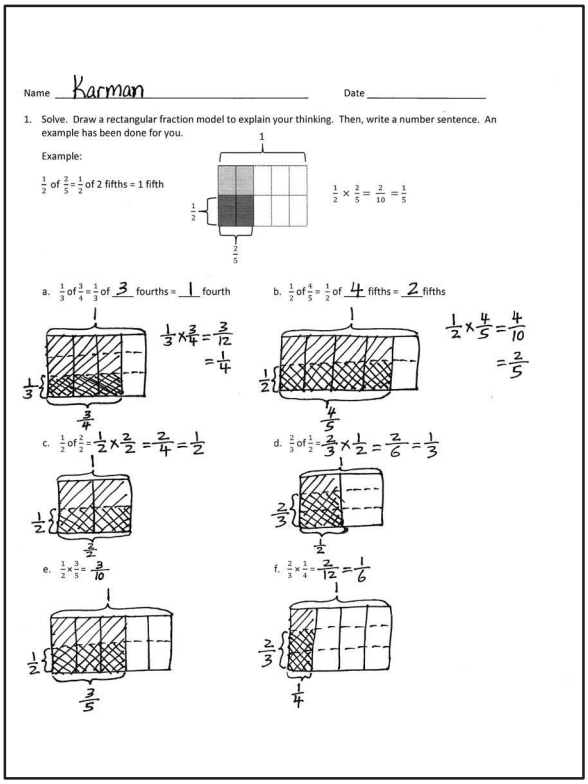
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Multiply unit fractions by non-unit fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


Name Karman Date

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a number sentence. An example has been done for you.

Example:

$\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{2}$ of 2 fifths = 1 fifth

$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$

a. $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{3}$ of 3 fourths = 1 fourth

$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$

b. $\frac{1}{2}$ of $\frac{4}{5} = \frac{1}{2}$ of 4 fifths = 2 fifths

$\frac{1}{2} \times \frac{4}{5} = \frac{4}{10} = \frac{2}{5}$

c. $\frac{1}{2}$ of $\frac{2}{2} = \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} = \frac{1}{2}$

d. $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

e. $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

f. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

- What is the relationship between Problems 1(a) and 1(b)? (Part (b) is double Part (a).)
- Share and explain your solution for Problem 1(c) to your partner. Why is taking 1 half of 2 halves equal to 1 half? Is it true for all numbers? 1 half of $\frac{6}{6}$? 1 half of $\frac{8}{8}$? 1 half of 8 wholes?
- How did you solve Problem 3? Explain your strategy to a partner.
- What kind of picture did you draw to solve Problem 4? Share and explain your solution to a partner.

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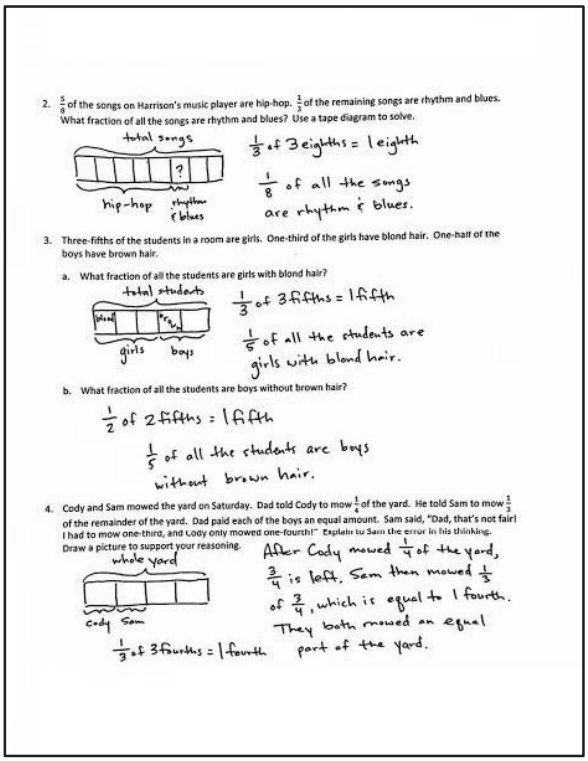
- We noticed some patterns when we wrote our multiplication sentences. Did you notice the same patterns in your Problem Set? (Students should note the multiplication of the numerators and denominators to produce the product.)
- Explore with students the commutative property in real-life situations. While the numeric product (fraction of the whole) is the same, are the situations also the same? (For example, $\frac{1}{3}\times\frac{1}{2}=\frac{1}{2}\times\frac{1}{3}$.) Is a class of fifth graders in which half are girls (a third of which wear glasses) the same as a class of fifth graders in which 1 third are girls (half of which wear glasses)?



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Number Correct: _____

A

Multiply a Fraction and a Whole Number

1.	$\frac{1}{5} \times 2 =$	
2.	$\frac{1}{5} \times 3 =$	
3.	$\frac{1}{5} \times 4 =$	
4.	$4 \times \frac{1}{5} =$	
5.	$\frac{1}{8} \times 3 =$	
6.	$\frac{1}{8} \times 5 =$	
7.	$\frac{1}{8} \times 7 =$	
8.	$7 \times \frac{1}{8} =$	
9.	$3 \times \frac{1}{10} =$	
10.	$7 \times \frac{1}{10} =$	
11.	$\frac{1}{10} \times 7 =$	
12.	$4 \div 2 =$	
13.	$4 \times \frac{1}{2} =$	
14.	$6 \div 3 =$	
15.	$\frac{1}{3} \times 6 =$	
16.	$10 \div 5 =$	
17.	$10 \times \frac{1}{5} =$	
18.	$\frac{1}{3} \times 9 =$	
19.	$\frac{2}{3} \times 9 =$	
20.	$\frac{1}{4} \times 8 =$	
21.	$\frac{3}{4} \times 8 =$	
22.	$\frac{1}{6} \times 12 =$	

23.	$\frac{5}{6} \times 12 =$	
24.	$\frac{1}{3} \times 15 =$	
25.	$\frac{2}{3} \times 15 =$	
26.	$15 \times \frac{2}{3} =$	
27.	$\frac{1}{5} \times 15 =$	
28.	$\frac{2}{5} \times 15 =$	
29.	$\frac{4}{5} \times 15 =$	
30.	$\frac{3}{5} \times 15 =$	
31.	$15 \times \frac{3}{5} =$	
32.	$18 \times \frac{1}{6} =$	
33.	$18 \times \frac{5}{6} =$	
34.	$\frac{5}{6} \times 18 =$	
35.	$24 \times \frac{1}{4} =$	
36.	$\frac{3}{4} \times 24 =$	
37.	$32 \times \frac{1}{8} =$	
38.	$32 \times \frac{3}{8} =$	
39.	$\frac{5}{8} \times 32 =$	
40.	$32 \times \frac{7}{8} =$	
41.	$\frac{5}{9} \times 54 =$	
42.	$63 \times \frac{7}{9} =$	
43.	$56 \times \frac{3}{7} =$	
44.	$\frac{6}{7} \times 49 =$	

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B

Number Correct: _____

Improvement: _____

Multiply a Fraction and a Whole Number

1.	$\frac{1}{7} \times 2 =$	
2.	$\frac{1}{7} \times 3 =$	
3.	$\frac{1}{7} \times 4 =$	
4.	$4 \times \frac{1}{7} =$	
5.	$\frac{1}{10} \times 3 =$	
6.	$\frac{1}{10} \times 7 =$	
7.	$\frac{1}{10} \times 9 =$	
8.	$9 \times \frac{1}{10} =$	
9.	$3 \times \frac{1}{8} =$	
10.	$5 \times \frac{1}{8} =$	
11.	$\frac{1}{8} \times 5 =$	
12.	$10 \div 5 =$	
13.	$10 \times \frac{1}{5} =$	
14.	$9 \div 3 =$	
15.	$\frac{1}{3} \times 9 =$	
16.	$10 \div 2 =$	
17.	$10 \times \frac{1}{2} =$	
18.	$\frac{1}{3} \times 6 =$	
19.	$\frac{2}{3} \times 6 =$	
20.	$\frac{1}{6} \times 12 =$	
21.	$\frac{5}{6} \times 12 =$	
22.	$\frac{1}{4} \times 8 =$	

23.	$\frac{3}{4} \times 8 =$	
24.	$\frac{1}{5} \times 15 =$	
25.	$\frac{2}{5} \times 15 =$	
26.	$\frac{4}{5} \times 15 =$	
27.	$\frac{3}{5} \times 15 =$	
28.	$15 \times \frac{3}{5} =$	
29.	$\frac{1}{3} \times 15 =$	
30.	$\frac{2}{3} \times 15 =$	
31.	$15 \times \frac{2}{3} =$	
32.	$24 \times \frac{1}{6} =$	
33.	$24 \times \frac{5}{6} =$	
34.	$\frac{5}{6} \times 24 =$	
35.	$20 \times \frac{1}{4} =$	
36.	$\frac{3}{4} \times 20 =$	
37.	$24 \times \frac{1}{8} =$	
38.	$24 \times \frac{3}{8} =$	
39.	$\frac{5}{8} \times 24 =$	
40.	$24 \times \frac{7}{8} =$	
41.	$\frac{5}{9} \times 63 =$	
42.	$54 \times \frac{7}{9} =$	
43.	$49 \times \frac{3}{7} =$	
44.	$\frac{6}{7} \times 56 =$	

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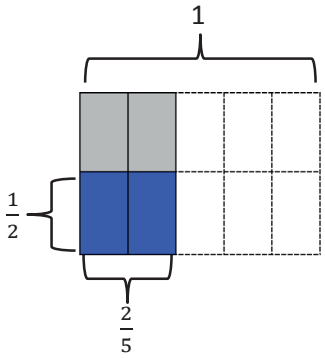
Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a number sentence. An example has been done for you.

Example:

$\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{2}$ of 2 fifths = 1 fifth(s)



$$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$$

a. $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{3}$ of ____ fourth(s) = ____ fourth(s)

b. $\frac{1}{2}$ of $\frac{4}{5} = \frac{1}{2}$ of ____ fifth(s) = ____ fifth(s)

c. $\frac{1}{2}$ of $\frac{2}{2} =$

d. $\frac{2}{3}$ of $\frac{1}{2} =$

e. $\frac{1}{2} \times \frac{3}{5} =$

f. $\frac{2}{3} \times \frac{1}{4} =$

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2. $\frac{5}{8}$ of the songs on Harrison's music player are hip-hop. $\frac{1}{3}$ of the remaining songs are rhythm and blues. What fraction of all the songs are rhythm and blues? Use a tape diagram to solve.

3. Three-fifths of the students in a room are girls. One-third of the girls have blond hair. One-half of the boys have brown hair.

   a. What fraction of all the students are girls with blond hair?

   b. What fraction of all the students are boys without brown hair?

4. Cody and Sam mowed the yard on Saturday. Dad told Cody to mow $\frac{1}{4}$ of the yard. He told Sam to mow $\frac{1}{3}$ of the remainder of the yard. Dad paid each of the boys an equal amount. Sam said, "Dad, that's not fair! I had to mow one-third, and Cody only mowed one-fourth!" Explain to Sam the error in his thinking. Draw a picture to support your reasoning.

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a number sentence.

   $\frac{1}{3}$ of $\frac{3}{7}$ =

2. In a cookie jar, $\frac{1}{4}$ of the cookies are chocolate chip, and $\frac{1}{2}$ of the rest are peanut butter. What fraction of all the cookies is peanut butter?

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking.

a. $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2}$ of ____ third(s) = ____ third(s)

b. $\frac{1}{2}$ of $\frac{4}{3} = \frac{1}{2}$ of ____ third(s) = ____ third(s)

c. $\frac{1}{3}$ of $\frac{3}{5} =$

d. $\frac{1}{2}$ of $\frac{6}{8} =$

e. $\frac{1}{3} \times \frac{4}{5} =$

f. $\frac{4}{5} \times \frac{1}{3} =$

2. Sarah has a photography blog. $\frac{3}{7}$ of her photos are of nature. $\frac{1}{4}$ of the rest are of her friends. What fraction of all of Sarah's photos is of her friends? Support your answer with a model.

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3. At Laurita's Bakery, $\frac{3}{5}$ of the baked goods are pies, and the rest are cakes. $\frac{1}{3}$ of the pies are coconut. $\frac{1}{6}$ of the cakes are angel food.

   a. What fraction of all of the baked goods at Laurita's Bakery are coconut pies?

   b. What fraction of all of the baked goods at Laurita's Bakery are angel food cakes?

4. Grandpa Mick opened a pint of ice cream. He gave his youngest grandchild $\frac{1}{5}$ of the ice cream and his middle grandchild $\frac{1}{4}$ of the remaining ice cream. Then, he gave his oldest grandchild $\frac{1}{3}$ of the ice cream that was left after serving the others.

   a. Who got the most ice cream? How do you know? Draw a picture to support your reasoning.

   b. What fraction of the pint of ice cream will be left if Grandpa Mick serves himself the same amount as the second grandchild?

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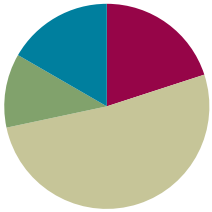
Lesson 15

Objective: Multiply non-unit fractions by non-unit fractions.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Multiply Fractions **5.NF.4** (4 minutes)
- Write Fractions as Decimals **5.NF.3** (4 minutes)
- Convert to Hundredths **4.NF.5** (4 minutes)

Multiply Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 13.

T: (Write $\frac{1}{2}$ of $\frac{1}{3}$.) Say the fraction of a set as a multiplication expression.
S: $\frac{1}{2} \times \frac{1}{3}$.
T: Draw an area model, and shade in 1 third.
S: (Draw an area model, partition it into 3 equal units, and shade 1 of the units.)
T: To show $\frac{1}{2}$ of $\frac{1}{3}$, how many parts do you need to break the 1 third into?
S: 2.
T: Shade 1 half of 1 third.
S: (Shade 1 of the 2 parts.)
T: How can we name this new unit?
S: Partition the other 2 thirds in half.
T: Show the new units.
S: (Partition the other thirds into 2 equal parts.)
T: How many new units do you have?

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S: 6 units.

T: Write the multiplication sentence.

S: (Write $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.)

Continue the process with the following possible sequence: $\frac{1}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{6}{5}$, and $\frac{3}{5}$ of $\frac{1}{2}$.

Write Fractions as Decimals (4 minutes)

Note: This fluency activity prepares students for Lessons 17 and 18.

T: (Write $\frac{1}{10}$.) Say the fraction.

S: 1 tenth.

T: Say it as a decimal.

S: Zero point one.

Continue with the following possible suggestions: $\frac{2}{10}$, $\frac{3}{10}$, $\frac{9}{10}$, and $\frac{6}{10}$.

T: (Write $\frac{1}{100}$ = _____.) Say the fraction.

S: 1 hundredth.

T: Say it as a decimal.

S: Zero point zero one.

Continue with the following possible sequence: $\frac{2}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{11}{100}$, $\frac{17}{100}$, and $\frac{53}{100}$.

T: (Write 0.01 = _____.) Say it as a fraction.

S: 1 hundredth.

T: (Write 0.01 = $\frac{1}{100}$.)

Continue with the following possible sequence: 0.02, 0.09, 0.13, and 0.37.

Convert to Hundredths (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for Lessons 17 and 18.

T: (Write $\frac{1}{5} = \frac{\quad}{100}$.) Write the equivalent fraction.

S: (Write $\frac{1}{5} = \frac{20}{100}$.)

T: (Write $\frac{1}{5} = \frac{20}{100}$ = _____.) Write 1 fifth as a decimal.

S: (Write $\frac{1}{5} = \frac{20}{100} = 0.2$.)

Continue with the following possible sequence: $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{50}$, $\frac{9}{50}$, $\frac{11}{50}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{25}$, $\frac{2}{25}$, $\frac{7}{25}$, and $\frac{12}{25}$.

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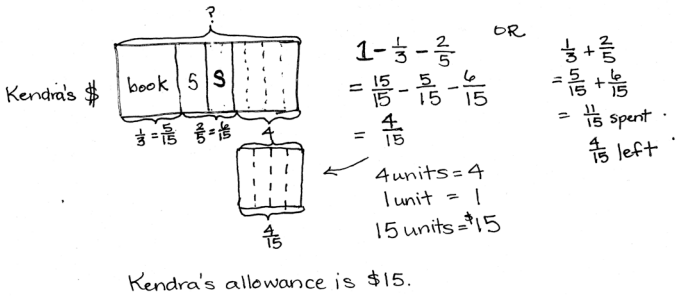
Application Problem (7 minutes)

Kendra spent $\frac{1}{3}$ of her allowance on a book and $\frac{2}{5}$ on a snack. If she had four dollars remaining after purchasing a book and snack, what was the total amount of her allowance?

Note: This problem reaches back to addition and subtraction of fractions, as well as fraction of a set. Keeping these skills fresh is an important goal of Application Problems.



Concept Development (31 minutes)

Materials: (S) Personal white board

Problem 1: $\frac{2}{3}$ **of** $\frac{3}{4}$

T: (Post Problem 1 on the board.) How is this problem different from the problems we did yesterday? Turn and talk.

S: In every problem we did yesterday, one factor had a numerator of 1. There are no numerators that are ones today. → Every problem multiplied a unit fraction by a non-unit fraction or a non-unit fraction by a unit fraction. These are two non-unit fractions.

T: (Write $\frac{1}{3}$ of 3 fourths.) What is 1 third of 3 fourths?

S: 1 fourth.

T: If 1 third of 3 fourths is 1 fourth, what is 2 thirds of 3 fourths? Discuss with your partner.

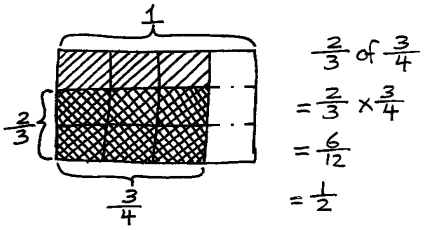
S: 2 thirds would just be double 1 third, so it would be 2 fourths. → 3 fourths is 3 equal parts, so $\frac{1}{3}$ of that would be 1 part or 1 fourth. We want $\frac{2}{3}$ this time, so that is 2 parts, or 2 fourths.

T: Name 2 fourths using halves.

S: 1 half.

T: So, 2 thirds of 3 fourths is 1 half. Let's draw a rectangular fraction model to show the product and check our thinking.

T: I'll draw it on the board, and you'll draw it on your personal white board. Let's draw 3 fourths and label it on the bottom. (Draw a rectangle, and cut it vertically into 4 units, and shade in 3 units.)



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Notice the dotted lines in the area model shown below.

If this were an actual pan partially full of brownies, the empty part of the pan would obviously not be cut! However, to name the unit represented by the double-shaded parts, the whole pan must show the same size or type of unit. Therefore, the empty part of the pan must also be partitioned as illustrated by the dotted lines.

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T: (Point to the 3 shaded units.) We now have to take 2 thirds of these 3 shaded units. What do I have to do? Turn and talk.

S: Cut each unit into thirds. → Cut it across into 3 equal parts, and shade in 2 parts.

T: Let's do that now. (Partition horizontally into thirds, shade in 2 thirds, and label.)

T: (Point to the whole rectangle.) What unit have we used to rename our whole?

S: Twelfths.

T: (Point to the 6 double-shaded units.) How many twelfths were double-shaded when we took $\frac{2}{3}$ of $\frac{3}{4}$?

S: 6 twelfths.

T: Compare our model to the product we thought about. Do they represent the same product, or have we made a mistake? Turn and talk.

S: The units are different, but the answer is the same. 2 fourths and 6 twelfths are both names for 1 half. → When we thought about it, we knew it would be 2 fourths. In the rectangular fraction model, there are 12 parts, and we shaded 6 of them. That's half.

T: Both of our approaches show that 2 thirds of 3 fourths is what simplified fraction?

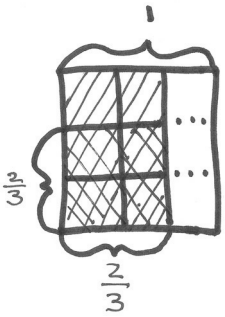
S: $\frac{1}{2}$.

T: Let's write this problem as a multiplication sentence. (Write $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$ on the board.) Turn and talk to your partner about the patterns you notice.

S: If you multiply the numerators, you get 6, and for the denominators, you get 12. That's 6 twelfths, just like the rectangular fraction model. → It's easy to solve a fraction of a fraction problem. Just multiply the top numbers to get the numerator and the bottom numbers to get the denominator. Sometimes you can simplify.

T: So, the product of the denominators tells us the total number of units—12 (point to the model). The product of the numerators tells us the total number of units selected—6 (point to the model).

Problem 2: $\frac{2}{3} \times \frac{2}{3}$



T: (Post Problem 2 on the board.) Let's solve 2 thirds of 2 thirds by drawing a rectangular fraction model, and then write a multiplication sentence.

S: (Work.)

T: Talk to your partner about whether the patterns are the same as the previous problem.

S: (Share.)

T: What patterns do you see between this problem and the last problem?

S: This problem has the same pattern as the previous problem. When you multiply the numerators, you get the numerator of the double-shaded part. When you multiply the denominators, you get the denominator of the double-shaded part. → It's pretty cool! The denominator of the product gives the area of the whole rectangle (3 by 3), and the numerator of the product gives the area of the double-shaded part (2 by 2).

T: Yes. We see from the model that the product of the denominators tells us the total number of units—9. The product of the numerator tells us the total number of units selected—4.

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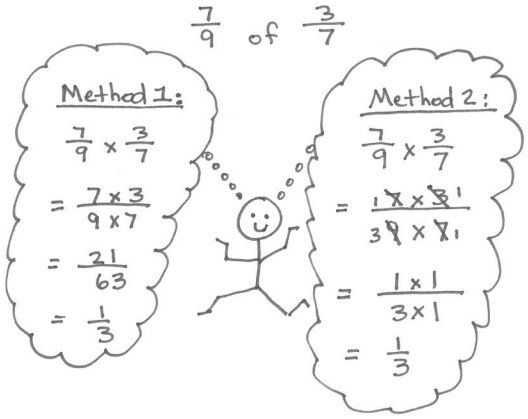
Problem 3: **a.** $\frac{7}{9}$ **of** $\frac{3}{7}$ **b.** $\frac{3}{10} \times \frac{5}{9}$ **c.** $\frac{5}{8} \times \frac{4}{15}$



T: (Post Problem 3(a) on the board.) How would this problem look if we drew a rectangular fraction model for it? Discuss with your partner.

S: We'd have to draw 3 sevenths first and then split each seventh into ninths. → We'd end up with a model showing sixty-thirds. It would be really hard to draw.

T: You are right. It's not really practical to draw a rectangular fraction model for a problem like this, because the units are so small. Could the pattern that we've noticed in the multiplication expressions help us? Turn and talk.

S: $\frac{7}{9}$ of $\frac{3}{7}$ is the same as $\frac{7}{9} \times \frac{3}{7}$. → Our pattern lets us just multiply the numerators and the denominators. → We can multiply and get 21 as the numerator and 63 as the denominator. Then, we can simplify and get 1 third.

T: Let me write what I hear you saying. (Write $\frac{7}{9} \times \frac{3}{7} = \frac{7 \times 3}{9 \times 7} = \frac{21}{63}$ on the board.)

T: What's the simplest form for $\frac{21}{63}$? Solve it on your personal white board.

S: $\frac{1}{3}$.

T: Let's use another strategy we learned recently and rename this fraction using larger units before we multiply. (Point to $\frac{7 \times 3}{9 \times 7}$.) Look for factors that are shared by the numerator and denominator. Turn and talk.

S: There's a 7 in both the numerator and denominator. → The numerator and denominator have a common factor of 7. → I know the 3 in the numerator can be divided by 3 to get 1, and the 9 in the denominator can be divided by 3 to get 3. → 7 divided by 7 is 1, so both sevens change to ones. The factors of 3 and 9 can both be divided by 3 and changed to 1 and 3.

T: We can rename this fraction by dividing both the numerators and denominators by common factors. 7 divided by 7 is 1 in both the numerator and denominator. (Cross out both sevens, and write ones next to them.) 3 divided by 3 is 1 in the numerator, and 9 divided by 3 is 3 in the denominator. (Cross out the 3 and 9, and write 1 and 3, respectively, next to them.)

T: What does the numerator show now?

S: 1 × 1.

T: What's the denominator?

S: 3 × 1.

T: Now, multiply. What is $\frac{7}{9}$ of $\frac{3}{7}$ equal to?

S: $\frac{1}{3}$.

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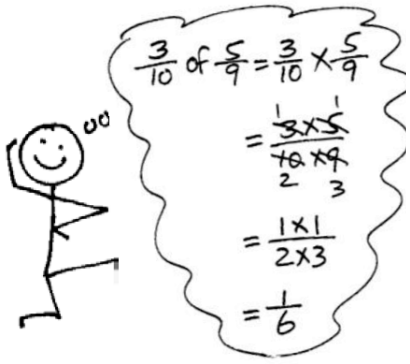
T: Look at the two strategies. Which one do you think is easier and more efficient to use? Turn and talk.

S: The first strategy of simplifying $\frac{21}{63}$ after I multiply is a little bit harder because I have to find the common factors between 21 and 63. → Simplifying first is a little easier. Before I multiply, the numbers are a little smaller, so it's easier to see common factors. Also, when I simplify first, the numbers I have to multiply are smaller, and my product is already expressed using the largest unit.



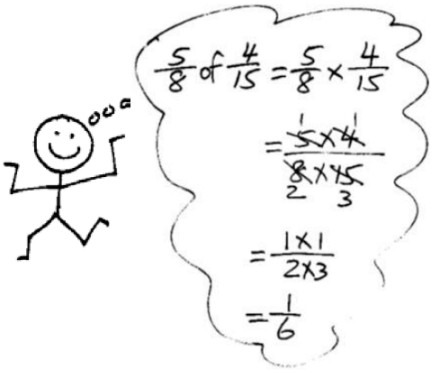
T: (Post Problem 3(b) on the board.) Let's practice using the strategy of simplifying first before we multiply. Work with a partner and solve. Remember, we are looking for common factors before we multiply. (Allow students time to work and share their answers.)

T: What is $\frac{3}{10}$ of $\frac{5}{9}$?

S: $\frac{1}{6}$.

T: Let's confirm that by multiplying first and then simplifying.

S: (Rework the problem to find $\frac{3}{10} \times \frac{5}{9} = \frac{15}{90} = \frac{1}{6}$.)



T: (Post Problem 3(c) on the board.) Solve independently. (Allow students time to solve the problem.)

T: What is $\frac{5}{8}$ of $\frac{4}{15}$?

S: $\frac{1}{6}$.

Problem 4

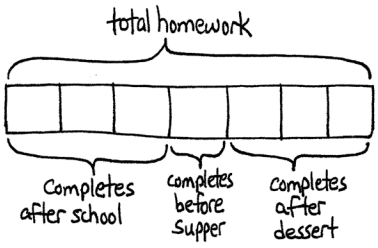
Nigel completes $\frac{3}{7}$ of his homework immediately after school and $\frac{1}{4}$ of the remaining homework before supper. He finishes the rest after dessert. What fraction of his work did he finish after dessert?

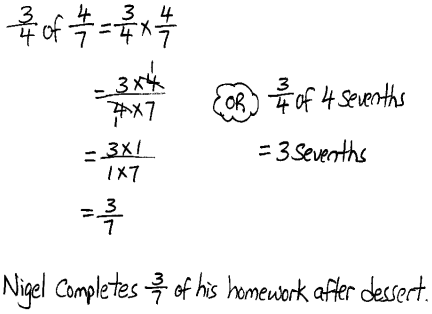
T: (Post the problem on the board, and read it aloud with students.) Let's solve using a tape diagram.

S/T: (Draw a tape diagram, and label it *total homework*.)

T: What fraction of his homework does Nigel finish immediately after school?

S: $\frac{3}{7}$.





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T: (Partition the diagram into sevenths, and label 3 of them *completes after school*.) What fraction of the homework does Nigel have remaining?

S: $\frac{4}{7}$.

T: What fraction of the remaining homework does Nigel finish before supper?

S: One-fourth of the remaining homework.

T: Nigel completes $\frac{1}{4}$ of 4 sevenths before supper. (Point to the remaining 4 units on the tape diagram.) What's $\frac{1}{4}$ of these 4 units?

S: 1 unit.

T: Then, what's $\frac{1}{4}$ of 4 sevenths? (Write $\frac{1}{4}$ of 4 sevenths = _____ sevenths on the board.)

S: 1 seventh. (Label 1 seventh of the diagram *completes before supper*.)

T: When does Nigel finish the rest? (Point to the remaining units.)

S: After dessert. (Label the remaining $\frac{3}{7}$ *completes after dessert*.)

T: Answer the question with a complete sentence.

S: Nigel completes $\frac{3}{7}$ of his homework after dessert.

T: Let's imagine that Nigel spent 70 minutes to complete all of his homework. Where would I place that information in the model?

S: Put 70 minutes above the diagram. → We just found out the whole, so we can label it above the tape diagram.

T: How could I find the number of minutes he worked on homework after dessert? Discuss with your partner, and then solve.

S: He finished $\frac{4}{7}$ already, so we can find $\frac{4}{7}$ of 70 minutes and then just subtract that from 70 to find how long he spent after dessert. → It's a fraction of a set. He does $\frac{3}{7}$ of his homework after dessert. We can multiply to find $\frac{3}{7}$ of 70. That'll be how long he worked after dessert. → We can first find the total minutes he spent after school by solving $\frac{3}{7}$ of 70. Then, we know each unit is 10 minutes. → We find what one unit is equal to, which is 10 minutes. Then, we know the time he spent after dessert is 3 units. 10 times 3 = 30.

T: Use your work to answer the question.

S: Nigel spends 30 minutes working after dessert.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

In these examples, students are simplifying the fractional factors before they multiply. This step may eliminate the need to simplify the product or make simplifying the product easier.

To help struggling students understand this procedure, it may help to use the commutative property to reverse the order of the factors. The following is an example:

$$\frac{3 \times 4}{4 \times 7} = \frac{4 \times 3}{4 \times 7}$$

In this example, students may now more readily see that $\frac{4}{4}$ is equivalent to $\frac{1}{1}$ and can be simplified before multiplying.

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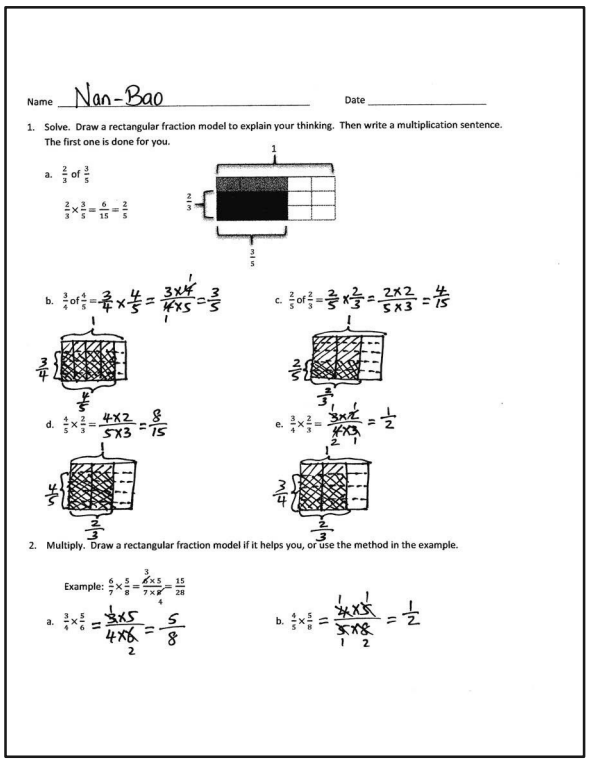
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Nan-Bao Date

1. Solve. Draw a rectangular fraction model to explain your thinking. Then write a multiplication sentence. The first one is done for you.

a. $\frac{2}{3}$ of $\frac{3}{5}$

$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

b. $\frac{3}{4}$ of $\frac{4}{5} = \frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{3}{5}$

c. $\frac{2}{5}$ of $\frac{2}{3} = \frac{2}{5} \times \frac{2}{3} = \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$

d. $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$

e. $\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{1}{2}$

2. Multiply. Draw a rectangular fraction model if it helps you, or use the method in the example.

Example: $\frac{6}{7} \times \frac{5}{8} = \frac{3}{\cancel{6} \times 5} = \frac{15}{28}$

a. $\frac{3}{4} \times \frac{5}{6} = \frac{1}{\cancel{3} \times 5} = \frac{5}{8}$

b. $\frac{4}{5} \times \frac{5}{8} = \frac{1}{\cancel{4} \times \cancel{5}} = \frac{1}{2}$

Student Debrief (10 minutes)

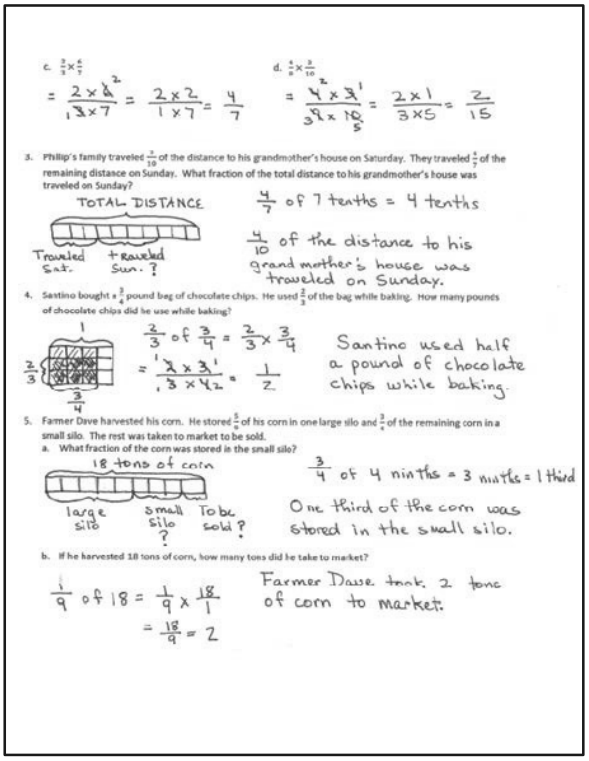
Lesson Objective: Multiply non-unit fractions by non-unit fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What is the relationship between Parts (c) and (d) of Problem 1? (Part (d) is double Part (c).)
- In Problem 2, how are Parts (b) and (d) different from Parts (a) and (c)? (Parts (b) and (d) each have two common factors.)
- Compare the picture you drew for Problem 3 with a partner. Explain your solution.
- In Problem 5, how is the information in the answer to Part (a) different from the information in the answer to Part (b)? What are the different approaches to solving, and is there one strategy that is more efficient than the others? (Using fraction of a set might be more efficient than subtraction.) Explain your strategy to a partner.


c. $\frac{2}{3} \times \frac{6}{7} = \frac{2 \times \cancel{6}^{2}}{\cancel{3} \times 7} = \frac{2 \times 2}{1 \times 7} = \frac{4}{7}$

d. $\frac{4}{5} \times \frac{3}{10} = \frac{\cancel{4}^{2} \times 3}{\cancel{5} \times \cancel{10}_{5}} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$

3. Phillip's family traveled $\frac{3}{10}$ of the distance to his grandmother's house on Saturday. They traveled $\frac{4}{7}$ of the remaining distance on Sunday. What fraction of the total distance to his grandmother's house was traveled on Sunday?

TOTAL DISTANCE

Traveled Sat. | + traveled Sun. ?

$\frac{4}{7}$ of 7 tenths = 4 tenths

$\frac{4}{10}$ of the distance to his grandmother's house was traveled on Sunday.

4. Santino bought a $\frac{3}{4}$ pound bag of chocolate chips. He used $\frac{2}{3}$ of the bag while baking. How many pounds of chocolate chips did he use while baking?

$\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{\cancel{3} \times \cancel{4}_{2}} = \frac{1}{2}$

Santino used half a pound of chocolate chips while baking.

5. Farmer Dave harvested his corn. He stored $\frac{3}{4}$ of his corn in one large silo and $\frac{1}{3}$ of the remaining corn in a small silo. The rest was taken to market to be sold.

a. What fraction of the corn was stored in the small silo?

18 tons of corn

large silo | small silo ? | To be sold ?

$\frac{3}{4}$ of 4 ninths = 3 ninths = 1 third

One third of the corn was stored in the small silo.

b. If he harvested 18 tons of corn, how many tons did he take to market?

$\frac{1}{9}$ of $18 = \frac{1}{9} \times \frac{18}{1} = \frac{18}{9} = 2$

Farmer Dave took 2 tons of corn to market.

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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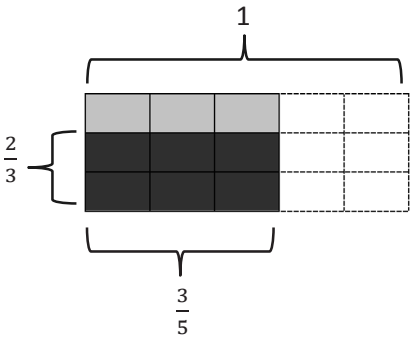
Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a multiplication sentence. The first one is done for you.

a. $\frac{2}{3}$ of $\frac{3}{5}$

$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$



b. $\frac{3}{4}$ of $\frac{4}{5} =$

c. $\frac{2}{5}$ of $\frac{2}{3} =$

d. $\frac{4}{5} \times \frac{2}{3} =$

e. $\frac{3}{4} \times \frac{2}{3} =$

2. Multiply. Draw a rectangular fraction model if it helps you, or use the method in the example.

Example: $\frac{6}{7} \times \frac{5}{8} = \frac{\overset{3}{\cancel{6}} \times 5}{7 \times \underset{4}{\cancel{8}}} = \frac{15}{28}$

a. $\frac{3}{4} \times \frac{5}{6}$

b. $\frac{4}{5} \times \frac{5}{8}$

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c. $\frac{2}{3} \times \frac{6}{7}$

d. $\frac{4}{9} \times \frac{3}{10}$

3. Phillip's family traveled $\frac{3}{10}$ of the distance to his grandmother's house on Saturday. They traveled $\frac{4}{7}$ of the remaining distance on Sunday. What fraction of the total distance to his grandmother's house was traveled on Sunday?

4. Santino bought a $\frac{3}{4}$-pound bag of chocolate chips. He used $\frac{2}{3}$ of the bag while baking. How many pounds of chocolate chips did he use while baking?

5. Farmer Dave harvested his corn. He stored $\frac{5}{9}$ of his corn in one large silo and $\frac{3}{4}$ of the remaining corn in a small silo. The rest was taken to market to be sold.

   a. What fraction of the corn was stored in the small silo?

   b. If he harvested 18 tons of corn, how many tons did he take to market?

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a multiplication sentence.

   a. $\frac{2}{3}$ of $\frac{3}{5} =$

   b. $\frac{4}{9} \times \frac{3}{8} =$

2. A newspaper's cover page is $\frac{3}{8}$ text, and photographs fill the rest. If $\frac{2}{5}$ of the text is an article about endangered species, what fraction of the cover page is the article about endangered species?

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Name _____ Date _____

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a multiplication sentence.

   a. $\frac{2}{3}$ of $\frac{3}{4} =$

   b. $\frac{2}{5}$ of $\frac{3}{4} =$

   c. $\frac{2}{5}$ of $\frac{4}{5} =$

   d. $\frac{4}{5}$ of $\frac{3}{4} =$

2. Multiply. Draw a rectangular fraction model if it helps you.

   a. $\frac{5}{6} \times \frac{3}{10}$

   b. $\frac{3}{4} \times \frac{4}{5}$

   c. $\frac{5}{6} \times \frac{5}{8}$

   d. $\frac{3}{4} \times \frac{5}{12}$

   e. $\frac{8}{9} \times \frac{2}{3}$

   f. $\frac{3}{7} \times \frac{2}{9}$

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3. Every morning, Halle goes to school with a 1-liter bottle of water. She drinks $\frac{1}{4}$ of the bottle before school starts and $\frac{2}{3}$ of the rest before lunch.

   a. What fraction of the bottle does Halle drink after school starts but before lunch?

   b. How many milliliters are left in the bottle at lunch?

4. Moussa delivered $\frac{3}{8}$ of the newspapers on his route in the first hour and $\frac{4}{5}$ of the rest in the second hour. What fraction of the newspapers did Moussa deliver in the second hour?

5. Rose bought some spinach. She used $\frac{3}{5}$ of the spinach on a pan of spinach pie for a party and $\frac{3}{4}$ of the remaining spinach for a pan for her family. She used the rest of the spinach to make a salad.

   a. What fraction of the spinach did she use to make the salad?

   b. If Rose used 3 pounds of spinach to make the pan of spinach pie for the party, how many pounds of spinach did Rose use to make the salad?

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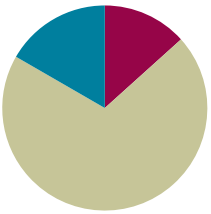
Lesson 16

Objective: Solve word problems using tape diagrams and fraction-by-fraction multiplication.

Suggested Lesson Structure

Fluency Practice	(8 minutes)
Concept Development	(42 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (8 minutes)

- Multiply Fractions **5.NF.4** (3 minutes)
- Multiply Whole Numbers by Decimals **5.NBT.7** (5 minutes)

Multiply Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 14 and 15.

T: (Write $\frac{1}{3}$ of $\frac{2}{5}$.) Write the fraction of a set as a multiplication expression.

S: $\frac{1}{3} \times \frac{2}{5}$.

T: Draw a rectangle, and shade in $\frac{2}{5}$.

S: (Draw a rectangle, partition it into 5 equal units, and shade 2 of the units.)

T: To show $\frac{1}{3}$ of $\frac{2}{5}$, how many equal parts do we need?

S: 3.

T: Show 1 third of 2 fifths.

S: (Partition the 2 fifths into thirds, and shade 1 third.)

T: Make the other units the same size as the double-shaded ones.

S: (Extend the horizontal thirds across the remaining units using dotted lines.)

T: What unit do we have now?

S: Fifteenths.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Sixty minutes are allotted for all lessons in Grade 5. All 60 minutes, however, do not need to be consecutive. For example, fluency activities can be completed as students wait in line or when they are transitioning between subjects.

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T: How many fifteenths are double-shaded?

S: 2.

T: Write the product, and say the sentence.

S: (Write $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.) $\frac{1}{3}$ of $\frac{2}{5}$ is 2 fifteenths.

Continue this process with the following possible sequence: $\frac{2}{3} \times \frac{2}{5}$, $\frac{3}{5} \times \frac{2}{3}$, and $\frac{3}{5} \times \frac{3}{4}$.

Multiply Whole Numbers by Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for Lessons 17 and 18.

T: (Write $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} =$ ____.) Say the repeated addition sentence with the answer.

S: $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$.

T: (Write 3 × ____ = $\frac{3}{10}$.) On your personal white board, write the number sentence, filling in the missing number.

S: (Write $3 \times \frac{1}{10} = \frac{3}{10}$.)

T: (Write $3 \times \frac{1}{10} = 3 \times 0.$____.) Fill in the missing digit.

S: (Write $3 \times \frac{1}{10} = 3 \times 0.1$.)

T: (Write 3 × 0.1 = 0.____.) Say the missing digit.

S: 3.

Continue with the following expression: $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$.

T: (Write 7 × 0.1 = ____.) On your personal white board, write the number sentence with the answer.

S: (Write 7 × 0.1 = 0.7.)

T: (Write 7 × 0.01 = ____.) Try this problem.

S: (Write 7 × 0.01 = 0.07.)

Continue this process with the following possible sequence: 9 × 0.1 and 9 × 0.01.

T: (Write $20 \times \frac{1}{10} =$ ____.) On your personal white board, write the number sentence with the answer.

S: (Write $20 \times \frac{1}{10} = \frac{20}{10} = 2$.)

T: (Write 20 × 0.1 = ____.) Try this problem.

S: (Write 20 × 0.1 = 2.)

T: (Write 20 × 0.01 = ____.) Try this problem.

S: (Write 20 × 0.01 = 0.2.)

Continue this process with the following possible sequence: 80 × 0.1 and 80 × 0.01.

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T: (Write $15 \times \frac{1}{10} =$ ____.) On your personal white board, write the number sentence with the answer.

S: (Write $15 \times \frac{1}{10} = \frac{15}{10}$.)

T: (Write $15 \times 0.1 =$ ____.) Write the number sentence, and answer as a decimal.

S: (Write $15 \times 0.1 = 1.5$.)

T: (Write $15 \times 0.01 =$ ____.) Try this problem.

S: (Write $15 \times 0.01 = 0.15$.)

Continue with the following possible sequence: 37×0.1 and 37×0.01.

Concept Development (42 minutes)

Materials: (S) Problem Set, personal white board

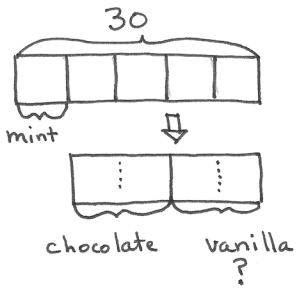
Note: Because today's lesson involves students in learning a new type of tape diagram, the time normally allotted to the Application Problem has been used in the Concept Development to allow students ample time to draw and solve the story problems.

Note: There are multiple approaches to solving these problems. Modeling for a few strategies is included here, but teachers should not discourage students from using other mathematically sound procedures for solving. The dialogues for the modeled problems are detailed as a scaffold for teachers unfamiliar with fraction tape diagrams.

Problem 2 from the Problem Set opens the lesson and is worked using two different fractions (first 1 fifth, then 2 fifths) so that diagramming of two different whole–part situations may be modeled.

Problem 2

Joakim is icing 30 cupcakes. He spreads mint icing on $\frac{1}{5}$ of the cupcakes and chocolate on $\frac{1}{2}$ of the remaining cupcakes. The rest will get vanilla frosting. How many cupcakes have vanilla frosting?



T: (Display Problem 2, and read it aloud with students.) Let's use a tape diagram to model this problem.

T: This problem is about Joakim's cupcakes. What does the first sentence tell us?

S: Joakim has 30 cupcakes.

T: (Draw a tape diagram, and label the whole as *30*.) Joakim is icing the cupcakes. What fraction of the cupcakes receives mint icing?

S: $\frac{1}{5}$ of the cupcakes.

T: How can I show fifths in my tape diagram?

S: Partition the whole into 5 equal units.

T: How many of those units have mint icing?

S: 1.

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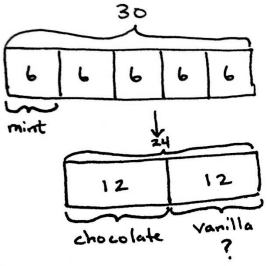
T: Let's show that now. (Partition the tape diagram into fifths, and label 1 unit *mint*.)
T: Read the next sentence.
S: (Read.)
T: Where are the remaining cupcakes in our tape diagram?
S: The unlabeled units.
T: Let's drop that part down and draw a new tape diagram to represent the remaining cupcakes. (Draw a new tape diagram underneath the original whole.)
T: What do we know about these remaining cupcakes?
S: Half of them get chocolate icing.
T: How can we represent that in our new tape diagram?
S: Cut it into 2 equal parts, and label 1 of them *chocolate*.
T: Let's do that now. (Partition the lower diagram into 2 units, and label 1 unit *chocolate*.) What about the rest of the remaining cupcakes?
S: They are vanilla.
T: Let's label the other half *vanilla*. (Model.) What is the question asking us?
S: How many are vanilla?
T: Place a question mark below the portion showing vanilla. (Put a question mark beneath *vanilla*.)
T: Let's look at our tape diagram to see if we can find how many cupcakes get vanilla icing. How many units does the model show? (Point to the original tape diagram.)
S: 5 units.
T: (Write 5 units.) How many cupcakes does Joakim have in all?

5 units = 30 cupcakes
1 unit = 6 cupcakes

S: 30 cupcakes.
T: (Write = 30 cupcakes.) If 5 units equal 30 cupcakes, how can we find the value of 1 unit? Turn and talk.
S: It's like 5 times what equals 30. 5 × 6 = 30, so 1 unit equals 6 cupcakes. → We can divide. 30 cupcakes ÷ 5 = 6 cupcakes.
T: What is 1 unit equal to? (Write 1 unit = ____.)
S: 6 cupcakes.
T: Let's write 6 in each unit to show its value. (Write 6 in each unit of the original diagram.) That means that 6 cupcakes get mint icing. How many cupcakes remain? (Point to 4 remaining units.) Turn and talk.
S: 30 – 6 = 24. → 6 + 6 + 6 + 6 = 24. → 4 units of 6 is 24. 4 × 6 = 24.
T: Let's label that on the diagram showing the remaining cupcakes. (Label *24* above the second tape diagram.) How can we find the number of cupcakes that get vanilla icing? Turn and talk.
S: Half of the 24 cupcakes get chocolate, and half get vanilla. Half of 24 is 12. → 24 ÷ 2 = 12.
T: What is half of 24?
S: 12.



24 ÷ 2 = 12

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T: (Write $\frac{24}{2} = 12$, and label *12* in each half of the second tape diagram.) Write a statement to answer the question.

S: 12 cupcakes have vanilla icing.

T: Let's think of this another way. When we labeled the 1 fifth for the mint icing, what fraction of the cupcakes were remaining?

S: $\frac{4}{5}$.

S: What does Joakim do with the remaining cupcakes?

S: $\frac{1}{2}$ of the remaining cupcakes get chocolate icing.

T: (Write $\frac{1}{2}$ of _____.) $\frac{1}{2}$ of what fraction?

S: 1 half of 4 fifths.

T: (Write 4 fifths.) What is $\frac{1}{2}$ of 4 fifths?

S: 2 fifths.

T: So, 2 fifths of all the cupcakes got chocolate, and 2 fifths of all the cupcakes got vanilla. The question asked us how many cupcakes got vanilla icing. Let's find 2 fifths of all the cupcakes, which is 2 fifths of 30. Work with your partner to solve.

$\frac{1}{2}$ of 4 fifths
= 2 fifths

$\frac{2}{5} \times 30$
$= \frac{2 \times 30^{6}}{5_{1}}$
$= 12$

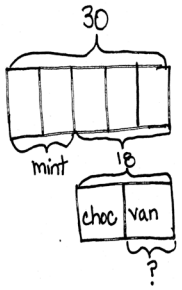
12 cupcakes have vanilla icing.

S: 1 fifth of 30 is 6, so 2 fifths of 30 is 12. → $\frac{2}{5} \times 30 = \frac{2 \times 30}{5} = \frac{60}{5} = 12$. → $\frac{2}{5} \times 30 = 2 \times \frac{30}{5} = 2 \times 6 = 12$.

T: So, using fraction multiplication, we got the same answer, 12 cupcakes.

T: This time, let's imagine that Joakim put mint icing on 2 fifths of the cupcakes. Draw another diagram to show that situation.

S: (Draw.)



5 units = 30
1 unit = 6

$\frac{18}{2} = 9$

or $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

$\frac{3}{10} \times 30$
$= \frac{3 \times 30^{3}}{10_{1}}$
$= 9$

9 cupcakes got vanilla icing.

T: What fraction of the cupcakes is remaining this time?

S: 3 fifths.

T: Let's draw a second tape diagram that is the same length as the remaining part of our whole. (Draw the second tape diagram below the first.) Has the value of one unit changed in our model? Why or why not?

S: The unit is still 6 because the whole is still 30, and we still have fifths. → Each unit is still 6 because we still divided 30 into 5 equal parts.

MP.4

T: So, how many remaining cupcakes are there this time?

S: 18.

T: Imagine that Joakim still put chocolate icing on half the remaining cupcakes, and the rest were still vanilla. How many cupcakes got vanilla icing this time? Work with a partner to model it in your tape diagram and answer the question with a complete sentence.

S: (Work.)

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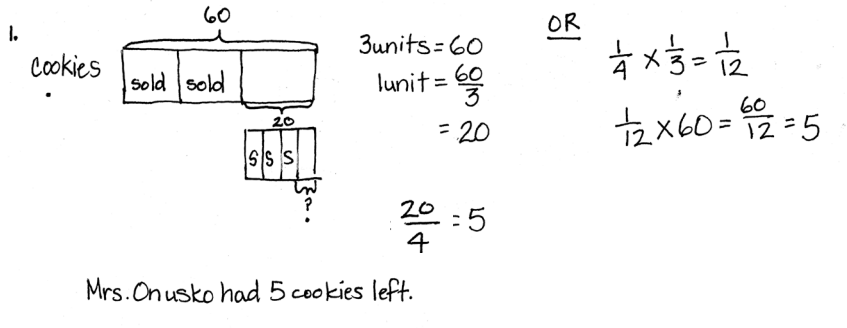
T: Let's confirm that there were 9 cupcakes that got vanilla icing by using fraction multiplication. How might we do this? Turn and talk.

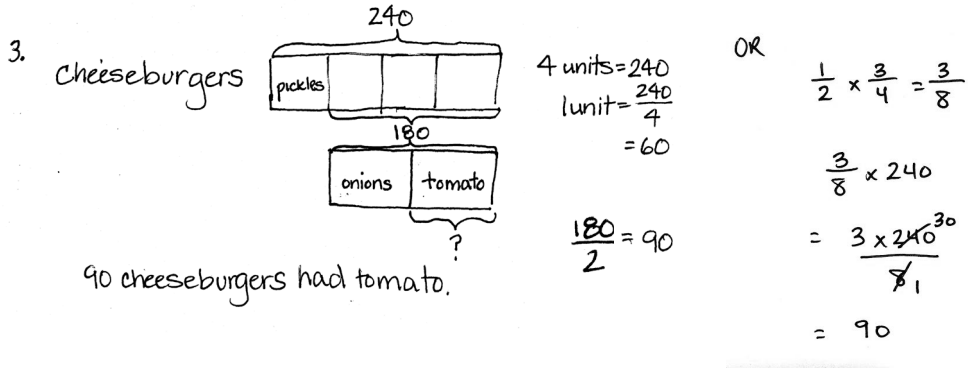
MP.4

S: We could just multiply $\frac{1}{2} \times \frac{3}{5}$ and get $\frac{3}{10}$. Then, we can find $\frac{3}{10}$ of 30. That's 9. → We can find $\frac{1}{2}$ of 3 fifths. That gives us the fraction of all the cupcakes that got vanilla icing. We need the number of cupcakes, not just the fraction, so we need to multiply $\frac{3}{10}$ and 30 to get 9 cupcakes. → Nine cupcakes got vanilla frosting.

T: Complete Problems 1 and 3 on the Problem Set. Check your work with a neighbor when you're finished. You may use either method to solve.

Solutions for Problem 1 and Problem 3

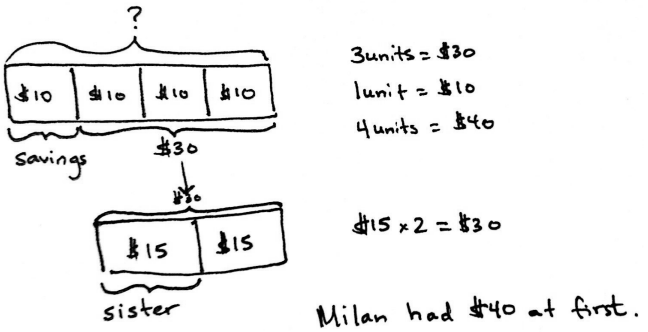




Problem 5

Milan puts $\frac{1}{4}$ of her lawn-mowing money in savings and uses $\frac{1}{2}$ of the remaining money to pay back her sister. If she has $15 left, how much did she have at first?

T: (Post Problem 5 on the board, and read it aloud with students.) How is this problem different from the ones we've just solved? Turn and discuss with your partner.



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S: In the others, we knew what the whole was; this time, we don't. → We know how much money she has left, but we have to figure out what she had at the beginning. It seems like we might have to work backward. → The other problems were whole-to-part problems. This one is part-to-whole.

T: Let's draw a tape diagram. (Draw a blank tape diagram.) What is the whole in this problem?

S: We don't know yet; we have to find it.

T: I'll put a question mark above our tape diagram to show that this is unknown. (Label the diagram with a question mark.) What fraction of her money does Milan put in savings?

S: $\frac{1}{4}$.

T: How can we show that on our tape diagram?

S: Cut the whole into 4 equal parts, and bracket one of them. → Cut it into fourths, and label 1 unit *savings*.

T: (Record on the tape diagram.) What part of our tape diagram shows the remaining money?

S: The other parts.

T: Let's draw another tape diagram to represent the remaining money. Notice that I will draw it exactly the same length as those last 3 parts. (Model.) What do we know about this remaining part?

S: Milan gives half of it to her sister.

T: How can we model that?

S: Cut the tape diagram into two equal parts, and label one of them. (Partition the second tape diagram in halves, and label one of them *sister*.)

T: What about the other half of the remaining money?

S: That's how much she has left. It's $15.

T: Let's label that. (Write $15 in the second equal part.) If this half is $15 (point to labeled half), what do we know about the amount she gave her sister, and what does that tell us about how much was remaining in all? Turn and talk.

S: If one half is $15, then the other half is $15, too. That makes $30. → $15 + $15 = $30. → $15 × 2 = $30.

T: If the lower tape diagram is worth $30, what do we know about these 3 units in the whole? (Point to the original tape diagram.) Turn and discuss.

S: The remaining money is the same as 3 units, so 3 units is equal to $30. → They represent the same money in two different parts of the tape diagram. 3 units is equal to $30.

T: (Label 3 units *$30*.) If 3 units = $30, what is the value of 1 unit?

S: (Work and show 1 unit = $10.)

T: Label *$10* inside each of the 3 units. (Model on the diagram.) If these 3 units are equal to $10 each, what is the value of this last unit? (Point to the unit that was labeled *savings*.)

S: $10.

T: (Label *$10* inside the savings' unit.) Look at our tape diagram. We have 4 units of $10 each. What is the value of the whole?

S: (Work and show 4 units = $40.)

T: Make a statement to answer the question.

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S: Milan had $40 at first.

T: Let's check our work using a fraction of a set. What multiplication sentence tells us what fraction of all her money Milan gave to her sister? What fraction did she give to her sister?

S: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

T: So, $15 should be 3 eighths of $40. Is that true? Let's see. Find $\frac{3}{8}$ of $40 with your partner.

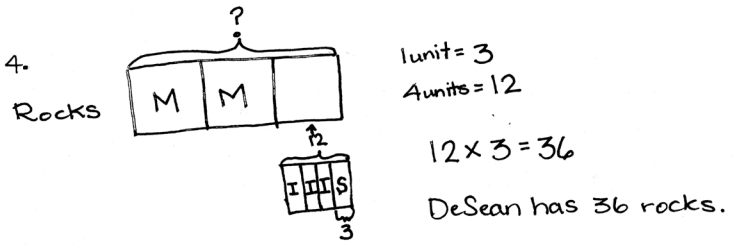
S: (Work and show $\frac{3}{8}$ of $40 = $15.)

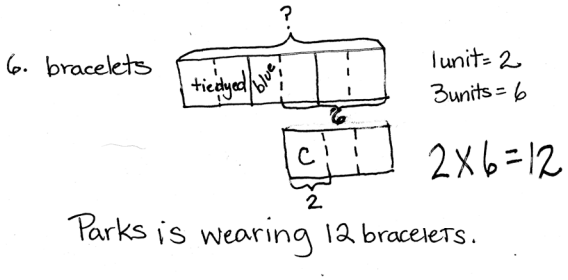
T: Does this confirm our answer of $40 as Milan's money at first?

S: Yes!

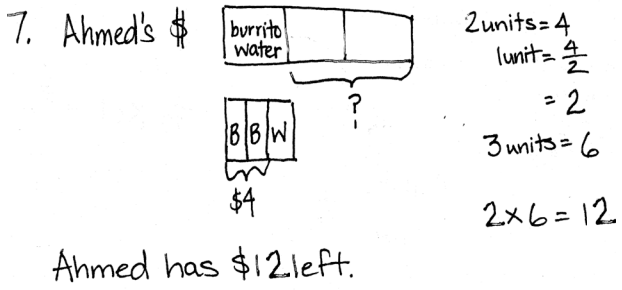
T: Complete Problems 4 and 6 on the Problem Set. Check your work with a neighbor when you've finished. You may use either method to solve.

Solutions for Problem 4 and Problem 6





Note: Problem 7 may be used as an extension for early finishers.



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Problem Set (10 minutes)

The Problem Set forms the basis for today's lesson. Please see the Concept Development for modeling suggestions.

Student Debrief (10 minutes)

Lesson Objective: Solve word problems using tape diagrams and fraction-by-fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

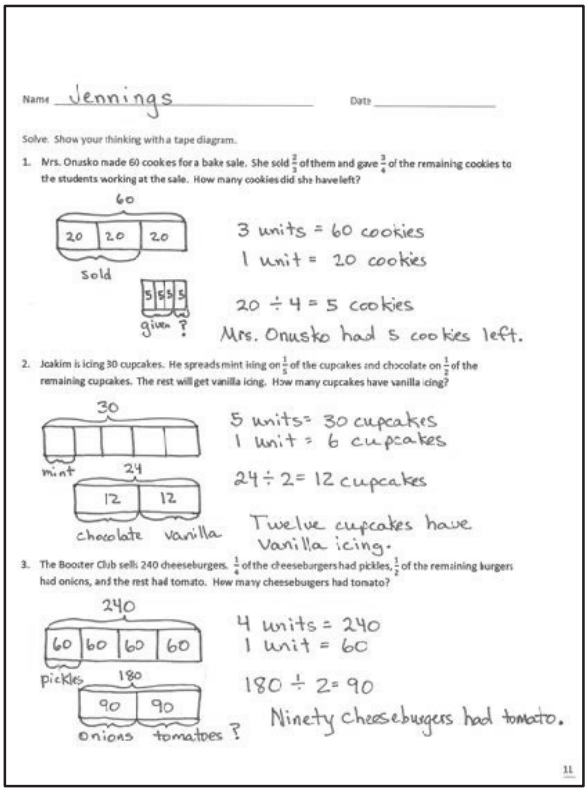
Any combination of the questions below may be used to lead the discussion.

- Did you use the same method for solving Problems 1 and 3? Why or why not?
- Did you use the same method for solving Problems 4 and 6? Why or why not?
- Were any alternate methods used? If so, explain what you did.
- How was setting up Problems 1 and 3 different from the process for solving Problems 4 and 6? What were your thoughts as you worked?
- Talk about how your tape diagrams helped you find the solutions. Give some examples of questions that you could have been able to answer using the information in your tape diagrams.
- Questions for further analysis of tape diagrams:
  - Problem 1: Half of the cookies sold were oatmeal raisin. How many oatmeal raisin cookies were sold?
  - Problem 3: What fraction of the burgers had onions? How many burgers had onions?

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Anticipate that some students may struggle with a homework assignment, and provide support in one of the following ways:

- Complete one of the problems or a portion of a problem as an example before the pages are duplicated for students.
- Staple the Problem Set to the homework as a reference.
- Provide a copy of completed homework as a reference.
- Differentiate homework by using some of these strategies for specific students or specifying that only certain problems be completed.



Name Jennings Date

Solve. Show your thinking with a tape diagram.

1. Mrs. Onusko made 60 cookies for a bake sale. She sold $\frac{2}{3}$ of them and gave $\frac{3}{4}$ of the remaining cookies to the students working at the sale. How many cookies did she have left?

60
20 20 20
sold
5 5 5 5
given ?

3 units = 60 cookies
1 unit = 20 cookies
20 ÷ 4 = 5 cookies
Mrs. Onusko had 5 cookies left.

2. Joakim is icing 30 cupcakes. He spreads mint icing on $\frac{1}{5}$ of the cupcakes and chocolate on $\frac{1}{2}$ of the remaining cupcakes. The rest will get vanilla icing. How many cupcakes have vanilla icing?

30
mint
24
12 12
chocolate vanilla

5 units = 30 cupcakes
1 unit = 6 cupcakes
24 ÷ 2 = 12 cupcakes
Twelve cupcakes have vanilla icing.

3. The Booster Club sells 240 cheeseburgers. $\frac{1}{4}$ of the cheeseburgers had pickles, $\frac{1}{2}$ of the remaining burgers had onions, and the rest had tomato. How many cheeseburgers had tomato?

240
60 60 60 60
pickles
180
90 90
onions tomatoes ?

4 units = 240
1 unit = 60
180 ÷ 2 = 90
Ninety cheeseburgers had tomato.

11

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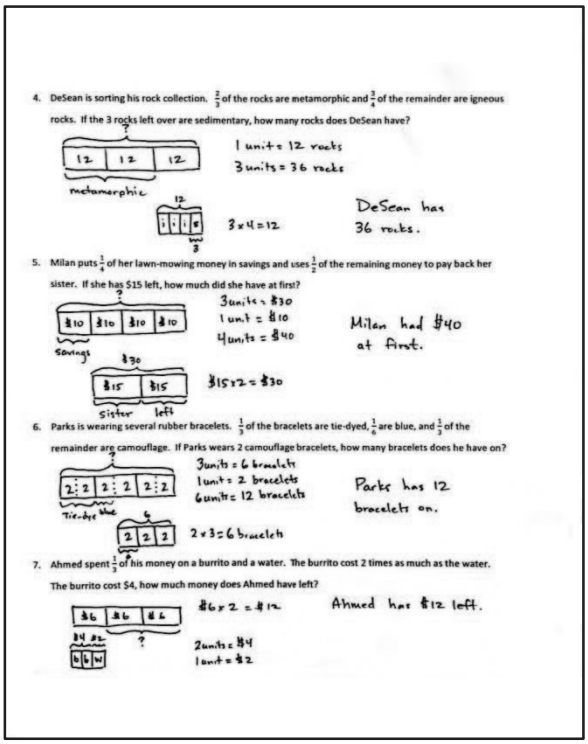
- Problem 4: How many more metamorphic rocks does DeSean have than igneous rocks?
- Problem 6: If Parks takes off 2 tie-dye bracelets and puts on 2 more camouflage bracelets, what fraction of all the bracelets would be camouflage?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



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Name _____ Date _____

Solve and show your thinking with a tape diagram.

1. Mrs. Onusko made 60 cookies for a bake sale. She sold $\frac{2}{3}$ of them and gave $\frac{3}{4}$ of the remaining cookies to the students working at the sale. How many cookies did she have left?

2. Joakim is icing 30 cupcakes. He spreads mint icing on $\frac{1}{5}$ of the cupcakes and chocolate on $\frac{1}{2}$ of the remaining cupcakes. The rest will get vanilla icing. How many cupcakes have vanilla icing?

3. The Booster Club sells 240 cheeseburgers. $\frac{1}{4}$ of the cheeseburgers had pickles, $\frac{1}{2}$ of the remaining burgers had onions, and the rest had tomato. How many cheeseburgers had tomato?

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4. DeSean is sorting his rock collection. $\frac{2}{3}$ of the rocks are metamorphic, and $\frac{3}{4}$ of the remainder are igneous rocks. If the 3 rocks left over are sedimentary, how many rocks does DeSean have?

5. Milan puts $\frac{1}{4}$ of her lawn-mowing money in savings and uses $\frac{1}{2}$ of the remaining money to pay back her sister. If she has $15 left, how much did she have at first?

6. Parks is wearing several rubber bracelets. $\frac{1}{3}$ of the bracelets are tie-dye, $\frac{1}{6}$ are blue, and $\frac{1}{3}$ of the remainder are camouflage. If Parks wears 2 camouflage bracelets, how many bracelets does he have on?

7. Ahmed spent $\frac{1}{3}$ of his money on a burrito and a water bottle. The burrito cost 2 times as much as the water. The burrito cost $4. How much money does Ahmed have left?

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Name _____ Date _____

Solve and show your thinking with a tape diagram.

Three-quarters of the boats in the marina are white, $\frac{4}{7}$ of the remaining boats are blue, and the rest are red. If there are 9 red boats, how many boats are in the marina?

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Name _____ Date _____

Solve and show your thinking with a tape diagram.

1. Anthony bought an 8-foot board. He cut off $\frac{3}{4}$ of the board to build a shelf and gave $\frac{1}{3}$ of the rest to his brother for an art project. How many inches long was the piece Anthony gave to his brother?

2. Riverside Elementary School is holding a school-wide election to choose a school color. Five-eighths of the votes were for blue, $\frac{5}{9}$ of the remaining votes were for green, and the remaining 48 votes were for red.

   a. How many votes were for blue?

   b. How many votes were for green?

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c. If every student got one vote, but there were 25 students absent on the day of the vote, how many students are there at Riverside Elementary School?

d. Seven-tenths of the votes for blue were made by girls. Did girls who voted for blue make up more than or less than half of all votes? Support your reasoning with a picture.

e. How many girls voted for blue?

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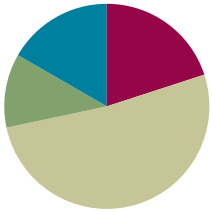
Lesson 17

Objective: Relate decimal and fraction multiplication.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Multiply Fractions **5.NF.4** (4 minutes)
- Write Fractions as Decimals **5.NF.3** (4 minutes)
- Multiply Whole Numbers by Decimals **5.NBT.7** (4 minutes)

Multiply Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 13–16.

T: (Write $\frac{1}{2} \times \frac{1}{3} =$ ____.) Say the number sentence with the answer.

S: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Continue the process with the following possible problems: $\frac{1}{2} \times \frac{1}{4}$ and $\frac{1}{2} \times \frac{1}{5}$.

T: (Write $\frac{3}{4} \times \frac{1}{2} =$ ____.) On your personal white board, write the number sentence with the answer.

S: (Write $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.)

T: (Write $\frac{3}{4} \times \frac{1}{3} =$ ____.) Try this problem.

S: (Write $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.)

Repeat the process with the following possible problems: $\frac{1}{2} \times \frac{2}{2}$, $\frac{1}{3} \times \frac{4}{5}$, and $\frac{2}{3} \times \frac{1}{3}$.

T: (Write $\frac{2}{5} \times \frac{2}{3} =$ ____.) Say the number sentence with the answer.

S: $\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$.

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Continue the process with the following possible problem: $\frac{3}{5} \times \frac{3}{4}$.

T: (Write $\frac{1}{5} \times \frac{2}{3} =$ ____.) On your personal white board, write the equation and solve.

S: (Write $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$.)

T: (Write $\frac{2}{3} \times \frac{3}{2} =$ ____.) Try this problem.

S: (Write $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.)

Continue the process with the following possible problems: $\frac{3}{4} \times \frac{2}{3}$, $\frac{3}{8} \times \frac{2}{3}$, and $\frac{2}{5} \times \frac{5}{8}$.

Write Fractions as Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lessons 17 and 18.

T: (Write $\frac{1}{10}$.) Say the fraction.

S: 1 tenth.

T: Write it as a decimal.

S: (Write 0.1.)

Continue with the following possible sequence: $\frac{2}{10}$, $\frac{8}{10}$, $\frac{3}{10}$, and $\frac{7}{10}$.

T: (Write $\frac{1}{100}$.) Say the fraction.

S: 1 hundredth.

T: Write it as a decimal.

S: (Write 0.01.)

Continue with the following possible sequence: $\frac{2}{100}$, $\frac{7}{100}$, $\frac{9}{100}$, $\frac{12}{100}$, $\frac{15}{100}$, $\frac{45}{100}$, and $\frac{93}{100}$.

T: (Write 0.01.) Say it as a fraction.

S: 1 hundredth.

T: Write it as a fraction.

S: (Write $\frac{1}{100}$.)

Continue with the following possible sequence: 0.03, 0.09, 0.11, and 0.87.

Multiply Whole Numbers by Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for Lessons 17 and 18. In the following dialogue, several possible student responses are represented.

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T: (Write 2 × 0.1 = ____.) What is 2 copies of 1 tenth?
S: (Write 2 × 0.1 = 2 tenths.)
T: (Write 0.2 in the number sentence above.)
T: (Erase the product, and replace the 2 with a 3.) What is 3 copies of 1 tenth?
S: (Write 3 × 0.1 = 3 tenths.)
T: Write it as a decimal on your personal white board.
S: (Write 0.3.)
T: 4 copies of 1 tenth?
S: (Write 0.4.)
T: 7 × 0.1?
S: (Write 0.7.)
T: (Write 7 × 0.01 = ____.) What is 7 copies of 1 hundredth?
S: (Write 7 hundredths.)
T: Write it as a decimal.
S: (Write 0.07.)
T: What is 5 copies of 1 hundredth? Write it as a decimal.
S: (Write 0.05.)
T: 5 × 0.01?
S: (Write 0.05.)
T: (Write 9 × 0.01 = ____.) On your personal white board, write the number sentence with the answer.
S: (Write 9 × 0.01 = 0.09.)
T: (Write 2 × 0.1 = ____.) Say the answer.
S: 2 tenths.
T: (Write 20 × 0.1 = ____.) What is 20 copies of 1 tenth?
S: (Write 20 tenths.)
T: Rename it using ones.
S: (Write 2 ones.)
T: (Write 20 × 0.01 = ____.) On your personal white board, write the number sentence with the answer. What is 20 copies of 1 hundredth?
S: (Write 20 × 0.01 = 0.20.) 20 hundredths.
T: Rename the product using tenths.
S: 2 tenths.

Continue this process with the following possible sequence, shifting between choral and personal white board responses: 30 × 0.1, 30 × 0.01, 80 × 0.01, and 80 × 0.1. If students are successful with the previous sequence, continue with the following: 83 × 0.1, 83 × 0.01, 53 × 0.01, 53 × 0.1, 64 × 0.01, and 37 × 0.1.

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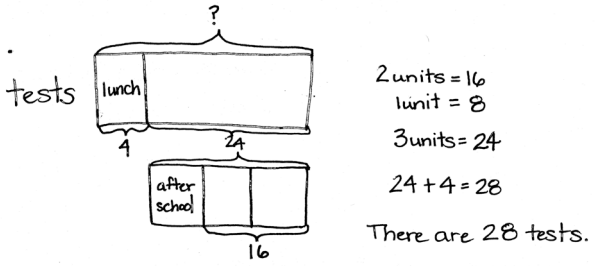
Application Problem (7 minutes)

Ms. Casey grades 4 tests during her lunch. She grades $\frac{1}{3}$ of the remainder after school. If she still has 16 tests to grade after school, how many tests are there?

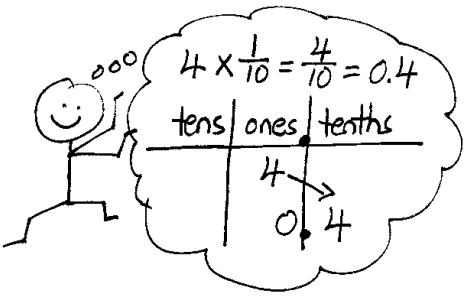
Note: Today's Application Problem recalls Lesson 16's work with tape diagrams. This is a challenging problem in that the value of a part is given and then the value of 2 thirds of the remainder. Possibly remind students to draw without concern initially for proportionality. They can rework the tape diagram if needed.



Concept Development (31 minutes)

Materials: (S) Personal white board, millions through thousandths place value chart (Template)

Problem 1: a. 0.1 × 4 b. 0.1 × 2 c. 0.01 × 6



T: (Post Problem 1(a) on the board.) Read this multiplication expression using unit form and the word *of*.

S: 1 tenth of 4.

T: Write this expression as a multiplication sentence using a fraction, and solve. Do not simplify your product.

S: (Write $\frac{1}{10} \times 4 = \frac{4}{10}$.)

T: Write this as a decimal on your personal white board.

S: (Write 0.4.)

T: (Write 0.1 × 4 = 0.4.) Let's compare the 4 ones that we started with to the product that we found, 4 tenths. Place 4 and 0.4 on a place value chart, and talk to your partner about what happened to the digit 4 when we multiplied by 1 tenth. Why did our answer get smaller?

S: This is like 4 copies of 1 tenth. → There are 40 tenths in 4 wholes. 1 tenth of 40 is 4. The unit is tenths, so the answer is 4 tenths. → The digit 4 stays the same because we are multiplying by 1 of something, but the unit is smaller, so the digit 4 shifted one space to the right. The answer is 4 tenths.

T: What about $\frac{1}{100}$ of 4? Multiply and then show your thinking on the place value chart.

S: (Work to show 4 hundredths or 0.04.)

T: What about $\frac{1}{1{,}000}$ of 4?

S: 4 thousandths. → 0.004.

Repeat the sequence with 0.1 × 2 and 0.01 × 6. Ask students to verbalize the patterns they notice.

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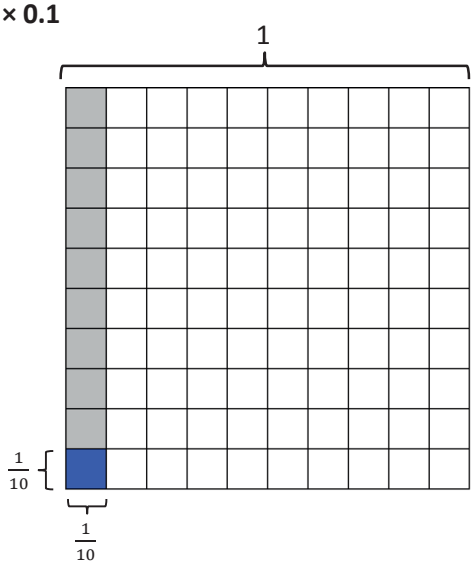
Problem 2: a. 0.1 × 0.1 b. 0.2 × 0.1 c. 1.2 × 0.1



T: (Post Problem 2(a) on the board.) Write this as a fraction multiplication sentence, and solve it with a partner.

S: (Write $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$.)

T: Let's draw an area model to see if this makes sense. What should I draw first? Turn and talk.

S: Draw a rectangle, and cut it vertically into 10 units, and shade one of them. (Draw and label $\frac{1}{10}$.)

T: What do I do next?

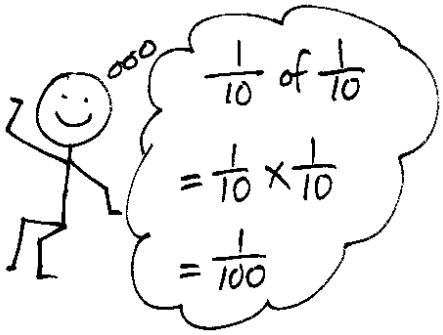
S: Cut each unit horizontally into 10 equal parts, and shade in 1 of those units.

T: (Cut and label $\frac{1}{10}$.) What units does our model show now?

MP.4

S: Hundredths.

T: Look at the double-shaded parts. What is $\frac{1}{10}$ of $\frac{1}{10}$? (Save this model for use again in Problem 3(a).)



S: 1 hundredth. → $\frac{1}{100}$.

T: Write the answer as a decimal.

S: 0.01.

T: Let's show this multiplication on the place value chart. When writing 1 tenth, where do we put the digit 1?

S: In the tenths place.

T: Turn and talk to your partner about what happened to the digit 1 that started in the tenths place when we took 1 tenth of it.

S: The digit shifted 1 place to the right. → We were taking a part of 1 tenth, so the answer is smaller than 1 tenth. It makes sense that the digit shifted to the right one place again because the answer got smaller.

T: (Post Problem 2(b) on the board.) Show me 2 tenths on your place value chart.

S: (Show the digit 2 in the tenths place.)

T: Explain to a partner what will happen to the digit 2 when you multiply it by 1 tenth.

S: Again, it will shift one place to the right. → Every time you multiply by a tenth, no matter what the digit, the value of the digit gets smaller. The 2 shifts one place over to the hundredths place.

T: Show this problem using fraction multiplication, and solve.

S: (Work and show $\frac{2}{10} \times \frac{1}{10} = \frac{2}{100}$.)

T: (Post Problem 2(c) on the board.) If we were to show this multiplication on the place value chart, visualize what would happen. Tell your partner what you see.

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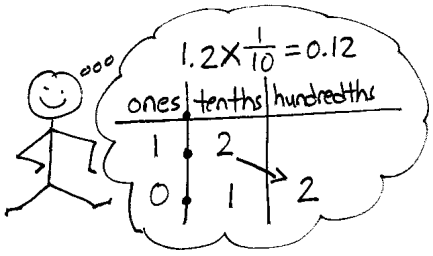
S: The 1 is in the ones place, and the 2 is in the tenths place. Both digits would shift one place to the right, so the 1 would be in the tenths place, and the 2 would be in the hundredths place. The answer would be 0.12. → Each digit shifts one place each. The answer is 12 hundredths.



T: How can we express 1.2 as a fraction greater than 1? Turn and talk.

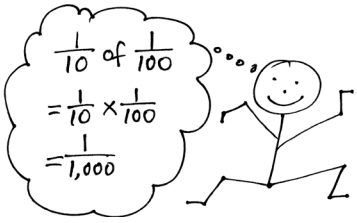
S: 1 and 2 tenths is the same as 12 tenths. 12 tenths as a fraction is just 12 over 10.

T: Show the solution to this problem using fraction multiplication.

S: (Work and show $\frac{12}{10} \times \frac{1}{10} = \frac{12}{100}$.)

Problem 3: a. 0.1 × 0.01 b. 0.5 × 0.01 c. 1.5 × 0.01

T: (Post Problem 3(a) on the board.) Work with a partner to show this as fraction multiplication.



S: (Work and show $\frac{1}{10} \times \frac{1}{100}$.)

T: What is $\frac{1}{10} \times \frac{1}{100}$?

S: $\frac{1}{1{,}000}$.

T: (Retrieve the model drawn in Problem 2(a).) Remember this model showed 1 tenth of 1 tenth, which is 1 hundredth. We just solved 1 tenth of 1 hundredth, which is 1 thousandth. Turn and talk with your partner about how that would look as a model.

S: If I had to draw it, I'd have to cut the whole vertically into 100 equal parts and just shade 1. Then, I'd have to cut just one of those tiny parts horizontally into 10 equal parts. If I did that to the rest of the parts, I'd end up with 1,000 equal parts, and only 1 of them would be double shaded! → It would be like taking that 1 tiny hundredth and dividing it into 10 parts to make thousandths. I'd need a really fine pencil point!

T: (Point to the tenths on the place value chart.) Put 1 tenth on the place value chart. I'm here in the tenths place, and I have to find 1 *hundredth* of this number. The digit 1 will shift in which direction, and why?

S: It will shift to the right because the product is smaller than what we started with.

T: How many places will it shift?

S: Two places.

T: Why two places? Turn and talk.

S: We shifted one place to the right when multiplying by a tenth, so it should be two places to the right when multiplying by a hundredth. Similarly, when we multiply by 10, the numbers shift one place to the left; they shift two places to the left when we multiply by 100. → Our model showed us that finding a hundredth of something is like finding a tenth of a tenth, so we have to shift one place two times.

T: Yes. (Move finger two places to the right to the thousandths place.) So, $\frac{1}{10} \times \frac{1}{100}$ is equal to $\frac{1}{1{,}000}$.

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T: (Post Problem 3(b) on the board.) Visualize a place value chart. When writing 0.5, where will the digit 5 be?

S: In the tenths place.

T: What will happen as we multiply by 1 hundredth?

S: The 5 will shift two places to the right to the thousandths place.

T: Say the answer.

S: 5 thousandths.

T: Show the solution to this problem using fraction multiplication.

S: (Write and solve $\frac{5}{10} \times \frac{1}{100} = \frac{5}{1{,}000}$.)

T: Show the answer as a decimal.

S: 0.005.

T: (Post Problem 3(c) on the board.) Express 1.5 as a fraction greater than 1.

S: $\frac{15}{10}$.

T: Show the solution to this problem using fraction multiplication.

S: (Write and show $\frac{15}{10} \times \frac{1}{100} = \frac{15}{1{,}000}$.)

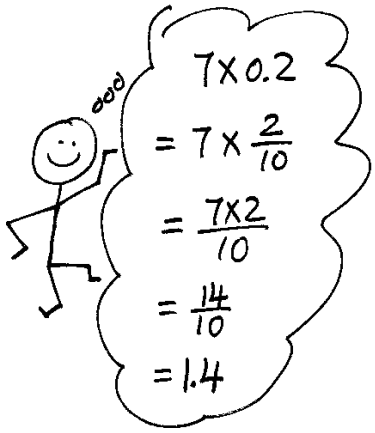
T: Write the answer as a decimal.

S: 0.015.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

It may be too taxing to ask some students to visualize a place value chart. As in previous problems, a place value chart can be displayed or provided. To provide further support for specific students, teachers can also provide place value disks.

Problem 4: a. 7 × 0.2 b. 0.7 × 0.2 c. 0.07 × 0.2



T: (Post Problem 4(a) on the board.) I'm going to rewrite this problem expressing the decimal as a fraction. (Write $7 \times \frac{2}{10}$.) Are these equivalent expressions? Turn and talk.

S: Yes. $0.2 = \frac{2}{10}$. So, they show the same thing.

T: When we multiply, what will the numerator show?

S: 7 × 2.

T: The denominator?

S: 10.

T: (Write $\frac{7 \times 2}{10}$.) Write the answer as a fraction.

S: (Write $\frac{14}{10}$.)

T: Write 14 tenths as a decimal.

S: (Write 1.4.)

T: Think about what we know about the place value chart and multiplying by tenths. Does our product make sense? Turn and talk.

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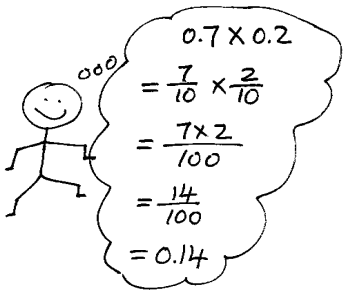
S: Sure! 7 times 2 is 14. So, 7 times 2 tenths is like 7 times 2 times 1 tenth. The answer should be one-tenth the size of 14. → It does make sense. It's like 7 times 2 equals 14, and then the digits in 14 both shift one place to the right because we took only 1 tenth of it. → I know it's like 2 tenths copied 7 times. Five copies of 2 tenths is 1, and then I added 2 more copies of 2 tenths.

T: (Post Problem 4(b) on the board.) Work with a partner, and show the solution using fraction multiplication.



S: (Write and solve $\frac{7}{10} \times \frac{2}{10} = \frac{14}{100}$.)

T: What's 14 hundredths as a decimal?

S: 0.14.

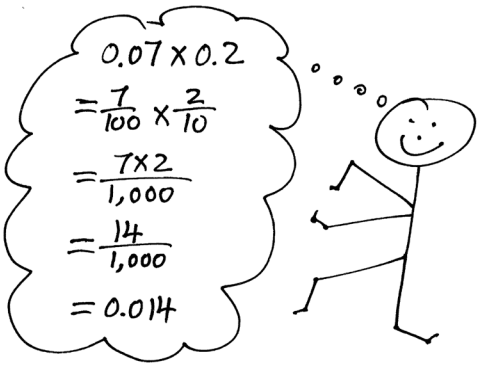
T: (Post Problem 4(c) on the board.) Solve this problem independently. Compare your answer with a partner when you're done. (Allow students time to work, and compare answers.)

T: Say the problem using fractions.

S: $\frac{7}{100} \times \frac{2}{10} = \frac{14}{1{,}000}$.

T: What's 14 thousandths as a decimal?

S: 0.014.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Teachers and parents alike may want to express multiplying by one-tenth as moving the decimal point one place to the left. Notice the instruction focuses on the movement of the digits in a number. Just like the ones place, the tens place, and all places on the place value chart, the decimal point does not move. It is in a fixed location separating the ones from the tenths.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

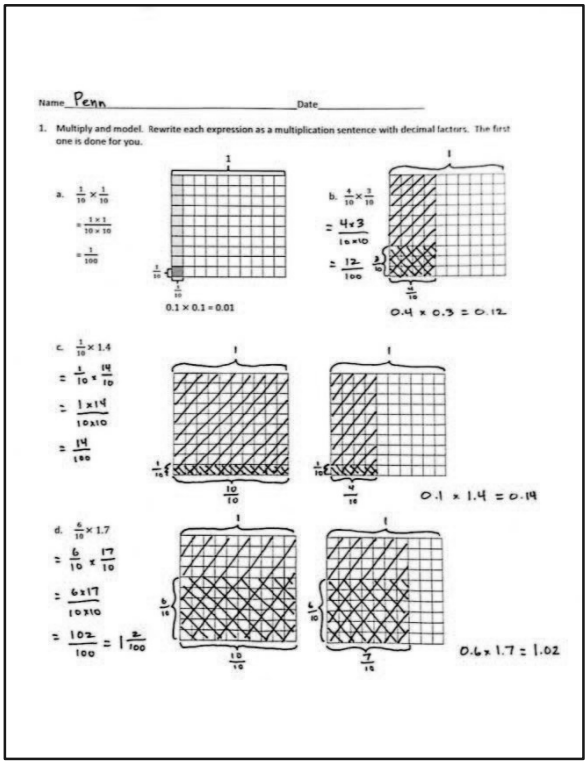
Lesson Objective: Relate decimal and fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 2, what pattern did you notice between (a), (b), and (c); (d), (e), and (f); and (g), (h), and (i)? (The product is to the tenths, hundredths, and thousandths.)
- Share and explain your solution to Problem 3 with a partner.
- Share your strategy for solving Problem 4 with a partner.
- Explain to your partner why $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ and $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1{,}000}$.
- We know that when we take one-tenth of 3, this shifts the digit 3 one place to the right on the place value chart because 3 tenths is 1 tenth of 3. How do the digits shift when we multiply by 1 tenth? By 1 hundredth?


Name Penn Date

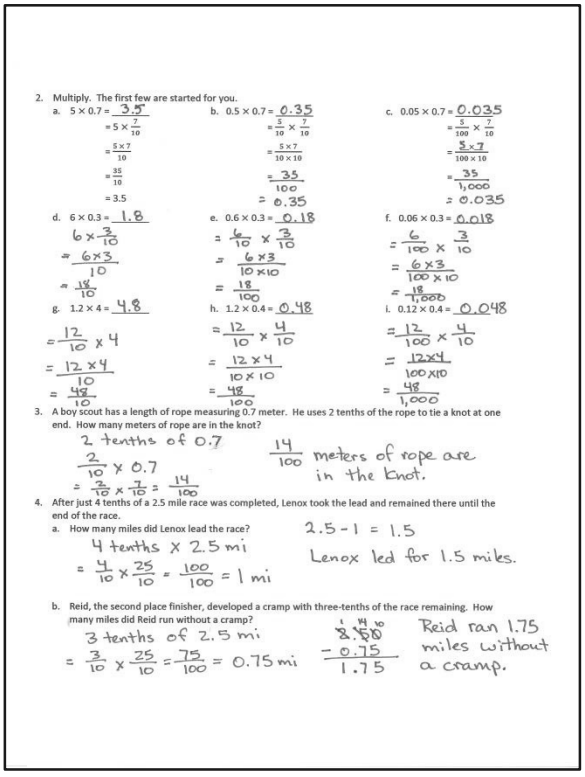
1. Multiply and model. Rewrite each expression as a multiplication sentence with decimal factors. The first one is done for you.

a. $\frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{1}{100}$ 0.1 × 0.1 = 0.01

b. $\frac{4}{10} \times \frac{3}{10} = \frac{4 \times 3}{10 \times 10} = \frac{12}{100}$ 0.4 × 0.3 = 0.12

c. $\frac{1}{10} \times 1.4 = \frac{1}{10} \times \frac{14}{10} = \frac{1 \times 14}{10 \times 10} = \frac{14}{100}$ 0.1 × 1.4 = 0.14

d. $\frac{6}{10} \times 1.7 = \frac{6}{10} \times \frac{17}{10} = \frac{6 \times 17}{10 \times 10} = \frac{102}{100} = 1\frac{2}{100}$ 0.6 × 1.7 = 1.02


2. Multiply. The first few are started for you.

a. 5 × 0.7 = 3.5 $= 5 \times \frac{7}{10} = \frac{5 \times 7}{10} = \frac{35}{10} = 3.5$

b. 0.5 × 0.7 = 0.35 $= \frac{5}{10} \times \frac{7}{10} = \frac{5 \times 7}{10 \times 10} = \frac{35}{100} = 0.35$

c. 0.05 × 0.7 = 0.035 $= \frac{5}{100} \times \frac{7}{10} = \frac{5 \times 7}{100 \times 10} = \frac{35}{1{,}000} = 0.035$

d. 6 × 0.3 = 1.8 $6 \times \frac{3}{10} = \frac{6 \times 3}{10} = \frac{18}{10}$

e. 0.6 × 0.3 = 0.18 $= \frac{6}{10} \times \frac{3}{10} = \frac{6 \times 3}{10 \times 10} = \frac{18}{100}$

f. 0.06 × 0.3 = 0.018 $= \frac{6}{100} \times \frac{3}{10} = \frac{6 \times 3}{100 \times 10} = \frac{18}{1{,}000}$

g. 1.2 × 4 = 4.8 $= \frac{12}{10} \times 4 = \frac{12 \times 4}{10} = \frac{48}{10}$

h. 1.2 × 0.4 = 0.48 $= \frac{12}{10} \times \frac{4}{10} = \frac{12 \times 4}{10 \times 10} = \frac{48}{100}$

i. 0.12 × 0.4 = 0.048 $= \frac{12}{100} \times \frac{4}{10} = \frac{12 \times 4}{100 \times 10} = \frac{48}{1{,}000}$

3. A boy scout has a length of rope measuring 0.7 meter. He uses 2 tenths of the rope to tie a knot at one end. How many meters of rope are in the knot?

2 tenths of 0.7 $\frac{2}{10} \times 0.7 = \frac{2}{10} \times \frac{7}{10} = \frac{14}{100}$ $\frac{14}{100}$ meters of rope are in the knot.

4. After just 4 tenths of a 2.5 mile race was completed, Lenox took the lead and remained there until the end of the race.

a. How many miles did Lenox lead the race?

4 tenths × 2.5 mi $= \frac{4}{10} \times \frac{25}{10} = \frac{100}{100} = 1$ mi

2.5 − 1 = 1.5 Lenox led for 1.5 miles.

b. Reid, the second place finisher, developed a cramp with three-tenths of the race remaining. How many miles did Reid run without a cramp?

3 tenths of 2.5 mi $= \frac{3}{10} \times \frac{25}{10} = \frac{75}{100} = 0.75$ mi

2.50 − 0.75 = 1.75 Reid ran 1.75 miles without a cramp.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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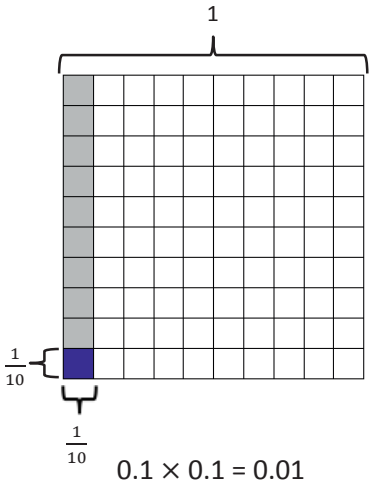
Name _____ Date _____

1. Multiply and model. Rewrite each expression as a multiplication sentence with decimal factors. The first one is done for you.

a. $\frac{1}{10} \times \frac{1}{10}$

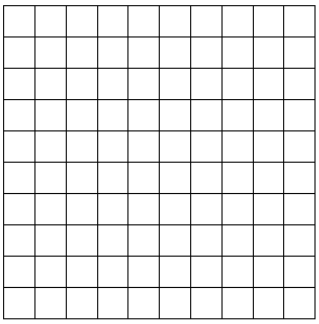
$= \frac{1 \times 1}{10 \times 10}$

$= \frac{1}{100}$

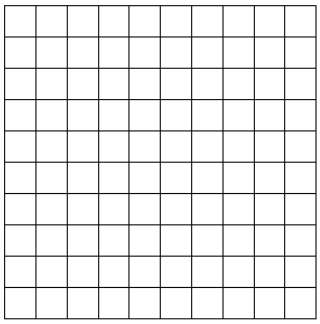


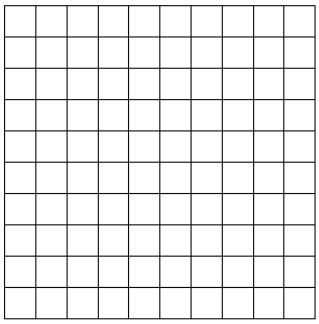
$0.1 \times 0.1 = 0.01$

b. $\frac{4}{10} \times \frac{3}{10}$

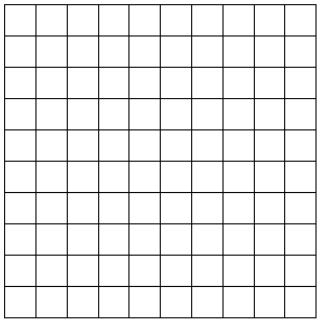


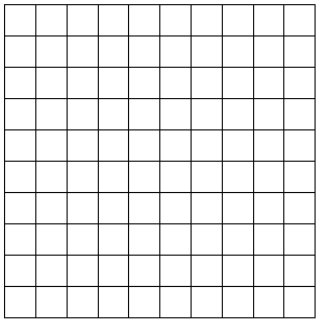
c. $\frac{1}{10} \times 1.4$





d. $\frac{6}{10} \times 1.7$





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2. Multiply. The first few are started for you.

a. $5 \times 0.7 =$ _____
$= 5 \times \frac{7}{10}$
$= \frac{5 \times 7}{10}$
$= \frac{35}{10}$
$= 3.5$

b. $0.5 \times 0.7 =$ _____
$= \frac{5}{10} \times \frac{7}{10}$
$= \frac{5 \times 7}{10 \times 10}$
$=$

c. $0.05 \times 0.7 =$ _____
$= \frac{5}{100} \times \frac{7}{10}$
$= \frac{__ \times __}{100 \times 10}$
$=$

d. $6 \times 0.3 =$ _____

e. $0.6 \times 0.3 =$ _____

f. $0.06 \times 0.3 =$ _____

g. $1.2 \times 4 =$ _____

h. $1.2 \times 0.4 =$ _____

i. $0.12 \times 0.4 =$ _____

3. A Boy Scout has a length of rope measuring 0.7 meter. He uses 2 tenths of the rope to tie a knot at one end. How many meters of rope are in the knot?

4. After just 4 tenths of a 2.5-mile race was completed, Lenox took the lead and remained there until the end of the race.

a. How many miles did Lenox lead the race?

b. Reid, the second-place finisher, developed a cramp with 3 tenths of the race remaining. How many miles did Reid run without a cramp?

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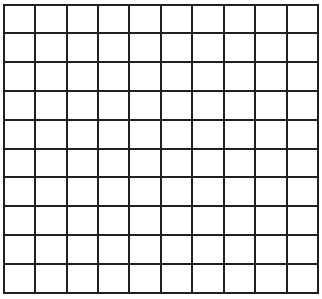
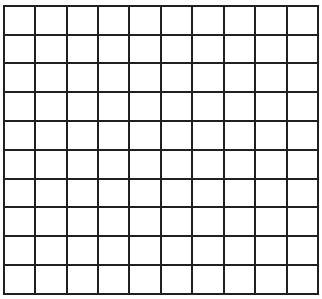
Name _____ Date _____

1. Multiply and model. Rewrite the expression as a number sentence with decimal factors.

$\frac{1}{10} \times 1.2$





2. Multiply.

a. $1.5 \times 3 =$ _____ b. $1.5 \times 0.3 =$ _____ c. $0.15 \times 0.3 =$ _____

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Name _____ Date _____

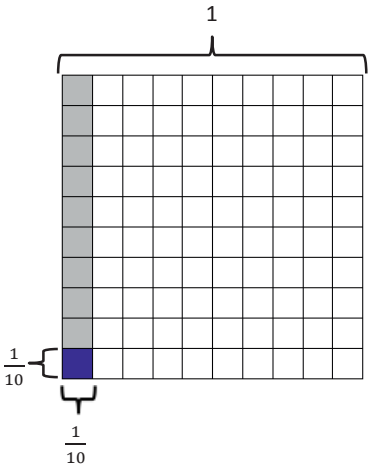
1. Multiply and model. Rewrite each expression as a number sentence with decimal factors. The first one is done for you.

a. $\frac{1}{10} \times \frac{1}{10}$

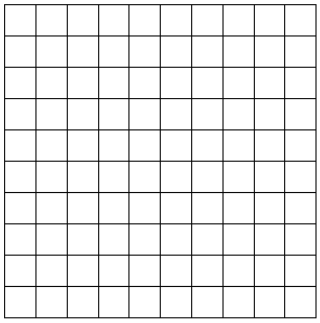
$= \frac{1 \times 1}{10 \times 10}$

$= \frac{1}{100}$

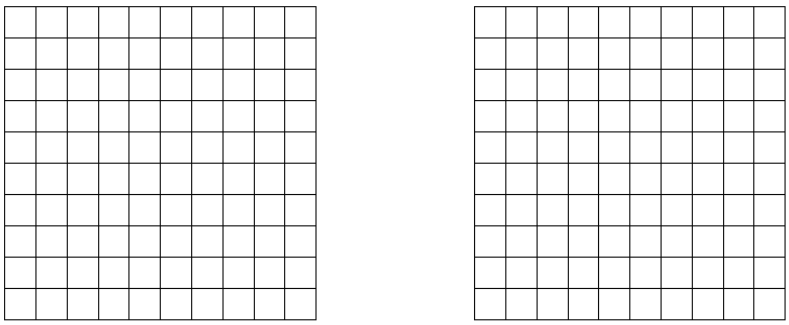
$0.1 \times 0.1 = 0.01$



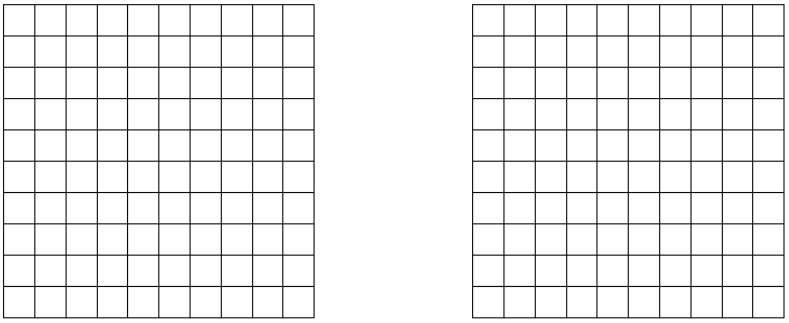
b. $\frac{6}{10} \times \frac{2}{10}$



c. $\frac{1}{10} \times 1.6$



d. $\frac{6}{10} \times 1.9$



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2. Multiply. The first few are started for you.

a. $4 \times 0.6 =$ _____

$= 4 \times \frac{6}{10}$

$= \frac{4 \times 6}{10}$

$= \frac{24}{10}$

$= 2.4$

b. $0.4 \times 0.6 =$ _____

$= \frac{4}{10} \times \frac{6}{10}$

$= \frac{4 \times 6}{10 \times 10}$

$=$

c. $0.04 \times 0.6 =$ _____

$= \frac{4}{100} \times \frac{6}{10}$

$= \frac{__ \times __}{100 \times 10}$

$=$

d. $7 \times 0.3 =$ _____

e. $0.7 \times 0.3 =$ _____

f. $0.07 \times 0.3 =$ _____

g. $1.3 \times 5 =$ _____

h. $1.3 \times 0.5 =$ _____

i. $0.13 \times 0.5 =$ _____

3. Jennifer makes 1.7 liters of lemonade. If she pours 3 tenths of the lemonade in the glass, how many liters of lemonade are in the glass?

4. Cassius walked 6 tenths of a 3.6-mile trail.

a. How many miles did Cassius have left to hike?

b. Cameron was 1.3 miles ahead of Cassius. How many miles did Cameron hike already?

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1,000,000	100,000	10,000	1,000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
							•			
							•			
							•			
							•			
							•			
							•			
							•			
							•			
							•			
							•			

millions through thousandths place value chart

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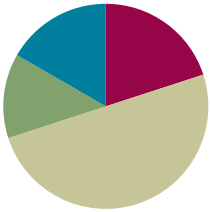
Lesson 18

Objective: Relate decimal and fraction multiplication.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Sprint: Multiply Fractions **5.NF.4** (8 minutes)
- Multiply Whole Numbers and Decimals **5.NBT.7** (4 minutes)

Sprint: Multiply Fractions (8 minutes)

Materials: (S) Multiply Fractions Sprint

Note: This fluency activity reviews Lesson 13.

Multiply Whole Numbers and Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 17.

3 × 2 = 6	3 × 0.2 = 0.6	0.3 × 0.2 = 0.06	0.03 × 0.2 = 0.006
2 × 7 = 14	2 × 0.7 = 1.4	0.2 × 0.7 = 0.14	0.02 × 0.7 = 0.014
5 × 3 = 15	0.5 × 3 = 1.5	0.5 × 0.3 = 0.15	0.5 × 0.03 = 0.015

T: (Write 3 × 2.) Say the number sentence with the answer.
S: 3 × 2 = 6.
T: (Write 3 × 0.2.) On your personal white board, write the number sentence, and solve.
S: (Write 3 × 0.2 = 0.6.)
T: (Write 0.3 × 0.2.) Try this problem.
S: (Write 0.3 × 0.2 = 0.06.)
T: (Write 0.03 × 0.2.) Try this problem.
S: (Write 0.03 × 0.2 = 0.006.)

Continue this process with the following possible sequence: 2 × 7, 2 × 0.7, 0.2 × 0.7, 0.02 × 0.7, 5 × 3, 0.5 × 3, 0.5 × 0.3, and 0.5 × 0.03.

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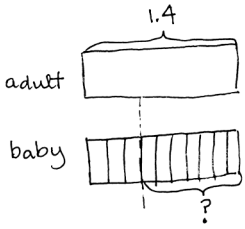
Application Problem (8 minutes)

An adult female gorilla is 1.4 meters tall when standing upright. Her daughter is 3 tenths as tall. How much more will the young female gorilla need to grow before she is as tall as her mother?



$\frac{3}{10} \times 1.4$

$= \frac{3}{10} \times \frac{14}{10}$

$= \frac{42}{100}$

1.40
- .42
0.98

The baby needs to grow 0.98 meters.

Note: This Application Problem reinforces that multiplying a decimal number by tenths can be interpreted in fraction or decimal form (as practiced in Lesson 17). Students who solve this problem by converting to smaller units (centimeters or millimeters) should be encouraged to compare their process to solving the problem using 1.4 meters.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

With reference to Table 2 of the Common Core Learning Standards, this Application Problem is classified as a *compare with unknown product* situation. Table 2 is a matrix that organizes story problems or situations into specific categories. Consider presenting this table in a student-friendly format as a tool to help students identify specific types of story problems.

Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1: a. 3.2 × 2.1 b. 3.2 × 0.44 c. 3.2 × 4.21

T: (Write 3.2 × 2.1 on the board.) Rewrite this problem as a fraction multiplication expression.

S: (Write $\frac{32}{10} \times \frac{21}{10}$.)

T: Before we multiply these two decimals, let's estimate what our product will be. Turn and talk.

S: 3.2 is pretty close to 3, and 2.1 is pretty close to 2. I'd say our answer will be around 6. → The product will be a little more than 6 because 3.2 is a little more than 3, and 2.1 is a little more than 2. → It's about twice as much as 3.

T: Now that we've estimated, let's solve. (Write $= \frac{32}{10} \times \frac{21}{10}$ on the board.) What do we get when we multiply tenths by tenths?

(a) $3.2 \times 2.1 = \frac{32}{10} \times \frac{21}{10}$

$= \frac{32 \times 21}{10 \times 10}$

$= \frac{672}{100}$

$= 6.72$

32 tenths × 21 tenths = 672 hundredths

= 6.72

32 tenths
× 21 tenths
32
640
672 hundredths
= 6.72

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S: Hundredths.

T: Let's use unit form to multiply 32 tenths and 21 tenths. Solve with your partner. (Allow students time to work and solve.)

T: (Write = $\frac{32 \times 21}{10 \times 10}$.) What is 32 tenths times 21 tenths?

S: 672 hundredths.

T: (Write = $\frac{672}{100}$.) Write this as a decimal.

S: (Write 6.72.)

T: Does this answer make sense given what we estimated the product to be earlier?

S: Yes.

T: (Write 3.2 × 0.44.) Before we solve this one, turn and talk with your partner to estimate the product.

S: We are still multiplying by 3.2, but this time, we want about 3 copies of almost 1 half. That's like 3 halves, so our answer will be around 1 and a half. → This is about 3 times more than 4 tenths, so the answer will be around 12 tenths. It will be a little more because 3.2 is a little more than 3 times as much.

T: Work with a partner, and rewrite this problem as a fraction multiplication expression.

S: (Work and show $\frac{32}{10} \times \frac{44}{100}$.)

T: What is 1 tenth of a hundredth?

S: 1 thousandth.

T: Work with a partner to multiply. Express your answer as a fraction and a decimal.

S: (Work and show $\frac{1{,}408}{1{,}000} = 1.408$.)

(b) $3.2 \times 0.44 = \frac{32}{10} \times \frac{44}{100}$

$= \frac{1{,}408}{1{,}000}$

$= 1.408$

32 tenths
× 44 hundredths
128
1280
1,408 thousandths

= 1.408

T: Does this product make sense given our estimates?

S: Yes! It's a little more than 1.2 and a little less than 1.5.

T: (Write 3.2 × 4.21 on the board.) Estimate this product with your partner.

S: Three times as much as 4 is 12. This will be a little more than that because 3.2 is a little more than 3, and 4.21 is a little more than 4. → It's still multiplying by something close to 3. This time, it's close to 4. 3 fours is 12.

T: Rewrite this problem as a fraction multiplication expression.

S: (Write $\frac{32}{10} \times \frac{421}{100}$.)

T: (Write = $\frac{32}{10} \times \frac{421}{100}$.) Solve independently. Express your answer as a fraction and a decimal.

S: (Write and solve $\frac{13{,}472}{1{,}000} = 13.472$.)

T: Does our answer make sense? Turn and talk. (Allow students time to discuss with their partners.)

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Problem 2: 2.6 × 0.4

T: (Write 2.6 × 0.4 on the board.) This time, let's rewrite this problem vertically in unit form first. 2.6 is equal to how many tenths?

S: 26 tenths.

T: (Write = 26 tenths.) 0.4 is equal to how many tenths?

S: 4 tenths.

T: (Write × 4 tenths.) Think. What does tenths times tenths result in?

S: Hundredths.

T: Our product will be named in hundredths. What is 26 times 4?

S: 104.

T: I'll record 104 as the product. (Write 104 in the algorithm.) 104 what? What is our unit?

S: 104 hundredths.

T: Write it in standard form.

S: (Write = 1.04.)

T: Use fraction multiplication to confirm our product. (Allow students time to work.)

MP.2

$2.6 \times 0.4 = \frac{26}{10} \times \frac{4}{10}$

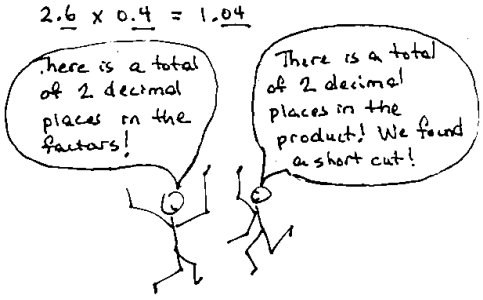
$= \frac{104}{100}$

$= 1.04$

26 tenths × 4 tenths = 104 hundredths

= 1.04

$2.6 \times 0.4 = 1.04$



T: What do you notice about the total number of decimal places in the factors and the number of decimal places in the product? Turn and talk.

S: There is one decimal place in each factor and two decimal places in the answer. → I see a total of two decimal places in the factors and two decimal places in the product. They match. → There is the same number of decimal places in both factors because there are decimal places in the product.

T: Keep this observation in mind as we continue our work. Let's see if it's always true.

Problem 3: a. 3.1 × 1.4 b. 0.31 × 1.4

T: (Write 3.1 × 1.4 on the board.) Estimate the product with your partner.

S: It should be something close to 3 because 3 times 1 is 3. → Something between 3 and 6 because 1.4 is close to the midpoint of 1 and 2. → It's close to 3 times 1 and a half. That's 4 and a half.

T: Let's use unit form with tenths to solve this. What will my multiplication expression be?

S: 31 tenths × 14 tenths.

T: What is the product?

S: 434 hundredths.

T: How do you know the unit is hundredths?

Unit form

(a) 3.1 × 1.4 = 31 tenths × 14 tenths

= 434 hundredths

= 4.34

Fraction form

$\frac{31}{10} \times \frac{14}{10} = \frac{434}{100}$

$= 4.34$

Short cut!

3.1 × 1.4 = 4.34

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S: Tenths times tenths is hundredths. → I could say 1 tenth of a tenth. That's 1 hundredth. → I can picture it on the place value chart.

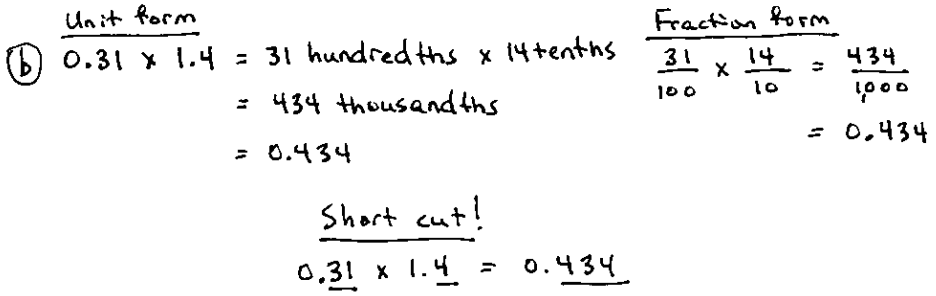
T: Name the product as a decimal.

S: 4 and 34 hundredths. → 4.34.

T: What do you notice about the decimal places in the factors and the product this time?

S: This is like before. We have a total of two decimal places in the factors and two decimal places in the answer. → We multiplied tenths times tenths and got hundredths in our answer. That's two decimal places. It's just like last time.

T: (Write 0.31 × 1.4 on the board.) Let's think of 0.31 and 1.4 as whole numbers of units. 0.31 is the same as 31 what? 1.4 is the same as 14 what?



S: 31 hundredths and 14 tenths.

T: If we were using fractions to multiply these two numbers, is 31 × 14 the numerator or denominator?

S: The numerator.

T: What does the numerator of a fraction tell us?

S: The number of units we have.

T: This whole number multiplication problem is the same as our last one. What is 31 times 14?

S: 434.

T: While these digits are the same as last time, will our product be the same? Why or why not? Turn and talk.

S: We are multiplying hundredths and tenths this time, so our unit in the answer has to be thousandths. The answer is 434 thousandths. → Last time, we had a total of two decimal places in our factors, so we had two decimal places in our product. This time, there is a total of three decimal places in the factors, so we should have thousandths in the answer. → Last time, we were multiplying by about 3 times as much as 1 and a half. This time, we want about 3 tenths of 1 and a half. So, our answer is going to be a lot smaller.

T: What is our product?

S: 434 thousandths.

T: Yes. Remember that 1 hundredth times 1 tenth gives us the unit of 1 thousandth, the denominator of our fraction.

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Problem 4: 4.2 × 0.12

T: (Write 4.2 × 0.12 on the board.) Work independently to solve this problem using either unit form or fraction form. When you're done, count the decimal places in both factors and decimal places in the product. Compare your work with a neighbor. Did you get the same answer with your chosen method as you did when counting the digits?

S: (Work and discuss.) Yes!

T: What is the product of 4.2 × 0.12?

S: 0.504.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Another strategy for multiplying decimal numbers is compensation. Here's an example.

2.3 × 4.5

(2.3 × **10**) × (4.5 × **10**) = 23 × 45 = 1,035

1,035 ÷ **10** ÷ **10** = 10.35

2.3 × 4.5 = 10.35

Each decimal factor was multiplied by a multiple of ten resulting in a whole number. To solve the original decimal multiplication, the whole number product had to be divided by the same number of multiples of 10.

Student Debrief (10 minutes)

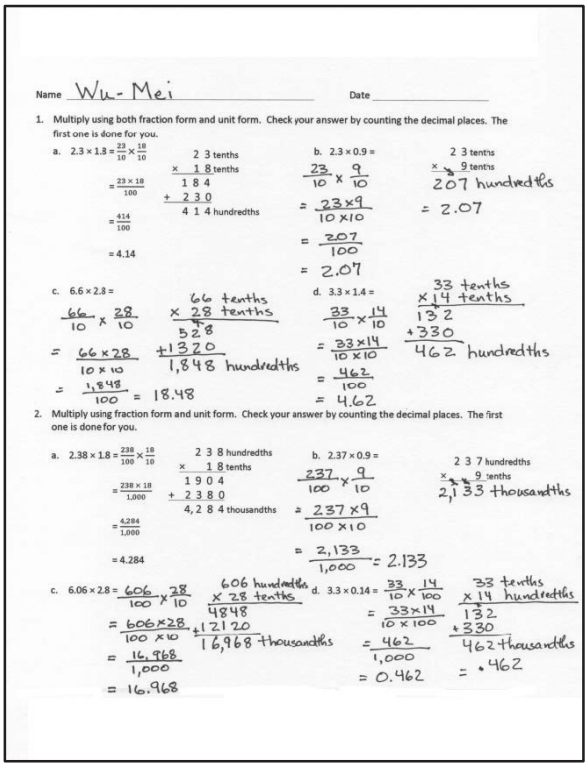
Lesson Objective: Relate decimal and fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what is the relationship between the answers for Parts (a) and (b) and the answers for Parts (c) and (d)? What pattern did you notice between 1(a) and 1(b)? (Part (a) is double Part (b). Part (c) is 4 times as large as Part (d).) Explain why that is the case.


Name Wu-Mei Date

1. Multiply using both fraction form and unit form. Check your answer by counting the decimal places. The first one is done for you.

a. 2.3 × 1.8 = 23/10 × 18/10 = 23 × 18/100 = 414/100 = 4.14
2 3 tenths × 1 8 tenths; 1 8 4 + 2 3 0; 4 1 4 hundredths

b. 2.3 × 0.9 = 23/10 × 9/10 = 23×9/10×10 = 207/100 = 2.07
2 3 tenths × 9 tenths; 207 hundredths = 2.07

c. 6.6 × 2.8 = 66/10 × 28/10 = 66×28/10×10 = 1,848/100 = 18.48
66 tenths × 28 tenths; 528 + 1320; 1,848 hundredths

d. 3.3 × 1.4 = 33/10 × 14/10 = 33×14/10×10 = 462/100 = 4.62
33 tenths × 14 tenths; 132 + 330; 462 hundredths

2. Multiply using fraction form and unit form. Check your answer by counting the decimal places. The first one is done for you.

a. 2.38 × 1.8 = 238/100 × 18/10 = 238 × 18/1,000 = 4,284/1,000 = 4.284
2 3 8 hundredths × 1 8 tenths; 1 9 0 4 + 2 3 8 0; 4,2 8 4 thousandths

b. 2.37 × 0.9 = 237/100 × 9/10 = 237×9/100×10 = 2,133/1,000 = 2.133
2 3 7 hundredths × 9 tenths; 2,133 thousandths

c. 6.06 × 2.8 = 606/100 × 28/10 = 606×28/100×10 = 16,968/1,000 = 16.968
606 hundredths × 28 tenths; 4848 + 12120; 16,968 thousandths

d. 3.3 × 0.14 = 33/10 × 14/100 = 33×14/10×100 = 462/1,000 = 0.462
33 tenths × 14 hundredths; 132 + 330; 462 thousandths = .462

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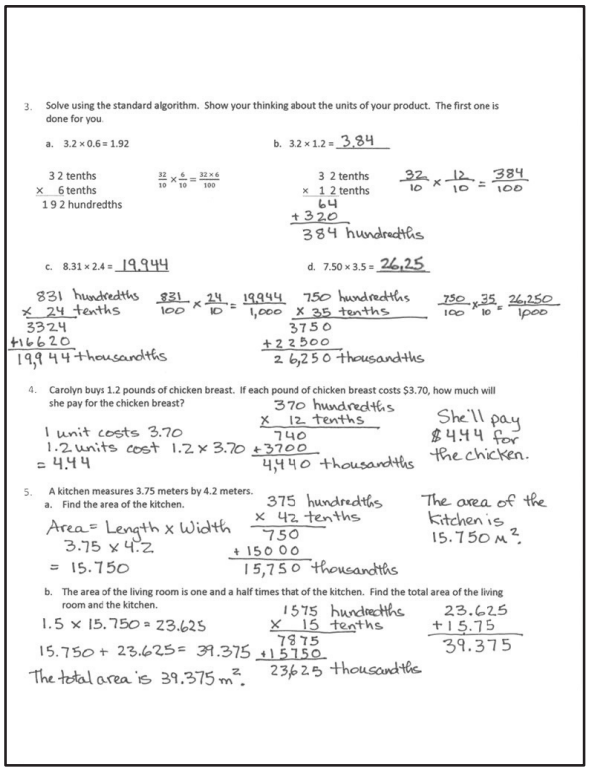
- Compare Problems 1(c) and 2(c). Why are the products not so different? Use estimation, and explain it to your partner.
- Compare Problems 1(d) and 2(d). Why do they have the same digits but a different product? Explain it to your partner.
- What do you notice about the relationship between 3(a) and 3(b)? (Part (a) is half of Part (b).)
- For Problem 5, compare and share your solutions with a partner. Explain how you solved.
- In one sentence, explain to your partner the pattern that we discovered today in the number of decimal places in our factors compared to the number of decimal places in our products.


3. Solve using the standard algorithm. Show your thinking about the units of your product. The first one is done for you.

a. $3.2 \times 0.6 = 1.92$

3 2 tenths × 6 tenths = 1 9 2 hundredths

$\frac{32}{10} \times \frac{6}{10} = \frac{32 \times 6}{100}$

b. $3.2 \times 1.2 = 3.84$

3 2 tenths × 1 2 tenths
64
+320
384 hundredths

$\frac{32}{10} \times \frac{12}{10} = \frac{384}{100}$

c. $8.31 \times 2.4 = 19.944$

831 hundredths × 24 tenths
3324
+16620
19,944 thousandths

$\frac{831}{100} \times \frac{24}{10} = \frac{19944}{1,000}$

d. $7.50 \times 3.5 = 26.25$

750 hundredths × 35 tenths
3750
+22500
26,250 thousandths

$\frac{750}{100} \times \frac{35}{10} = \frac{26,250}{1000}$

4. Carolyn buys 1.2 pounds of chicken breast. If each pound of chicken breast costs $3.70, how much will she pay for the chicken breast?

1 unit costs 3.70
1.2 units cost $1.2 \times 3.70 = 4.44$

370 hundredths × 12 tenths
740
+3700
4440 thousandths

She'll pay $4.44 for the chicken.

5. A kitchen measures 3.75 meters by 4.2 meters.

a. Find the area of the kitchen.

Area = Length x Width
$3.75 \times 4.2 = 15.750$

375 hundredths × 42 tenths
750
+15000
15,750 thousandths

The area of the kitchen is $15.750\ m^2$.

b. The area of the living room is one and a half times that of the kitchen. Find the total area of the living room and the kitchen.

$1.5 \times 15.750 = 23.625$
$15.750 + 23.625 = 39.375$
The total area is $39.375\ m^2$.

1575 hundredths × 15 tenths
7875
+15750
23625 thousandths

23.625
+15.75
39.375

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Number Correct: _____

A

Multiply Fractions

1.	$\frac{1}{2} \times \frac{1}{2} =$	
2.	$\frac{1}{2} \times \frac{1}{3} =$	
3.	$\frac{1}{2} \times \frac{1}{4} =$	
4.	$\frac{1}{2} \times \frac{1}{7} =$	
5.	$\frac{1}{7} \times \frac{1}{2} =$	
6.	$\frac{1}{3} \times \frac{1}{2} =$	
7.	$\frac{1}{3} \times \frac{1}{3} =$	
8.	$\frac{1}{3} \times \frac{1}{6} =$	
9.	$\frac{1}{3} \times \frac{1}{5} =$	
10.	$\frac{1}{5} \times \frac{1}{3} =$	
11.	$\frac{1}{5} \times \frac{2}{3} =$	
12.	$\frac{2}{5} \times \frac{2}{3} =$	
13.	$\frac{1}{4} \times \frac{1}{3} =$	
14.	$\frac{1}{4} \times \frac{2}{3} =$	
15.	$\frac{3}{4} \times \frac{2}{3} =$	
16.	$\frac{1}{6} \times \frac{1}{3} =$	
17.	$\frac{5}{6} \times \frac{1}{3} =$	
18.	$\frac{5}{6} \times \frac{2}{3} =$	
19.	$\frac{5}{4} \times \frac{2}{3} =$	
20.	$\frac{1}{5} \times \frac{1}{5} =$	
21.	$\frac{2}{5} \times \frac{2}{5} =$	
22.	$\frac{2}{5} \times \frac{3}{5} =$	
23.	$\frac{2}{5} \times \frac{5}{3} =$	
24.	$\frac{3}{5} \times \frac{5}{2} =$	
25.	$\frac{1}{3} \times \frac{1}{3} =$	
26.	$\frac{1}{3} \times \frac{2}{3} =$	
27.	$\frac{2}{3} \times \frac{2}{3} =$	
28.	$\frac{2}{3} \times \frac{3}{2} =$	
29.	$\frac{2}{3} \times \frac{4}{3} =$	
30.	$\frac{2}{3} \times \frac{5}{3} =$	
31.	$\frac{3}{2} \times \frac{3}{5} =$	
32.	$\frac{3}{4} \times \frac{1}{5} =$	
33.	$\frac{3}{4} \times \frac{4}{5} =$	
34.	$\frac{3}{4} \times \frac{5}{5} =$	
35.	$\frac{3}{4} \times \frac{6}{5} =$	
36.	$\frac{1}{4} \times \frac{6}{5} =$	
37.	$\frac{1}{7} \times \frac{1}{7} =$	
38.	$\frac{1}{8} \times \frac{3}{5} =$	
39.	$\frac{5}{6} \times \frac{1}{4} =$	
40.	$\frac{3}{4} \times \frac{3}{4} =$	
41.	$\frac{2}{3} \times \frac{6}{6} =$	
42.	$\frac{3}{4} \times \frac{6}{2} =$	
43.	$\frac{7}{8} \times \frac{7}{9} =$	
44.	$\frac{7}{12} \times \frac{9}{8} =$	

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B

Number Correct: _____

Improvement: _____

Multiply Fractions

1.	$\frac{1}{2} \times \frac{1}{3} =$	
2.	$\frac{1}{2} \times \frac{1}{4} =$	
3.	$\frac{1}{2} \times \frac{1}{5} =$	
4.	$\frac{1}{2} \times \frac{1}{9} =$	
5.	$\frac{1}{9} \times \frac{1}{2} =$	
6.	$\frac{1}{5} \times \frac{1}{2} =$	
7.	$\frac{1}{5} \times \frac{1}{3} =$	
8.	$\frac{1}{5} \times \frac{1}{7} =$	
9.	$\frac{1}{5} \times \frac{1}{3} =$	
10.	$\frac{1}{3} \times \frac{1}{5} =$	
11.	$\frac{1}{3} \times \frac{2}{5} =$	
12.	$\frac{2}{3} \times \frac{2}{5} =$	
13.	$\frac{1}{3} \times \frac{1}{4} =$	
14.	$\frac{1}{3} \times \frac{3}{4} =$	
15.	$\frac{2}{3} \times \frac{3}{4} =$	
16.	$\frac{1}{3} \times \frac{1}{6} =$	
17.	$\frac{2}{3} \times \frac{1}{6} =$	
18.	$\frac{2}{3} \times \frac{5}{6} =$	
19.	$\frac{3}{2} \times \frac{3}{4} =$	
20.	$\frac{1}{5} \times \frac{1}{5} =$	
21.	$\frac{3}{5} \times \frac{3}{5} =$	
22.	$\frac{3}{5} \times \frac{4}{5} =$	
23.	$\frac{3}{5} \times \frac{5}{4} =$	
24.	$\frac{4}{5} \times \frac{5}{3} =$	
25.	$\frac{1}{4} \times \frac{1}{4} =$	
26.	$\frac{1}{4} \times \frac{3}{4} =$	
27.	$\frac{3}{4} \times \frac{3}{4} =$	
28.	$\frac{3}{4} \times \frac{4}{3} =$	
29.	$\frac{3}{4} \times \frac{5}{4} =$	
30.	$\frac{3}{4} \times \frac{6}{4} =$	
31.	$\frac{4}{3} \times \frac{4}{6} =$	
32.	$\frac{2}{3} \times \frac{1}{5} =$	
33.	$\frac{2}{3} \times \frac{4}{5} =$	
34.	$\frac{2}{3} \times \frac{5}{5} =$	
35.	$\frac{2}{3} \times \frac{6}{5} =$	
36.	$\frac{1}{3} \times \frac{6}{5} =$	
37.	$\frac{1}{9} \times \frac{1}{9} =$	
38.	$\frac{1}{5} \times \frac{3}{8} =$	
39.	$\frac{3}{4} \times \frac{1}{6} =$	
40.	$\frac{2}{3} \times \frac{2}{3} =$	
41.	$\frac{3}{4} \times \frac{8}{8} =$	
42.	$\frac{2}{3} \times \frac{6}{3} =$	
43.	$\frac{6}{7} \times \frac{8}{9} =$	
44.	$\frac{7}{12} \times \frac{8}{7} =$	

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Name _____ Date _____

1. Multiply using both fraction form and unit form. Check your answer by counting the decimal places. The first one is done for you.

a. $2.3 \times 1.8 = \frac{23}{10} \times \frac{18}{10}$

$= \frac{23 \times 18}{100}$

$= \frac{414}{100}$

$= 4.14$

2 3 tenths
× 1 8 tenths
1 8 4
+ 2 3 0
4 1 4 hundredths

b. $2.3 \times 0.9 =$

2 3 tenths
× 9 tenths

c. $6.6 \times 2.8 =$

d. $3.3 \times 1.4 =$

2. Multiply using fraction form and unit form. Check your answer by counting the decimal places. The first one is done for you.

a. $2.38 \times 1.8 = \frac{238}{100} \times \frac{18}{10}$

$= \frac{238 \times 18}{1{,}000}$

$= \frac{4{,}284}{1{,}000}$

$= 4.284$

2 3 8 hundredths
× 1 8 tenths
1 9 0 4
+ 2 3 8 0
4, 2 8 4 thousandths

b. $2.37 \times 0.9 =$

2 3 7 hundredths
× 9 tenths

c. $6.06 \times 2.8 =$

d. $3.3 \times 0.14 =$

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3. Solve using the standard algorithm. Show your thinking about the units of your product. The first one is done for you.

a. 3.2 × 0.6 = 1.92

3 2 tenths
× 6 tenths
1 9 2 hundredths

$\frac{32}{10} \times \frac{6}{10} = \frac{32 \times 6}{100}$

b. 3.2 × 1.2 = _____

3 2 tenths
× 1 2 tenths

c. 8.31 × 2.4 = _____

d. 7.50 × 3.5 = _____

4. Carolyn buys 1.2 pounds of chicken breast. If each pound of chicken breast costs $3.70, how much will she pay for the chicken breast?

5. A kitchen measures 3.75 meters by 4.2 meters.

a. Find the area of the kitchen.

b. The area of the living room is one and a half times that of the kitchen. Find the total area of the living room and the kitchen.

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Name _____ Date _____

Multiply. Do at least one problem using unit form and at least one problem using fraction form.

a. $3.2 \times 1.4 =$

b. $1.6 \times 0.7 =$

c. $2.02 \times 4.2 =$

d. $2.2 \times 0.42 =$

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Name _____ Date _____

1. Multiply using fraction form and unit form. Check your answer by counting the decimal places. The first one is done for you.

a. $3.3 \times 1.6 = \frac{33}{10} \times \frac{16}{10}$

$= \frac{33 \times 16}{100}$

$= \frac{528}{100}$

$= 5.28$

3 3 tenths
× 1 6 tenths
1 9 8
+ 3 3 0
5 2 8 hundredths

b. $3.3 \times 0.8 =$

3 3 tenths
× 8 tenths

c. $4.4 \times 3.2 =$

d. $2.2 \times 1.6 =$

2. Multiply using fraction form and unit form. The first one is partially done for you.

a. $3.36 \times 1.4 = \frac{336}{100} \times \frac{14}{10}$

$= \frac{336 \times 14}{1{,}000}$

$= \frac{4{,}704}{1{,}000}$

$= 4.704$

3 3 6 hundredths
× 1 4 tenths

b. $3.35 \times 0.7 =$

3 3 5 hundredths
× 7 tenths

c. $4.04 \times 3.2 =$

d. $4.4 \times 0.16 =$

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3. Solve using the standard algorithm. Show your thinking about the units of your product. The first one is done for you.

a. 3.2 × 0.6 = 1.92

3 2 tenths
× 6 tenths
1 9 2 hundredths

$$\frac{32}{10} \times \frac{6}{10} = \frac{32 \times 6}{100}$$

b. 2.3 × 2.1 = _____

2 3 tenths
× 2 1 tenths

c. 7.41 × 3.4 = _____

d. 6.50 × 4.5 = _____

4. Erik buys 2.5 pounds of cashews. If each pound of cashews costs $7.70, how much will he pay for the cashews?

5. A swimming pool at a park measures 9.75 meters by 7.2 meters.

a. Find the area of the swimming pool.

b. The area of the playground is one and a half times that of the swimming pool. Find the total area of the swimming pool and the playground.

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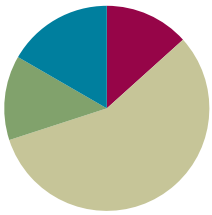
Lesson 19

Objective: Convert measures involving whole numbers, and solve multi-step word problems.

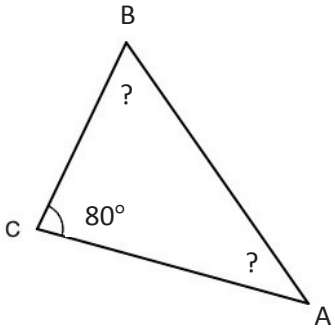
Suggested Lesson Structure

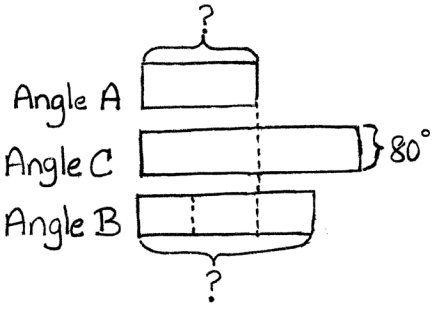


Application Problem	(8 minutes)
Fluency Practice	(8 minutes)
Concept Development	(34 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Application Problem (8 minutes)

Angle A of a triangle is $\frac{1}{2}$ the size of angle C. Angle B is $\frac{3}{4}$ the size of angle C. If angle C measures 80 degrees, what are the measures of angle A and angle B?





$\frac{1}{2} \times 80° = 40°$

$\frac{3}{4} \times 80° = 60°$

Angle A measures 40°.
Angle B measures 60°.

Note: Because today's fluency activity asks students to recall the content of Lesson 18, this Application Problem asks students to recall previous learning to find a fraction of a set. The presence of a third angle increases complexity.

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Fluency Practice (8 minutes)

- Multiply Decimals **5.NBT.7** (4 minutes)
- Convert Measures **4.MD.1** (4 minutes)

Multiply Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 17 and 18.

4 × 2 = 8	4 × 0.2 = 0.8	0.4 × 0.2 = 0.08	0.04 × 0.2 = 0.008
2 × 9 = 18	2 × 0.9 = 1.8	0.2 × 0.9 = 0.18	0.02 × 0.9 = 0.018
4 × 3 = 12	0.4 × 3 = 1.2	0.4 × 0.3 = 0.12	0.4 × 0.03 = 0.012

T: (Write 4 × 2 = ____.) Say the number sentence with the answer.
S: 4 × 2 = 8.
T: (Write 4 × 0.2 = ____.) On your personal white board, write the number sentence and the answer.
S: (Write 4 × 0.2 = 0.8.)
T: (Write 0.4 × 0.2 = ____.) Try this problem.
S: (Write 0.4 × 0.2 = 0.08.)

Continue this process with the following possible sequence: 2 × 9, 2 × 0.9, 0.2 × 0.9, 0.02 × 0.9, 4 × 3, 0.4 × 3, 0.4 × 0.3, and 0.4 × 0.03.

Convert Measures (4 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency activity prepares students for Lesson 19. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without looking at it.

T: (Write 1 yd = ____ ft.) How many feet are equal to 1 yard?
S: 3 feet.
T: (Write 1 yd = 3 ft. Below it, write 10 yd = ____ ft.) 10 yards?
S: 30 feet.

Continue with the following possible sequence: 1 pint = 2 cups, 8 pints = 16 cups, 1 ft = 12 in, 4 ft = 48 in, 1 gal = 4 qt, and 8 gal = 32 qt.

T: (Write 2 c = ____ pt.) How many pints are equal to 2 cups?
S: 1 pint.
T: (Write 2 c = 1 pt. Below it, write 16 c = ____ pt.) 16 cups?
S: 8 pints.

Continue with the following possible sequence: 12 in = 1 ft, 48 in = 4 ft, 3 ft = 1 yd, 24 ft = 8 yd, 4 qt = 1 gal, and 24 qt = 6 gal.

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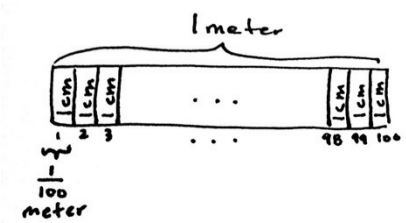
Concept Development (34 minutes)

Materials: (S) Personal white board

Problem 1: 30 centimeters = _____ meters



$30 \text{ cm} = 30 \times 1\text{cm}$
$= 30 \times \frac{1}{100}\text{m}$
$= \frac{30}{100}\text{m}$
$= \frac{3}{10}\text{m}$

T: (Post Problem 1 on the board.) Which is a larger unit, centimeters or meters?

S: Meters.

T: So, we are expressing a smaller unit in terms of a larger unit. Is 30 centimeters more or less than 1 meter?

S: Less than 1 meter.

T: Is it more than or less than half a meter? Talk to your partner about how you know.

S: It's less than half a meter because 50 cm is half a meter, and this is only 30 cm. → It's less than half a meter because 30 out of a hundred is less than half.

T: Let's keep that in mind as we work. We want to rename these centimeters using meters.

T: (Write 30 cm = 30 × 1 cm.) We know that 30 cm is the same as 30 copies of 1 cm. Let's rename 1 cm as a fraction of a meter. What fraction of a meter is 1 cm? Turn and talk.

S: It takes 100 cm to make a meter, so 1 cm would be 1 hundredth of a meter. → 100 cm = 1 meter, so 1 cm = $\frac{1}{100}$ meter. → 100 out of 100 cm makes 1 whole meter. We're looking at 1 out of 100 cm, so that is 1 hundredth of a meter.

T: (Write = $30 \times \frac{1}{100}$ m.) How do you know this is true?

S: It's true because we just renamed the centimeter as the same amount in meters. → One centimeter is the same thing as 1 hundredth of a meter.

T: Now, we have 30 copies of $\frac{1}{100}$ meter. How many hundredths of a meter is that in all?

S: 30 hundredths of a meter.

T: Write it as a fraction on your personal white board, and then work with a neighbor to express it in simplest form.

S: (Work.)

T: Answer the question in simplest form.

S: 30 cm = $\frac{3}{10}$ m.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide students with meter sticks and centimeter rulers to help answer these important first questions of this Concept Development. These tools help students see the relationship of centimeters to meters and meters to centimeters.

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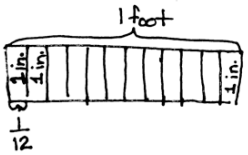
T: (Write = $\frac{3}{10}$ m.) Think about our estimate. Does this answer make sense?

S: Yes. We thought it would be less than a half meter, and $\frac{3}{10}$ meter is less than half a meter.

Problem 2: 9 inches = _____ foot

T: (Write 9 inches = 9 × 1 inch on the board.) 9 inches is 9 copies of 1 inch. What fraction of a foot is 1 inch? Draw a tape diagram if it helps you.



S: 1 twelfth of a foot.

T: Before we rename 1 inch, let's estimate. Will 9 inches be more than half a foot or less than half a foot? Turn and tell your partner how you know.

S: Half a foot is 6 inches. Nine inches is more than 6 inches, so it will be more than half a foot.

T: (Write = 9 × $\frac{1}{12}$ foot.) Let's rename 1 inch as a fraction of a foot. Now, we have written 9 copies of $\frac{1}{12}$ foot. Are these expressions equivalent?

S: Yes.

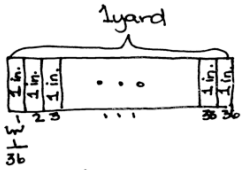
T: Multiply. How many feet is the same amount as 9 inches?

S: 9 twelfths of a foot. → 3 fourths of a foot.

T: Does this answer make sense? Turn and talk.

9 inches = 9 × 1 inch
= 9 × $\frac{1}{12}$ foot
= $\frac{9}{12}$ ft or $\frac{3}{4}$ ft

Repeat sequence for 24 inches = _____ yard.



24 inches = 24 × 1 inch.
= 24 × $\frac{1}{36}$ yd.
= $\frac{24}{36}$ yard or $\frac{2}{3}$ yard

Problem 3:

Koalas will often sleep for 20 hours a day. For what fraction of a day does a koala often sleep?

T: (Post Problem 3 on the board.) What will we need to do to solve this problem? Turn and talk.

S: We'll need to express hours in days. → We'll need to convert 20 hours into a fraction of a day.

T: Work with a partner to solve. Express your answer in its simplest form.

S: (Work, share, and show 20 hours = $\frac{5}{6}$ day.)

20 hours = 20 × 1 hour
= 20 × $\frac{1}{24}$ day
= $\frac{5}{\cancel{20} \times 1}$ day
$\cancel{24}_{6}$
= $\frac{5}{6}$ day

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Problem 4: 15 inches = _____ feet

T: (Post Problem 4 on the board.) Compare this conversion to the others we've done. Turn and talk.

S: We're still converting from a smaller unit to a larger unit. → The last one converted something smaller than a whole day. This is converting something more than a whole foot. Fifteen inches is more than a foot, so our answer will be greater than 1. → We still have to think about what fraction of a foot is 1 inch.

T: Yes. The process of converting will be the same, but our answer will be greater than 1. Let's keep that in mind as we work. Write an equation showing how many copies of 1 inch we have.

S: (Work and show 15 inches = 15 × 1 inch.)

T: What fraction of a foot is 1 inch? Turn and talk.

S: It takes 12 inches to make a foot, so 1 inch would be 1 twelfth of a foot. → 12 inches = 1 foot, so 1 inch = $\frac{1}{12}$ foot.

T: Now, we have 15 copies of $\frac{1}{12}$ foot. How many twelfths of a foot is that in all?

S: $\frac{15}{12}$ feet.

T: Work with a neighbor to express $\frac{15}{12}$ in its simplest form.

S: (Work and show 15 inches = $1\frac{1}{4}$ feet.)

15 inches = 15 × 1 inch
= 15 × $\frac{1}{12}$ feet
= $\frac{15}{12}$ feet
= $1\frac{3}{12}$ feet
= $1\frac{1}{4}$ feet

Problem 5: 24 ounces = _____ pounds

T: (Post Problem 5 on the board.) Work independently to solve this conversion problem.

S: (Work.)

T: Show the conversion in its simplest form.

S: (Show 24 ounces = $1\frac{1}{2}$ pounds.)

24 ounces = 24 × 1 ounce
= 24 × $\frac{1}{16}$ Pound
= $\frac{24}{16}$ pounds
= $1\frac{8}{16}$ pounds
= $1\frac{1}{2}$ pounds

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

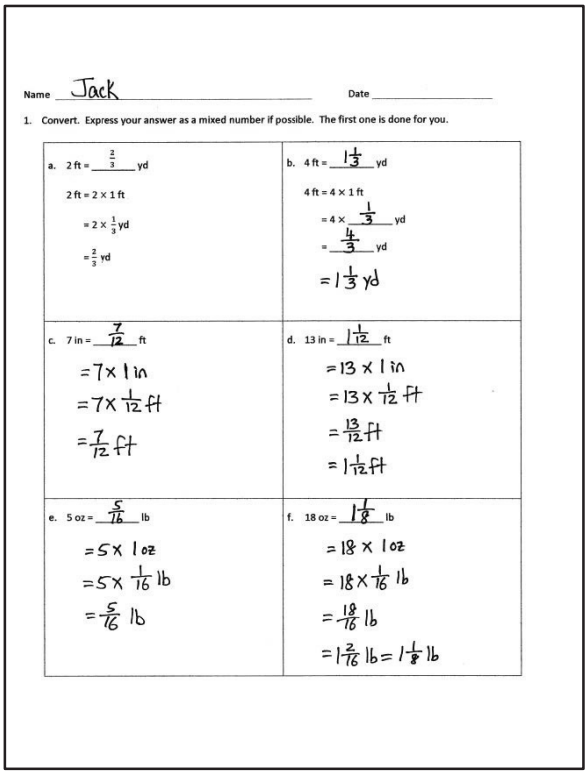
Lesson Objective: Convert measures involving whole numbers, and solve multi-step word problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what did you notice about all of the problems in the left-hand column? The right-hand column? Did you solve the problems differently as a result?
- Explain your process for solving Problem 4. How did you convert from cups to gallons? What is a cup expressed as a fraction of a gallon? How did you figure that out?
- In Problem 2, you were asked to find the fraction of a yard of craft trim Regina bought. Tell your partner how you solved this problem.
- How did today's second fluency activity, Convert Measures, help prepare you for this lesson?
- Look back at Problem 1(e). Five ounces is equal to how many pounds? What would 6 ounces be equal to? 7 ounces? 8 ounces? 9 ounces? Think carefully. $\frac{10}{16}$ pound equals how many ounces? $\frac{11}{16}$ pound? $\frac{12}{16}$ pound? $\frac{13}{16}$ pound? Talk about your thinking as you answered those questions.



Name Jack Date

1. Convert. Express your answer as a mixed number if possible. The first one is done for you.

a. 2 ft = $\frac{2}{3}$ yd
2 ft = 2 × 1 ft
= 2 × $\frac{1}{3}$ yd
= $\frac{2}{3}$ yd

b. 4 ft = $1\frac{1}{3}$ yd
4 ft = 4 × 1 ft
= 4 × $\frac{1}{3}$ yd
= $\frac{4}{3}$ yd
= $1\frac{1}{3}$ yd

c. 7 in = $\frac{7}{12}$ ft
= 7 × 1 in
= 7 × $\frac{1}{12}$ ft
= $\frac{7}{12}$ ft

d. 13 in = $1\frac{1}{12}$ ft
= 13 × 1 in
= 13 × $\frac{1}{12}$ ft
= $\frac{13}{12}$ ft
= $1\frac{1}{12}$ ft

e. 5 oz = $\frac{5}{16}$ lb
= 5 × 1 oz
= 5 × $\frac{1}{16}$ lb
= $\frac{5}{16}$ lb

f. 18 oz = $1\frac{1}{8}$ lb
= 18 × 1 oz
= 18 × $\frac{1}{16}$ lb
= $\frac{18}{16}$ lb
= $1\frac{2}{16}$ lb = $1\frac{1}{8}$ lb

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students may struggle as they try to articulate their ideas. Some strategies that may be used to support these students are below.

- Ask students to repeat the teacher's thinking in their own words.
- Ask students to add on to either the teacher's thinking or another student's thoughts.
- Give students time to practice with their partners before answering in a larger group.
- Pose a question, and ask students to use specific vocabulary in their answers.

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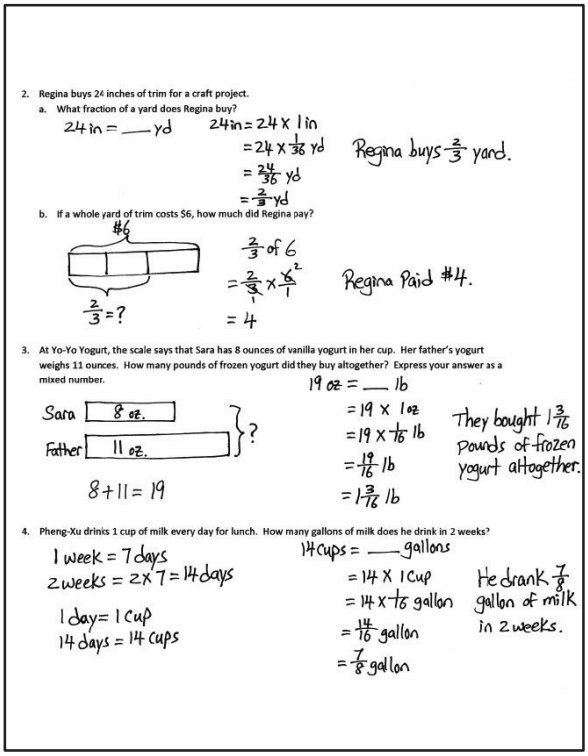
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



2. Regina buys 24 inches of trim for a craft project.
   a. What fraction of a yard does Regina buy?
   24 in = ___ yd
   24 in = 24 × 1 in
   = 24 × $\frac{1}{36}$ yd
   = $\frac{24}{36}$ yd
   = $\frac{2}{3}$ yd
   Regina buys $\frac{2}{3}$ yard.
   b. If a whole yard of trim costs $6, how much did Regina pay?
   #6
   $\frac{2}{3}$ = ?
   $\frac{2}{3}$ of 6
   = $\frac{2}{3} \times \frac{6}{1}$
   = 4
   Regina Paid #4.

3. At Yo-Yo Yogurt, the scale says that Sara has 8 ounces of vanilla yogurt in her cup. Her father's yogurt weighs 11 ounces. How many pounds of frozen yogurt did they buy altogether? Express your answer as a mixed number.
   Sara 8 oz.
   Father 11 oz.
   ?
   8 + 11 = 19
   19 oz = ___ lb
   = 19 × 1 oz
   = 19 × $\frac{1}{16}$ lb
   = $\frac{19}{16}$ lb
   = $1\frac{3}{16}$ lb
   They bought $1\frac{3}{16}$ pounds of frozen yogurt altogether.

4. Pheng-Xu drinks 1 cup of milk every day for lunch. How many gallons of milk does he drink in 2 weeks?
   1 week = 7 days
   2 weeks = 2 × 7 = 14 days
   1 day = 1 cup
   14 days = 14 cups
   14 cups = ___ gallons
   = 14 × 1 cup
   = 14 × $\frac{1}{16}$ gallon
   = $\frac{14}{16}$ gallon
   = $\frac{7}{8}$ gallon
   He drank $\frac{7}{8}$ gallon of milk in 2 weeks.

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Name _____ Date _____

1. Convert. Express your answer as a mixed number, if possible. The first one is done for you.

a. 2 ft = $\frac{2}{3}$ yd 2 ft = 2 × 1 ft = 2 × $\frac{1}{3}$ yd = $\frac{2}{3}$ yd	b. 4 ft = _____ yd 4 ft = 4 × 1 ft = 4 × _____ yd = _____ yd =
c. 7 in = _____ ft	d. 13 in = _____ ft
e. 5 oz = _____ lb	f. 18 oz = _____ lb

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2. Regina buys 24 inches of trim for a craft project.

   a. What fraction of a yard does Regina buy?

   b. If a whole yard of trim costs $6, how much did Regina pay?

3. At Yo-Yo Yogurt, the scale says that Sara has 8 ounces of vanilla yogurt in her cup. Her father's yogurt weighs 11 ounces. How many pounds of frozen yogurt did they buy altogether? Express your answer as a mixed number.

4. Pheng-Xu drinks 1 cup of milk every day for lunch. How many gallons of milk does he drink in 2 weeks?

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Name _____ Date _____

Convert. Express your answer as a mixed number, if possible.

a. 5 in = _____ ft

b. 13 in = _____ ft

c. 9 oz = _____ lb

d. 18 oz = _____ lb

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Name _____ Date _____

1. Convert. Express your answer as a mixed number, if possible.

a. 2 ft = $\frac{2}{3}$ yd 2 ft = 2 × 1 ft = 2 × $\frac{1}{3}$ yd = $\frac{2}{3}$ yd	b. 6 ft = _____ yd 6 ft = 6 × 1 ft = 6 × _____ yd = _____ yd
c. 5 in = _____ ft	d. 14 in = _____ ft
e. 7 oz = _____ lb	f. 20 oz = _____ lb
g. 1 pt = _____ qt	h. 4 pt = _____ qt

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2. Marty buys 12 ounces of granola.

   a. What fraction of a pound of granola did Marty buy?

   b. If a whole pound of granola costs $4, how much did Marty pay?

3. Sara and her dad visit Yo-Yo Yogurt again. This time, the scale says that Sara has 14 ounces of vanilla yogurt in her cup. Her father's yogurt weighs half as much. How many pounds of frozen yogurt did they buy altogether on this visit? Express your answer as a mixed number.

4. An art teacher uses 1 quart of blue paint each month. In one year, how many gallons of paint will she use?

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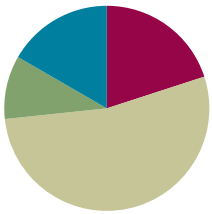
Lesson 20

Objective: Convert mixed unit measurements, and solve multi-step word problems.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Count by Fractions **5.NF.7** (3 minutes)
- Convert Measures **4.MD.1** (3 minutes)
- Multiply Decimals **5.NBT.7** (3 minutes)
- Find the Unit Conversion **5.MD.2** (3 minutes)

Count by Fractions (3 minutes)

Note: This fluency activity prepares students for Lesson 21.

1	2	3	4	5	6	7	8	9	10
$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{6}{2}$	$\frac{7}{2}$	$\frac{8}{2}$	$\frac{9}{2}$	$\frac{10}{2}$
$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5
$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5

T: Count by ones to 10. (Write as students count.)

S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

T: Count by halves to 10 halves. (Write as students count.)

S: 1 half, 2 halves, 3 halves, 4 halves, 5 halves, 6 halves, 7 halves, 8 halves, 9 halves, 10 halves.

T: Let's count by halves again. This time, when we arrive at a whole number, say the whole number. (Write as students count.)

S: 1 half, 1 whole, 3 halves, 2 wholes, 5 halves, 3 wholes, 7 halves, 4 wholes, 9 halves, 5 wholes.

T: Let's count by halves again. This time, change improper fractions to mixed numbers. (Write as students count.)

S: 1 half, 1, 1 and 1 half, 2, 2 and 1 half, 3, 3 and 1 half, 4, 4 and 1 half, 5.

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Convert Measures (3 minutes)

Materials: (S) Personal white board, Grade 5 Mathematics Reference Sheet (Lesson 8 Reference Sheet)

Note: This fluency exercise reviews Lessons 19 and 20. Allow students to use the conversion reference sheet if they are confused, but encourage them to answer questions without referring to it.

T: (Write 1 ft = ____ in.) How many inches are equal to 1 foot?
S: 12 inches.
T: (Write 1 ft = 12 in. Below it, write 2 ft = ____ in.) 2 feet?
S: 24 inches.
T: (Write 2 ft = 24 in. Below it, write 4 ft = ____ in.) 4 feet?
S: 48 inches.

Continue with the following possible sequence: 1 pint = 2 cups, 7 pints = 14 cups, 1 yard = 3 feet, 6 yards = 18 feet, 1 gallon = 4 quarts, and 9 gallons = 36 quarts.

T: (Write 2 c = ____ pt.) How many pints are equal to 2 cups?
S: 1 pint.
T: (Write 2 c = 1 pt. Below it, write 4 c = ____ pt.) 4 cups?
S: 2 pints.
T: (Write 4 c = 2 pt. Below it, write 10 c = ____ pt.) 10 cups?
S: 5 pints.

Continue with the following possible sequence: 12 in = 1 ft, 36 in = 3 ft, 3 ft = 1 yd, 12 ft = 4 yd, 4 qt = 1 gal, and 28 qt = 7 gal.

Multiply Decimals (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 17 and 18.

3 × 3 = 9	3 × 0.3 = 0.9	0.3 × 0.3 = 0.09	0.03 × 0.3 = 0.009
2 × 8 = 16	2 × 0.8 = 1.6	0.2 × 0.8 = 0.16	0.02 × 0.8 = 0.016
5 × 5 = 25	0.5 × 5 = 2.5	0.5 × 0.5 = 0.25	0.5 × 0.05 = 0.025

T: (Write 3 × 3 = ____.) Say the multiplication sentence with the answer.
S: 3 × 3 = 9.
T: (Write 3 × 0.3 = ____.) On your personal white board, write the number sentence and the answer.
S: (Write 3 × 0.3 = 0.9.)
T: (Write 0.3 × 0.3 = ____.) Try this problem.
S: (Write 0.3 × 0.3 = 0.09.)
T: (Write 0.03 × 0.3 = ____.) Try this problem.
S: (Write 0.03 × 0.3 = 0.009.)

Continue this process with the following possible sequence: 2 × 8, 2 × 0.8, 0.2 × 0.8, 0.02 × 0.8; 5 × 5, 0.5 × 5, 0.5 × 0.5, 0.5 × 0.05.

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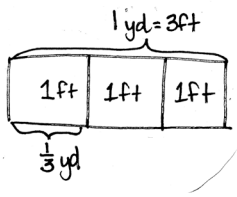
Find the Unit Conversion (3 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 12.

T: How many feet are in 1 yard?
S: 3 feet.
T: (Write 3 ft = 1 yd. Below it, write 1 ft = ____ yd.) What fraction of 1 yard is 1 foot?
S: 1 third.
T: On your personal white board, draw a tape diagram to explain your thinking.



1ft = 1 x 1ft
= 1 x $\frac{1}{3}$ yd
= $\frac{1}{3}$ yd

Continue with the following possible sequence: 2 ft = ____ yd, 5 in = ____ ft, 1 in = ____ ft, 1 oz = ____ lb, 9 oz = ____ lb, 1 pt = ____ qt, 3 pt = ____ qt, 4 days = ____ week, and 18 hours = ____ day.

Application Problem (6 minutes)

A recipe calls for $\frac{3}{4}$ lb of cream cheese. A small tub of cream cheese at the grocery store weighs 12 oz. Is this enough cream cheese for the recipe?

Note: This Application Problem builds on previous lessons involving unit conversions and multiplication of a fraction and a whole number. In addition to the method shown, students may also simply realize that $\frac{3}{4}$ is equal to $\frac{12}{16}$.

Need $\frac{3}{4}$ lb:

12oz = ___lb

12oz = 12 x 1oz
= 12 x $\frac{1}{16}$ lb
= $\frac{12}{16}$ lb
= $\frac{3}{4}$ lb

There is enough cream cheese for the recipe.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Another approach to this Application Problem is to think of it as a comparison problem. (See Table 2 of the *Common Core State Standards for Mathematics*.) Students can draw two bars, one showing the amount needed for the recipe and another showing the amount sold in the small tub. The tape diagram would help students recognize the need to convert one of the amounts so that like units can be compared.

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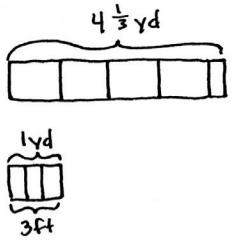
Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: Convert larger units to smaller units.

$4\frac{1}{3}$ yd = ____ ft



$4\frac{1}{3}$ yd = ____ ft
$4\frac{1}{3}$ yd = $4\frac{1}{3} \times 1$ yd
= $4\frac{1}{3} \times 3$ ft
= $\frac{13}{3} \times 3$ ft
= $\frac{13 \times 3}{1 \cancel{3}}$ ft
= 13 ft

T: (Write $4\frac{1}{3}$ yd = ____ ft on the board.) Which units are larger, yards or feet?

S: Yards.

T: Compare this problem with the conversions we worked on yesterday. What do you notice about the units? Turn and talk.

S: This is starting with larger units and converting to smaller ones. → Yesterday every conversion we did was small units to large units. This is large to small conversion.

T: Let's draw a tape diagram to model this problem. We want to name $4\frac{1}{3}$ yards using feet. (Draw a bar, and label it $4\frac{1}{3}$ yd.) Let's partition the bar into 4 equal units to represent the 4 whole yards and 1 smaller unit to represent $\frac{1}{3}$ of a yard. $4\frac{1}{3}$ yards is the same as $4\frac{1}{3} \times 1$ yard. (Write on the board.) How many feet are in 1 yard?

S: 3 feet.

T: On your personal white board, draw a tape diagram to explain your thinking.

S: (Draw.)

T: (Show 1 yard is equal to 3 feet below the tape diagram.) Write a new expression to rename the yard in feet.

S: (Write $4\frac{1}{3} \times 3$ ft.)

T: Before we multiply, let's express $4\frac{1}{3}$ as an improper fraction. How many thirds are in 1 whole?

S: 3 thirds.

T: So, how many thirds are in 4 wholes?

S: 12 thirds.

T: How many thirds are in 4 and 1 third?

S: 13 thirds.

T: Write a new multiplication expression that uses the improper fraction we just found.

S: (Write $\frac{13}{3} \times 3$ ft.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The lessons in this topic require students to know basic conversions, such as 16 ounces are combined to make 1 pound. Teachers can provide this background knowledge in the form of posters or other graphic organizers.

Students may also need support with some of the abbreviations used. For example, the abbreviation for pound (lb) and the abbreviation for ounce (oz) may seem confusing to some students. Teachers can post this information on a poster or provide reference sheets for all students.

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T: Work with a partner to find the product of 13 thirds and 3.

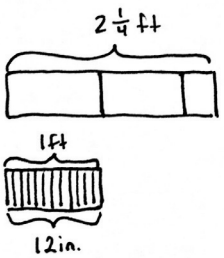
S: (Work.)

T: (Point to the original problem.) Fill in the blank using a complete sentence.

S: $4\frac{1}{3}$ yd is equal to 13 ft.

As necessary, repeat the same process with the following problems.

$2\frac{1}{4}$ ft = _____ in



$2\frac{1}{4}$ ft = ____ in

$2\frac{1}{4}$ ft $= 2\frac{1}{4} \times 1$ ft

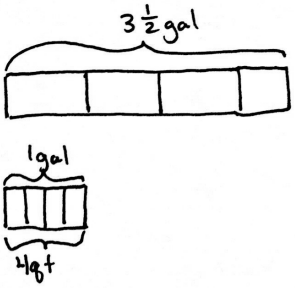
$= 2\frac{1}{4} \times 12$ in

$= \frac{9}{4} \times 12$ in

$= \frac{9 \times 12^{3}}{1\cancel{4}}$ in

$= 27$ in

$3\frac{1}{2}$ gal = _____ qt



$3\frac{1}{2}$ gal = ____ qt

$3\frac{1}{2}$ gal $= 3\frac{1}{2} \times 1$ gal

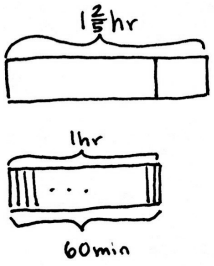
$= 3\frac{1}{2} \times 4$ qt

$= \frac{7}{2} \times 4$ qt

$= \frac{7 \times \cancel{4}^{2}}{\cancel{2}_{1}}$ qt

$= 14$ qt

$1\frac{2}{5}$ hr = _____ min



$1\frac{2}{5}$ hr = ____ min

$1\frac{2}{5}$ hr $= 1\frac{2}{5} \times 1$ hr

$= 1\frac{2}{5} \times 60$ min

$= \frac{7}{5} \times 60$ min.

$= \frac{7 \times \cancel{60}^{12}}{1\cancel{8}}$ min.

$= 84$ min.

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Problem 2: Convert smaller units to larger units.

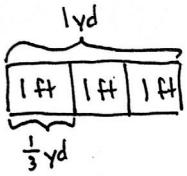
11 ft = ____ yd

T: (Write 11 ft = ____ yd on the board.) Which units are larger, feet or yards?

S: Yards.

T: Compare this problem to the others we've solved. Turn and share with a partner.

S: This one gives us the measurement in smaller units and wants the amount of larger units. → This one goes from small units to large units such as the ones we did yesterday.



11 ft = ____ yd

11 ft = 11 × 1 ft

= 11 × $\frac{1}{3}$ yd

= $\frac{11 \times 1}{3}$ yd

= $\frac{11}{3}$ yd

= $3\frac{2}{3}$ yd

T: What fraction of 1 yard is 1 foot?

S: 1 third.

T: On your personal white board, draw a tape diagram to show the relationship between feet and yards.

S: (Draw.)

T: What two whole numbers of yards will 11 feet fall between? Turn and talk.

S: 9 feet is 3 yards, and 12 feet is 4 yards, so 11 feet must be somewhere between 3 and 4 yards.

T: We know that 11 ft = 11 × 1 ft. (Write on the board.) Write a multiplication sentence that is equivalent to this one using yards.

S: (Work.)

T: Let's record the equivalent expression beneath our first one. (Record as shown.) What is $11 \times \frac{1}{3}$?

S: 11 thirds.

T: Express 11 thirds as a mixed number.

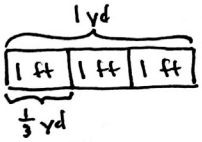
S: $3\frac{2}{3}$.

T: Express your answer as yards.

S: 3 and 2 thirds yards.

Repeat the same process with the following as necessary.

$5\frac{1}{2}$ ft = ____ yd



$5\frac{1}{2}$ ft = ____ yd

$5\frac{1}{2}$ ft = $5\frac{1}{2} \times 1$ ft

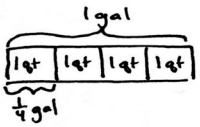
= $5\frac{1}{2} \times \frac{1}{3}$ yd

= $\frac{11}{2} \times \frac{1}{3}$ yd

= $\frac{11}{6}$ yd

= $1\frac{5}{6}$ yd

$3\frac{1}{3}$ qt = ____ gal



$3\frac{1}{3}$ qt = ____ gal

$3\frac{1}{3}$ qt = $3\frac{1}{3} \times 1$ qt

= $3\frac{1}{3} \times \frac{1}{4}$ gal

= $\frac{10}{3} \times \frac{1}{4}$ gal

= $\frac{10}{12}$ gal

= $\frac{5}{6}$ gal

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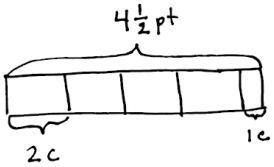
Note: If time permits, Problem 3 may be explored with students.

Problem 3: A container can hold $4\frac{1}{2}$ pints of water. How many cups of water can 2 containers hold?

T: (Post Problem 3 on the board, and read it out loud with students.) How do you solve this problem? Turn and share your idea with a partner.

S: It's a two-step problem. → I first have to convert $4\frac{1}{2}$ pints to cups, and then I'll have to double it.

T: Let's draw a tape diagram for $4\frac{1}{2}$ pt. I'll do it on the board, and you draw it on your personal white board.



$4\frac{1}{2}$ pt = $4\frac{1}{2} \times 1$ pt
= $4\frac{1}{2} \times 2$ c
= $\frac{9}{2} \times 2$ c
= $\frac{18}{2}$ c
= 9 c

9 c × 2 = 18 c
Two containers can hold 18 cups.

MP.2 S: (Draw and label.)

T: Say the multiplication expression to convert $4\frac{1}{2}$ pints to cups.

S: $4\frac{1}{2} \times 2$ cups.

T: Express $4\frac{1}{2}$ as an improper fraction, and restate the expression.

S: $\frac{9}{2} \times 2$ cups.

T: What's the answer?

S: $\frac{18}{2}$ cups.

T: How many whole cups is that?

S: 9 cups.

T: Finish by finding the amount of water in two containers. Turn and talk.

S: We have to find the water in 2 containers. → Since 1 container holds 9 cups, then we'll have to double it. 9 cups + 9 cups = 18 cups. → To find the amount 2 containers hold, we have to multiply. 2 × 9 cups = 18 cups.

Name Smith Date

1. Convert. Show your work. Express your answer as a mixed number. (Draw a tape diagram if it helps you.) The first one is done for you.

a. $2\frac{2}{3}$ yd = 8 ft
$2\frac{2}{3}$ yd = $2\frac{2}{3} \times 1$ yd
= $2\frac{2}{3} \times 3$ ft
= $\frac{8}{3} \times 3$ ft
= $\frac{24}{3}$ ft
= 8 ft

b. $1\frac{1}{2}$ qt = 3/8 gal
$1\frac{1}{2}$ qt = $1\frac{1}{2} \times 1$ qt
= $1\frac{1}{2} \times \frac{1}{4}$ gal
= $\frac{3}{2} \times \frac{1}{4}$ gal
= $\frac{3}{8}$ gal

c. $4\frac{2}{3}$ ft = 56 in
$4\frac{2}{3}$ ft = $4\frac{2}{3} \times 1$ ft
= $4\frac{2}{3} \times 12$ in
= $\frac{14}{3} \times 12$ in
= $\frac{168}{3}$ in
= 56 in

d. $9\frac{1}{2}$ pt = 4 3/4 qt
$9\frac{1}{2}$ pt = $9\frac{1}{2} \times 1$ pt
= $9\frac{1}{2} \times \frac{1}{2}$ qt
= $\frac{19}{2} \times \frac{1}{2}$ qt
= $\frac{19}{4}$ qt
= $4\frac{3}{4}$ qt

e. $3\frac{3}{5}$ hr = 216 min
$3\frac{3}{5}$ hr = $3\frac{3}{5} \times 1$ hr
= $3\frac{3}{5} \times 60$ min.
= $\frac{18}{5} \times 60$ min
= $\frac{1080}{5}$ min
= 216 min

f. $3\frac{2}{3}$ ft = 1 2/9 yd
$3\frac{2}{3}$ ft = $3\frac{2}{3} \times 1$ ft
= $3\frac{2}{3} \times \frac{1}{3}$ yd
= $\frac{11}{3} \times \frac{1}{3}$ yd
= $\frac{11}{9}$ yd
= $1\frac{2}{9}$ yd

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Student Debrief (10 minutes)

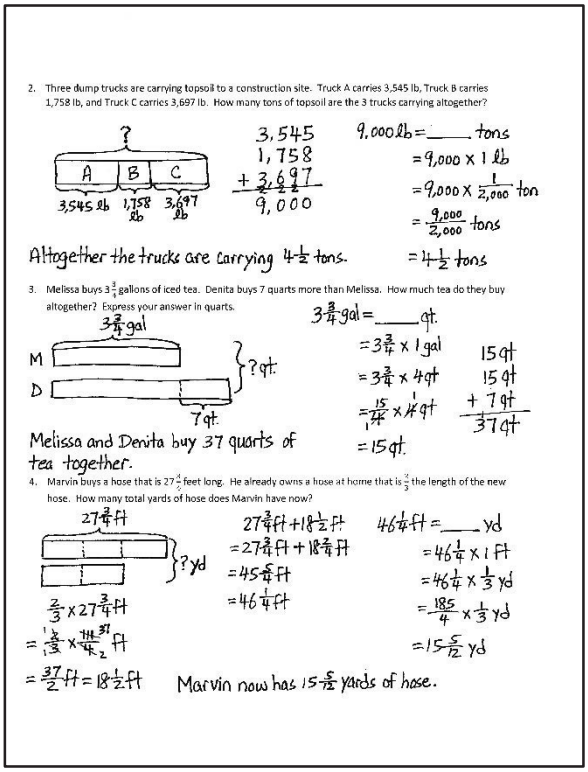
Lesson Objective: Convert mixed unit measurements, and solve multi-step word problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Compare and share your solutions for Problem 1 with your partner.
- Explain to your partner how to solve Problem 3. Did you have a different strategy than your partner?
- How did you solve for Problem 4? Explain your strategy to a partner.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. Convert. Show your work. Express your answer as a mixed number. (Draw a tape diagram if it helps you.) The first one is done for you.

a. $2\frac{2}{3}$ yd = 8 ft

$2\frac{2}{3}$ yd $= 2\frac{2}{3} \times 1$ yd

$= 2\frac{2}{3} \times 3$ ft

$= \frac{8}{3} \times 3$ ft

$= \frac{24}{3}$ ft

$= 8$ ft

b. $1\frac{1}{2}$ qt = _____ gal

$1\frac{1}{2}$ qt $= 1\frac{1}{2} \times 1$ qt

$= 1\frac{1}{2} \times \frac{1}{4}$ gal

$= \frac{3}{2} \times \frac{1}{4}$ gal

=

c. $4\frac{2}{3}$ ft = _____ in

d. $9\frac{1}{2}$ pt = _____ qt

e. $3\frac{3}{5}$ hr = _____ min

f. $3\frac{2}{3}$ ft = _____ yd

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2. Three dump trucks are carrying topsoil to a construction site. Truck A carries 3,545 lb, Truck B carries 1,758 lb, and Truck C carries 3,697 lb. How many tons of topsoil are the 3 trucks carrying altogether?

3. Melissa buys $3\frac{3}{4}$ gallons of iced tea. Denita buys 7 quarts more than Melissa. How much tea do they buy altogether? Express your answer in quarts.

4. Marvin buys a hose that is $27\frac{3}{4}$ feet long. He already owns a hose at home that is $\frac{2}{3}$ the length of the new hose. How many total yards of hose does Marvin have now?

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Name _____ Date _____

Convert. Express your answer as a mixed number.

a. $2\frac{1}{6}$ ft = _____ in

b. $3\frac{3}{4}$ ft = _____ yd

c. $2\frac{1}{2}$ c = _____ pt

d. $3\frac{2}{3}$ years = _____ months

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Name _____ Date _____

1. Convert. Show your work. Express your answer as a mixed number. The first one is done for you.

a. $2\frac{2}{3}$ yd = 8 ft

$2\frac{2}{3}$ yd $= 2\frac{2}{3} \times 1$ yd

$= 2\frac{2}{3} \times 3$ ft

$= \frac{8}{3} \times 3$ ft

$= \frac{24}{3}$ ft

$= 8$ ft

b. $1\frac{1}{4}$ ft = _____ yd

$1\frac{1}{4}$ ft $= 1\frac{1}{4} \times 1$ ft

$= 1\frac{1}{4} \times \frac{1}{3}$ yd

$= \frac{5}{4} \times \frac{1}{3}$ yd

=

c. $3\frac{5}{6}$ ft = _____ in

d. $7\frac{1}{2}$ pt = _____ qt

e. $4\frac{3}{10}$ hr = _____ min

f. 33 months = _____ years

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2. Four members of a track team run a relay race in 165 seconds. How many minutes did it take them to run the race?

3. Horace buys $2\frac{3}{4}$ pounds of blueberries for a pie. He needs 48 ounces of blueberries for the pie. How many more pounds of blueberries does he need to buy?

4. Tiffany is sending a package that may not exceed 16 pounds. The package contains books that weigh a total of $9\frac{3}{8}$ pounds. The other items to be sent weigh $\frac{3}{5}$ the weight of the books. Will Tiffany be able to send the package?

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Topic F

Multiplication with Fractions and Decimals as Scaling and Word Problems

5.NF.5, 5.NF.6

Focus Standards:	5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
	5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Instructional Days:	4	
Coherence -Links from:	G4–M3	Multi-Digit Multiplication and Division
	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions
	G6–M4	Expressions and Equations

Students interpret multiplication in Grade 3 as equal groups, and in Grade 4, students begin understanding multiplication as comparison. Here, in Topic F, students once again extend their understanding of multiplication to include scaling (**5.NF.5**). Students compare the product to the size of one factor, given the size of the other factor (**5.NF.5a**) without calculation (e.g., 486 × 1,327.45 is twice as large as 243 × 1,327.45 because 486 = 2 × 243). This reasoning, along with the other work of this module, sets the stage for students to reason about the size of products when quantities are multiplied by 1, by numbers larger than 1, and numbers smaller than 1. Students relate their previous work with equivalent fractions to interpreting multiplication by $\frac{n}{n}$ as multiplication by 1 (**5.NF.5b**).

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Students build on their new understanding of fraction equivalence as multiplication by $\frac{n}{n}$ to convert fractions to decimals and decimals to fractions. For example, $\frac{3}{25}$ is easily renamed in hundredths as $\frac{12}{100}$ using multiplication of $\frac{4}{4}$. The word form of *twelve hundredths* is then used to notate this quantity as a decimal. Conversions between fractional forms are limited to fractions whose denominators are factors of 10, 100, or 1,000. Students apply the concepts of the topic to real world, multi-step problems (**5.NF.6**).

A Teaching Sequence Toward Mastery of Multiplication with Fractions and Decimals as Scaling and Word Problems	
Objective 1:	**Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1. (Lesson 21)**
Objective 2:	**Compare the size of the product to the size of the factors. (Lessons 22–23)**
Objective 3:	**Solve word problems using fraction and decimal multiplication. (Lesson 24)**

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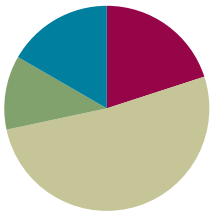
Lesson 21

Objective: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint: Multiply Decimals **5.NBT.7** (8 minutes)
- Find the Unit Conversion **5.MD.2** (4 minutes)

Sprint: Multiply Decimals (8 minutes)

Materials: (S) Multiply Decimals Sprint

Note: This fluency activity reviews Lessons 17–18.

Find the Unit Conversion (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 20.

T: (Write $2\frac{1}{3}$ yd = ____ ft.) How many feet are in 1 yard?

S: 3 feet.

T: Express $2\frac{1}{3}$ yards as an improper fraction.

S: $\frac{7}{3}$ yards.

T: Write an expression using the improper fraction and feet. Then, solve.

S: (Write $\frac{7}{3} \times 3$ feet = 7 feet.)

T: $2\frac{1}{3}$ yards equals how many feet? Answer in a complete sentence.

S: $2\frac{1}{3}$ yards equals 7 feet.

$2\frac{1}{3}\text{ yd} = ____\text{ ft}$

$= 2\frac{1}{3} \times 1\text{ yd}$

$= \frac{7}{3} \times 3\text{ ft}$

$= \frac{21}{3}\text{ ft}$

$= 7\text{ ft}$

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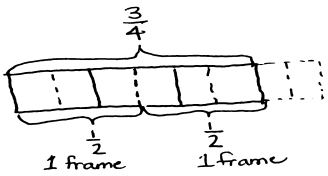
Continue with one or more of the following possible problems: $2\frac{1}{4}$ gal = ____ qt, $2\frac{3}{4}$ ft = ____ in, and $7\frac{1}{2}$ pt = ____ c.

Application Problem (7 minutes)

Carol had $\frac{3}{4}$ yard of ribbon. She wanted to use it to decorate two picture frames. If she uses half the ribbon on each frame, how many feet of ribbon will she use for one frame? Use a tape diagram to show your thinking.

Note: This Application Problem draws on fraction multiplication concepts taught in previous lessons in this module.



$\frac{1}{2}$ of $\frac{3}{4}$ yd
$= \frac{1}{2} \times \frac{3}{4}$ yd
$= \frac{3}{8}$ yd

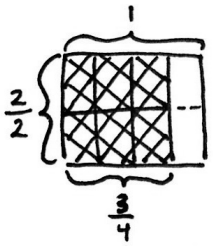
$\frac{3}{8}$ yd = ____ ft
$= \frac{3}{8} \times 1$ yd
$= \frac{3}{8} \times 3$ ft
$= \frac{9}{8}$ ft
$= 1\frac{1}{8}$ ft

She will use $1\frac{1}{8}$ feet of ribbon for one frame.

Concept Development (31 minutes)

Materials: (S) Personal white board

Problem 1: $\frac{2}{2}$ of $\frac{3}{4}$



$\frac{2}{2}$ of $\frac{3}{4} = \frac{6}{8}$

T: (Post Problem 1 on the board.) Write a multiplication expression for this problem.

S: (Write $\frac{2}{2} \times \frac{3}{4}$.)

T: Work with a partner to find the product of 2 halves and 3 fourths.

S: (Work.)

T: Say the product.

S: $\frac{6}{8}$.

T: (Write $= \frac{6}{8}$.) Let's draw an area model to verify our solution. (Draw a rectangle and label it 1.) What are we taking 2 halves of?

S: $\frac{3}{4}$.

T: (Partition the model vertically into fourths and shade 3 of them.) How do we show 2 halves?

S: Split each fourth unit into 2 equal parts, and shade both of them.

T: (Partition fourths horizontally, and shade both halves, or 6 eighths.) What is the product?

S: 6 eighths.

T: How does the size of the product, $\frac{6}{8}$, compare to the size of the original fraction, $\frac{3}{4}$? Turn and talk.

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S: They're exactly the same amount. 6 eighths and 3 fourths are equal. → They're the same. 3 fourths is just 6 eighths in simplest form. → Eighths are a smaller unit than fourths, but we have twice as many of them, so really, the two fractions are equal.

T: I hear you saying that the product, $\frac{6}{8}$, is equal to the amount we had at first, $\frac{3}{4}$. We multiplied. How is it possible that our quantity has not changed? Turn and talk.

S: We multiplied by 2 halves, which is a whole. So, I'm thinking we showed $\frac{3}{4}$ just using a different name. → 2 halves is equal to 1, so really, we just multiplied 3 fourths by 1. Anything times 1 will result in itself. → The fraction two-over-two is equivalent to 1. We just created an equivalent fraction by multiplying the numerator and denominator by a common factor.

T: It sounds like you think that our beginning amount (point to $\frac{3}{4}$) didn't change because we multiplied by one. Name some other fractions that are equal to 1.

S: 3 thirds. → 4 fourths. → 10 tenths. → 1 million millionths!

T: Let's test your hypothesis. Work with a partner to find $\frac{3}{3}$ of $\frac{3}{4}$. One of you can multiply the fractions, while the other draws an area model.

S: (Work and share.)

MP.3 T: What did you find out?

S: It happened again. The product is 9 twelfths, which is still equal to 3 fourths. → We were right: 3 thirds is equal to 1, so we got another product that is equal to 3 fourths. → My area model shows it very clearly. Even though twelfths are a smaller unit, 9 twelfths is equal to 3 fourths.

T: Show some other fraction multiplication expressions involving 3 fourths that would give us a product equal in size to 3 fourths.

S: (Show $\frac{3}{4}\times\frac{5}{5}$. → $\frac{8}{8}\times\frac{3}{4}$. → $\frac{100}{100}\times\frac{3}{4}$.)

T: Is $\frac{3}{4}$ equal to $\frac{18}{24}$? Turn and talk.

S: Yes, if we multiplied 3 fourths by 6 sixths, we'd get $\frac{18}{24}$. → Sure, $\frac{3}{4}$ is $\frac{18}{24}$ in simplest form. I can divide 18 and 24 by 6.

T: Is $\frac{1}{4}$ equal to $\frac{25}{100}$? Work with a partner to write a multiplication sentence and share your thinking.

S: Yes. I know 25 cents is 1 fourth of 100 cents. → It is equal because if we multiply 1 fourth and 25 twenty-fifths, that renames the same amount just using hundredths. It's like all the others we've done today.

$\frac{1}{4} \times \frac{25}{25} = \frac{25}{100}$

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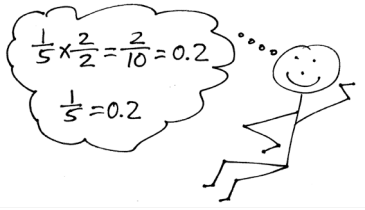
Problem 2: Express fractions as an equivalent decimal.

a. $\frac{1}{5} \times \frac{2}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$



T (Write Problem 1(a), $\frac{1}{5} \times \frac{2}{2}$, on the board.) Show the product.

S: $\frac{2}{10}$.

T: (Write = $\frac{2}{10}$.) What are some other ways to express $\frac{2}{10}$? Turn and talk.

S: We could write it in unit form, like 2 tenths. → One-fifth. → Tenths. That's a decimal. We could write it as 0.2.

T: Express $\frac{2}{10}$ as a decimal on your personal white board.

S: (Write 0.2.)

T: (Write = 0.2.) We multiplied one-fifth by a fraction equal to 1. Did that change the value of one-fifth?

S: No.

T: So, if $\frac{1}{5}$ is equal to $\frac{2}{10}$, and $\frac{2}{10}$ is equal to 0.2. Can we say that $\frac{1}{5}$ = 0.2? (Write $\frac{1}{5}$ = 0.2.) Turn and talk.

S: They are the same. We multiplied one-fifth by 1 to reach $\frac{2}{10}$, so they must be the same.

T: Let's try 3 fifths. How can we change 3 fifths to a decimal?

S: We could multiply by $\frac{2}{2}$ again. → Since we know one-fifth is equal to 0.2, 3 fifths is just 3 times more than that, so we could triple 0.2.

T: Work with a partner to express 3 fifths as a decimal.

S: (Work and share.)

T: Say $\frac{3}{5}$ as a decimal.

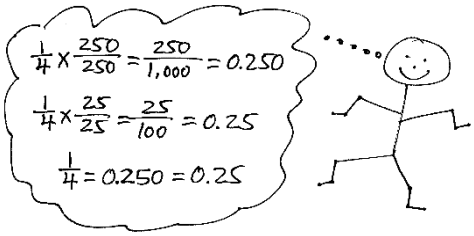
S: 0.6.

T: (Write Problem 2(b), $\frac{1}{4}$, on the board.) All of the fractions we have worked with so far have been related to tenths. Let's think about 1 fourth. We just agreed a moment ago that 1 fourth was equal to 25 hundredths. Write 25 hundredths as a decimal.

S: 0.25.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Once students are comfortable renaming fractions using decimal units, make a connection to the powers of 10 concepts learned back in Module 1. Students can be challenged to see that tenths can be notated as $\frac{1}{10^1}$, hundredths as $\frac{1}{10^2}$, and thousandths as $\frac{1}{10^3}$.



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T: Fourths were renamed as hundredths in this decimal. Could we have easily renamed fourths as tenths? Why or why not? Turn and talk.

S: We can't rename fourths as tenths because 4 isn't a factor of 10. → There's no whole number we can use to get from 4 to 10 using multiplication. → We could name 1 fourth as tenths, but that would be 2 and a half tenths, which is weird.

T: Since tenths are not possible, what unit did we use, and how did we get there?

S: We used hundredths. → We multiplied by 25 twenty-fifths.

T: Is 25 hundredths the only decimal name for 1 fourth? Is there another unit that would rename fourths as a decimal? Turn and talk.

S: We could multiply 25 hundredths by 10 tenths; that would be 250 thousandths. So, we could do it in two steps. → If we multiply 1 fourth by 250 over 250, that would get us to 250 thousandths. → Four 250s is equal to a thousand.

T: Work with a neighbor to express $\frac{1}{4}$ as a decimal, showing your work with multiplication sentences. One of you multiply by $\frac{25}{25}$, and the other multiply by $\frac{250}{250}$. Compare your work when you're done.

S: (Work and share.)

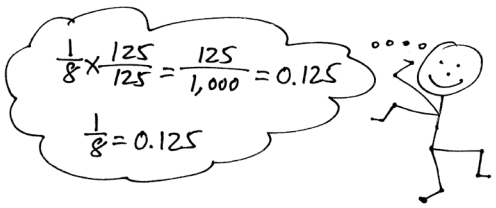
T: What did you find? Are the products the same?

S: Some of us got 25 hundredths, and some of us got 250 thousandths. They look different, but they're equal. I got 0.25, which looks like 25 cents; that's a quarter. Wow, that must be why we call $\frac{1}{4}$ a quarter!

T: (Write $\frac{1}{4} = 0.250 = 0.25$.) What about $\frac{2}{4}$? How could we express that as a decimal? Tell a neighbor what you think, and then show $\frac{2}{4}$ as a decimal.

S: We could multiply by $\frac{25}{25}$ again. → 2 fourths is a half. 1 half is 0.5. → 2 fourths is twice as much as 1 fourth. We could just double 0.25. (Show $\frac{2}{4} = 0.5$.)

T: (Write Problem 2(c), $\frac{1}{8}$, on the board.) Are eighths a unit we can express directly as a decimal, or do we need to multiply by a fraction equal to 1 first?



S: We'll need to multiply first.

T: What fraction equal to 1 will help us rename eighths? Discuss with your neighbor.

S: Eight isn't a factor of 10 or 100. I'm not sure. → I don't know if 1,000 can be divided by 8 without a remainder. I'll divide. Hey, it works!

T: Jonah, what did you find out?

S: 1,000 ÷ 8 = 125. We can multiply by $\frac{125}{125}$.

T: Work independently, and try Jonah's strategy. Show your work when you're done.

S: (Work and show $\frac{1}{8} = 0.125$.)

T: How would you express $\frac{2}{8}$ as a decimal? Tell a neighbor.

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S: We could multiply by $\frac{125}{125}$ again. → We could just double 0.125 and get 0.250. → $\frac{2}{8}$ is equal to $\frac{1}{4}$. We already solved that as 0.250 or 0.25.

T: Work independently to show $\frac{2}{8}$ as a decimal.

S: (Show $\frac{2}{8}$ = 0.250 or $\frac{2}{8}$ = 0.25.)

T: It's a good idea to remember some of these common fraction–decimal equivalencies, such as fourths and eighths; you will use them often in your future math work.

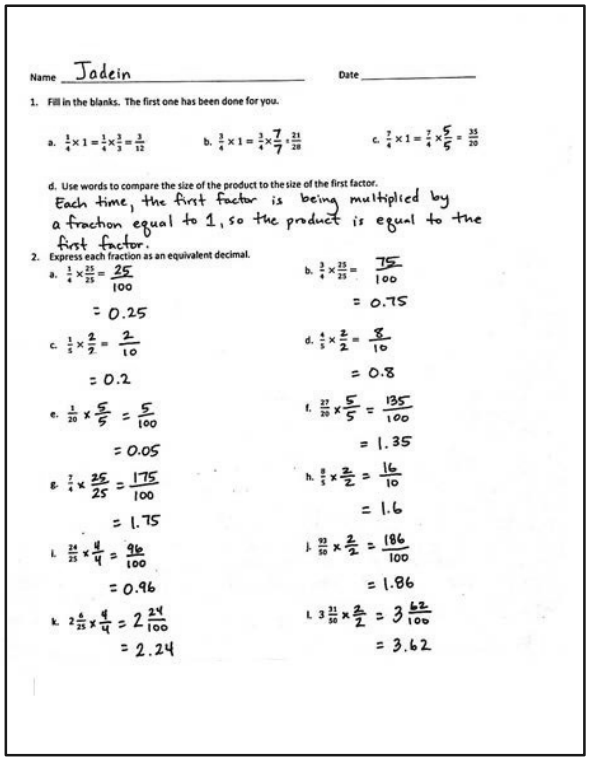
Continue with the following possible sequence: $\frac{1}{20}$, $1\frac{1}{20}$, $\frac{6}{25}$, and $\frac{51}{50}$.

NOTES ON PROPERTIES OF OPERATIONS:

After completing this lesson, it may be interesting to some students to know the name of the property they have been studying: multiplicative identity property of 1. Consider asking students if they can think of any other identity properties. Ideally, they will say that zero added to any number keeps the same value. This is the additive identity property of 0. (See Table 3 of the *Common Core State Standards for Mathematics*.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.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1. Fill in the blanks. The first one has been done for you.

a. $\frac{1}{4}\times 1=\frac{1}{4}\times\frac{3}{3}=\frac{3}{12}$ b. $\frac{3}{4}\times 1=\frac{3}{4}\times\frac{7}{7}=\frac{21}{28}$ c. $\frac{7}{4}\times 1=\frac{7}{4}\times\frac{5}{5}=\frac{35}{20}$

d. Use words to compare the size of the product to the size of the first factor.
Each time, the first factor is being multiplied by a fraction equal to 1, so the product is equal to the first factor.

2. Express each fraction as an equivalent decimal.

a. $\frac{1}{4}\times\frac{25}{25}=\frac{25}{100}$ = 0.25

b. $\frac{3}{4}\times\frac{25}{25}=\frac{75}{100}$ = 0.75

c. $\frac{1}{5}\times\frac{2}{2}=\frac{2}{10}$ = 0.2

d. $\frac{4}{5}\times\frac{2}{2}=\frac{8}{10}$ = 0.8

e. $\frac{1}{20}\times\frac{5}{5}=\frac{5}{100}$ = 0.05

f. $\frac{27}{20}\times\frac{5}{5}=\frac{135}{100}$ = 1.35

g. $\frac{7}{4}\times\frac{25}{25}=\frac{175}{100}$ = 1.75

h. $\frac{8}{5}\times\frac{2}{2}=\frac{16}{10}$ = 1.6

i. $\frac{24}{25}\times\frac{4}{4}=\frac{96}{100}$ = 0.96

j. $\frac{93}{50}\times\frac{2}{2}=\frac{186}{100}$ = 1.86

k. $2\frac{6}{25}\times\frac{4}{4}=2\frac{24}{100}$ = 2.24

l. $3\frac{31}{50}\times\frac{2}{2}=3\frac{62}{100}$ = 3.62

Student Debrief (10 minutes)

Lesson Objective: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Share your response to Problem 1(d) with a partner.
- In Problem 2, what is the relationship between Parts (a) and (b), (c) and (d), (e) and (f), (i) and (k), and (j) and (l)? (They have the same denominator.)

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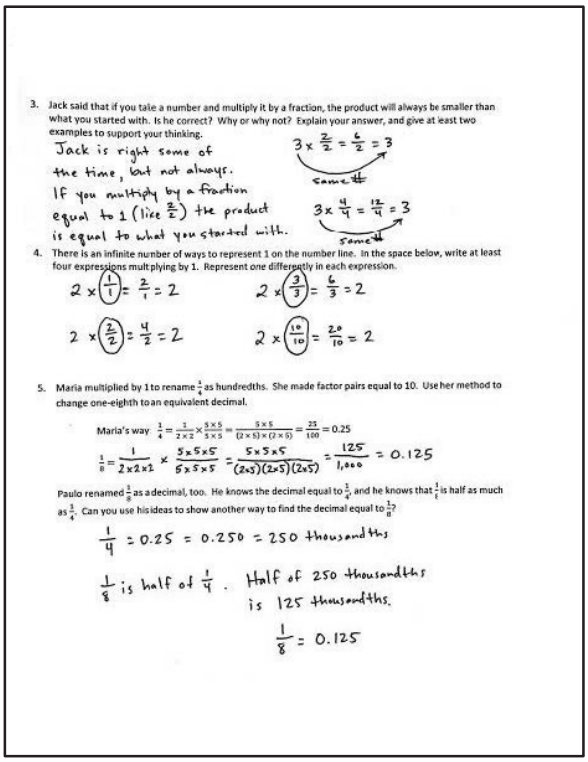
- In Problem 2, what did you notice about Parts (f), (g), (h), and (j)? (The fractions are greater than 1; thus, the answers will be more than one whole.)
- In Problem 2, what did you notice about Parts (k) and (l)? (The fractions are mixed numbers; thus, the answers will be more than one whole.)
- Share and explain your thought process for answering Problem 3.
- In Problem 4, did you have the same expressions to represent one on the number line as your partner's? Can you think of more expressions?
- How did you solve Problem 5? Share your strategy and solution with a partner.


3. Jack said that if you take a number and multiply it by a fraction, the product will always be smaller than what you started with. Is he correct? Why or why not? Explain your answer, and give at least two examples to support your thinking.

Jack is right some of the time, but not always. If you multiply by a fraction equal to 1 (like $\frac{2}{2}$) the product is equal to what you started with.

$3 \times \frac{2}{2} = \frac{6}{2} = 3$ same

$3 \times \frac{4}{4} = \frac{12}{4} = 3$ same

4. There is an infinite number of ways to represent 1 on the number line. In the space below, write at least four expressions multiplying by 1. Represent one differently in each expression.

$2 \times \frac{1}{1} = \frac{2}{1} = 2$ $\quad 2 \times \frac{3}{3} = \frac{6}{3} = 2$

$2 \times \frac{2}{2} = \frac{4}{2} = 2$ $\quad 2 \times \frac{10}{10} = \frac{20}{10} = 2$

5. Maria multiplied by 1 to rename $\frac{1}{4}$ as hundredths. She made factor pairs equal to 10. Use her method to change one-eighth to an equivalent decimal.

Maria's way $\frac{1}{4} = \frac{1}{2\times2} \times \frac{5\times5}{5\times5} = \frac{5\times5}{(2\times5)\times(2\times5)} = \frac{25}{100} = 0.25$

$\frac{1}{8} = \frac{1}{2\times2\times2} \times \frac{5\times5\times5}{5\times5\times5} = \frac{5\times5\times5}{(2\times5)(2\times5)(2\times5)} = \frac{125}{1{,}000} = 0.125$

Paulo renamed $\frac{1}{8}$ as a decimal, too. He knows the decimal equal to $\frac{1}{4}$ and he knows that $\frac{1}{8}$ is half as much as $\frac{1}{4}$. Can you use his ideas to show another way to find the decimal equal to $\frac{1}{8}$?

$\frac{1}{4} = 0.25 = 0.250 = 250$ thousandths

$\frac{1}{8}$ is half of $\frac{1}{4}$. Half of 250 thousandths is 125 thousandths.

$\frac{1}{8} = 0.125$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Number Correct: _____

A

Multiply Decimals

1.	3 × 2 =	
2.	3 × 0.2 =	
3.	3 × 0.02 =	
4.	3 × 3 =	
5.	3 × 0.3 =	
6.	3 × 0.03 =	
7.	2 × 4 =	
8.	2 × 0.4 =	
9.	2 × 0.04 =	
10.	5 × 3 =	
11.	5 × 0.3 =	
12.	5 × 0.03 =	
13.	7 × 2 =	
14.	7 × 0.2 =	
15.	7 × 0.02 =	
16.	4 × 3 =	
17.	4 × 0.3 =	
18.	0.4 × 3 =	
19.	0.4 × 0.3 =	
20.	0.4 × 0.03 =	
21.	0.3 × 0.04 =	
22.	6 × 2 =	

23.	0.6 × 2 =	
24.	0.6 × 0.2 =	
25.	0.6 × 0.02 =	
26.	0.2 × 0.06 =	
27.	5 × 7 =	
28.	0.5 × 7 =	
29.	0.5 × 0.7 =	
30.	0.5 × 0.07 =	
31.	0.7 × 0.05 =	
32.	2 × 8 =	
33.	9 × 0.2 =	
34.	3 × 7 =	
35.	8 × 0.03 =	
36.	4 × 6 =	
37.	0.6 × 7 =	
38.	0.7 × 0.7 =	
39.	0.8 × 0.06 =	
40.	0.09 × 0.6 =	
41.	6 × 0.8 =	
42.	0.7 × 0.9 =	
43.	0.08 × 0.8 =	
44.	0.9 × 0.08 =	

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B

Number Correct: _____

Improvement: _____

Multiply Decimals

1.	4 × 2 =	
2.	4 × 0.2 =	
3.	4 × 0.02 =	
4.	2 × 3 =	
5.	2 × 0.3 =	
6.	2 × 0.03 =	
7.	3 × 3 =	
8.	3 × 0.3 =	
9.	3 × 0.03 =	
10.	4 × 3 =	
11.	4 × 0.3 =	
12.	4 × 0.03 =	
13.	9 × 2 =	
14.	9 × 0.2 =	
15.	9 × 0.02 =	
16.	5 × 3 =	
17.	5 × 0.3 =	
18.	0.5 × 3 =	
19.	0.5 × 0.3 =	
20.	0.5 × 0.03 =	
21.	0.3 × 0.05 =	
22.	8 × 2 =	

23.	0.8 × 2 =	
24.	0.8 × 0.2 =	
25.	0.8 × 0.02 =	
26.	0.2 × 0.08 =	
27.	5 × 9 =	
28.	0.5 × 9 =	
29.	0.5 × 0.9 =	
30.	0.5 × 0.09 =	
31.	0.9 × 0.05 =	
32.	2 × 6 =	
33.	7 × 0.2 =	
34.	3 × 8 =	
35.	9 × 0.03 =	
36.	4 × 8 =	
37.	0.7 × 6 =	
38.	0.6 × 0.6 =	
39.	0.6 × 0.08 =	
40.	0.06 × 0.9 =	
41.	8 × 0.6 =	
42.	0.9 × 0.7 =	
43.	0.07 × 0.7 =	
44.	0.8 × 0.09 =	

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Name _____ Date _____

1. Fill in the blanks. The first one has been done for you.

a. $\frac{1}{4} \times 1 = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

b. $\frac{3}{4} \times 1 = \frac{3}{4} \times _ = \frac{21}{28}$

c. $\frac{7}{4} \times 1 = \frac{7}{4} \times _ = \frac{35}{20}$

d. Use words to compare the size of the product to the size of the first factor.

2. Express each fraction as an equivalent decimal.

a. $\frac{1}{4} \times \frac{25}{25} =$

b. $\frac{3}{4} \times \frac{25}{25} =$

c. $\frac{1}{5} \times _ =$

d. $\frac{4}{5} \times _ =$

e. $\frac{1}{20}$

f. $\frac{27}{20}$

g. $\frac{7}{4}$

h. $\frac{8}{5}$

i. $\frac{24}{25}$

j. $\frac{93}{50}$

k. $2\frac{6}{25}$

l. $3\frac{31}{50}$

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3. Jack said that if you take a number and multiply it by a fraction, the product will always be smaller than what you started with. Is he correct? Why or why not? Explain your answer, and give at least two examples to support your thinking.

4. There is an infinite number of ways to represent 1 on the number line. In the space below, write at least four expressions multiplying by 1. Represent *one* differently in each expression.

5. Maria multiplied by 1 to rename $\frac{1}{4}$ as hundredths. She made factor pairs equal to 10. Use her method to change one-eighth to an equivalent decimal.

   Maria's way: $\frac{1}{4} = \frac{1}{2 \times 2} \times \frac{5 \times 5}{5 \times 5} = \frac{5 \times 5}{(2 \times 5) \times (2 \times 5)} = \frac{25}{100} = 0.25$

   $\frac{1}{8} =$

   Paulo renamed $\frac{1}{8}$ as a decimal, too. He knows the decimal equal to $\frac{1}{4}$, and he knows that $\frac{1}{8}$ is half as much as $\frac{1}{4}$. Can you use his ideas to show another way to find the decimal equal to $\frac{1}{8}$?

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Name _____ Date _____

1. Fill in the blanks to make the equation true.

$$\frac{9}{4} \times 1 = \frac{9}{4} \times _ = \frac{45}{20}$$

2. Express the fractions as equivalent decimals.

a. $\frac{1}{4} =$

b. $\frac{2}{5} =$

c. $\frac{3}{25} =$

d. $\frac{5}{20} =$

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Name _____ Date _____

1. Fill in the blanks.

a. $\frac{1}{3} \times 1 = \frac{1}{3} \times \frac{3}{3} = \frac{\quad}{9}$

b. $\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\quad}{\quad} = \frac{14}{21}$

c. $\frac{5}{2} \times 1 = \frac{5}{2} \times \frac{\quad}{\quad} = \frac{25}{\quad}$

d. Compare the first factor to the value of the product.

2. Express each fraction as an equivalent decimal. The first one is partially done for you.

a. $\frac{3}{4} \times \frac{25}{25} = \frac{3 \times 25}{4 \times 25} = \frac{\quad}{100} =$

b. $\frac{1}{4} \times \frac{25}{25} =$

c. $\frac{2}{5} \times \frac{\quad}{\quad} =$

d. $\frac{3}{5} \times \frac{\quad}{\quad} =$

e. $\frac{3}{20}$

f. $\frac{25}{20}$

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g. $\frac{23}{25}$

h. $\frac{89}{50}$

i. $3\frac{11}{25}$

j. $5\frac{41}{50}$

3. $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. How can you use this to help you write $\frac{6}{8}$ as a decimal? Show your thinking to solve.

4. A number multiplied by a fraction is not always smaller than the original number. Explain this and give at least two examples to support your thinking.

5. Elise has $\frac{3}{4}$ of a dollar. She buys a stamp that costs 44 cents. Change both numbers into decimals, and tell how much money Elise has after paying for the stamp.

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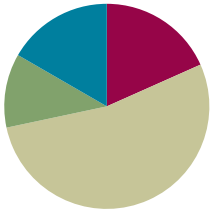
Lesson 22

Objective: Compare the size of the product to the size of the factors.

Suggested Lesson Structure



Fluency Practice	(11 minutes)
Application Problem	(7 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (11 minutes)

- Find the Unit Conversion **5.MD.2** (5 minutes)
- Multiply Fractions by Whole Numbers **5.NF.4** (4 minutes)
- Group Count by Multiples to 100 **5.NBT.2** (2 minutes)

Find the Unit Conversion (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 20.

T: (Write $3\frac{1}{4}$ gal = ____ qt and $3\frac{1}{4}$ gal = $3\frac{1}{4} \times 1$ gal.) How many quarts are in 1 gallon?

S: 4 quarts.

T: Write an equivalent multiplication sentence using an improper fraction and quarts.

S: (Write = $\frac{13}{4} \times 4$ qt.)

T: Solve and show.

S: (Work and hold up personal white boards.)

$3\frac{1}{4}$ gal = ____ qt

$= 3\frac{1}{4} \times 1$ gal

$= \frac{13}{4} \times 4$ qt

$= \frac{13 \times 4}{4}$ qt

$= 13$ qt

Continue with one or more of the following possible problems: $2\frac{2}{3}$ yd = ____ ft, $2\frac{5}{6}$ ft = ____ yd, $5\frac{1}{2}$ pt = ____ c, and $\frac{1}{2}$ c = ____ pt.

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Multiply Fractions by Whole Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 21.

T: (Write $\frac{1}{2} \times 10 =$ ____.) Say the multiplication sentence with the answer.

S: $\frac{1}{2} \times 10 = 5$.

T: (Write $10 \times \frac{1}{2} =$ ____.) Try this problem.

S: $10 \times \frac{1}{2} = 5$.

Continue the process with the following possible problems: $\frac{1}{3} \times 12$, $12 \times \frac{1}{3}$, $15 \times \frac{1}{5}$, and $\frac{1}{5} \times 15$.

T: (Write $\frac{1}{2} \times 6 =$ ____.) On your personal white boards, write the number sentence and the answer.

S: (Write $\frac{1}{2} \times 6 = 3$.)

T: (Write $\frac{2}{2} \times 6 =$ ____.) Write the multiplication sentence. Below it, rewrite the multiplication sentence as a whole number times 6.

S: (Write $\frac{2}{2} \times 6 =$ ____. Below it, write $1 \times 6 = 6$.)

T: (Write $\frac{3}{2} \times 6 =$ ____.) Write the number sentence and the answer.

S: (Write $\frac{3}{2} \times 6 = 9$.)

Continue with the following possible sequence: $8 \times \frac{1}{4}$, $8 \times \frac{4}{4}$, and $8 \times \frac{5}{4}$.

Group Count by Multiples to 100 (2 minutes)

Note: This fluency activity prepares students for Lesson 22.

T: Count by tens to 100. (Extend a finger each time a multiple is counted.)
S: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.
T: (Show 10 extended fingers.) How many tens are in 100?
S: 10.
T: (Write $10 \times 10 = 100$.) Count by twenties to 100. (Extend a finger each time a multiple is counted.)
S: 20, 40, 60, 80, 100.
T: (Show 5 extended fingers.) How many twenties are in 100?
S: 5.
T: (Write $20 \times 5 = 100$. Below it, write $5 \times$ ____ = 100.) How many fives are in 100?
S: 20.

Repeat the process by counting multiples of 4 and 25, as well as 2 and 50.

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Application Problem (7 minutes)

To test her math skills, Isabella's father told her he would give her $\frac{6}{8}$ of a dollar if she could tell him how much money it is, as well as the money amount in decimal form. What should Isabella tell her father? Show your calculations.

Note: This Application Problem reviews Lesson 21's Concept Development. Among other strategies, students might convert the eighths to fourths, and then multiply by $\frac{25}{25}$, or they may remember the decimal equivalent of 1 eighth and multiply by 6.

$\frac{6}{8} = \frac{3}{4}$

$\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = .75$

Isabella should tell her father that $\frac{6}{8}$ of a dollar is the same as 75 cents.

Concept Development (32 minutes)

Materials: (T) 12-inch string (S) Personal white board

Problem 1: a. $\frac{4}{4} \times$ **12 inches** **b.** $\frac{3}{4} \times$ **12 inches** **c.** $\frac{5}{4} \times$ **12 inches**

T: (Post Problem 1(a–c) on the board.) Find the products of these expressions.

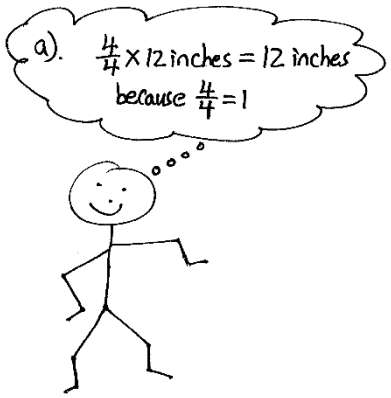
S: (Work.)

T: Let's compare the size of the products you found to the size of this factor. (Point to 12 inches.) Did multiplying 12 inches by 4 fourths change the length of this string? (Hold up the string.) Why or why not? Turn and talk.

S: The product is equal to 12 inches. → We multiplied and got 48 fourths, but that's just another name for 12 using a different unit. → It's 4 fourths of the string, all of it. → Multiplying by 1 means just 1 copy of the number, so it stays the same. → The other factor just named 1 as a fraction, but it is still just multiplying by 1, so the size of 12 won't change.

T: (Write $\frac{4}{4} \times 12 = 12$ under Problem 1(a).) Next consider Problem 1(b). Did multiplying by 3 fourths change the size of our other factor—12 inches? If so, how? Turn and talk.

S: The string became shorter because we only took 3 of 4 parts of it. → We got almost all of 12 inches, but not quite. We wanted 3 fourths of it rather than 4 fourths, so the factor became smaller after we multiplied. → We got 9 inches this time instead of 12 inches.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Whenever students calculate problems involving measurements, they benefit from having established mental benchmarks of each increment. For example, students should be able to think about 12 inches not just as a foot, but also as a specific length (perhaps a length just a little longer than a sheet of paper). Although teachers can give benchmarks for specific increments, it is probably better if students discover benchmarks on their own. Establishing mental benchmarks may be essential for English language learners' understanding.

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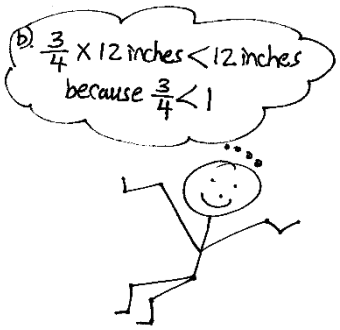
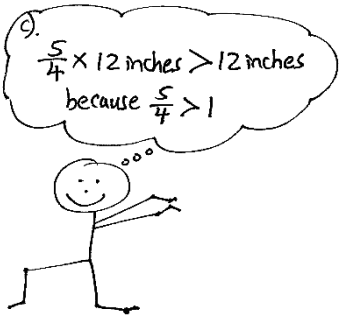


T: (Write $\frac{3}{4} \times 12 < 12$ under Problem 1(b).) I hear you saying that 12 inches was shortened—resized to 9 inches. How can it be that multiplying made 12 inches smaller when I thought multiplication always made numbers become larger? Turn and talk.

S: We took only part of 12 inches. When you take just a part of something, it is smaller than what you start with. → We ended up with 3 of the 4 parts, not the whole thing. → Adding $\frac{3}{4}$ twelve times is going to be smaller than adding one the same number of times.

T: So, 9 inches is 3 fourths as much as 12 inches. True or false?

S: True.



T: Let's consider our last expression—Problem 1(c). How did multiplying by 5 fourths change or not change the size of the other factor, 12 inches? How would it change the length of the string? Turn and talk.

MP.2 S: The answer to this one was greater than 12 inches because it's more than 4 fourths of it. → The product was greater than 12 inches. → $\frac{5}{4} \times 12 = \frac{60}{4} = 15$. → We copied a number greater than 1 twelve times. The answer had to be greater than copying 1 the same number of times. → 5 fourths of the string would be 1 fourth longer than the string is now.

T: (Write $\frac{5}{4} \times 12 > 12$ under Problem 1(c).) So, 15 inches is 5 fourths as much as 12 inches. True or false?

S: True.

T: 15 inches is 1 and $\frac{1}{4}$ times as much as 12 inches. True or false?

S: True.

T: We've compared our products to one factor, 12 inches, in each of these expressions. We explained the changes we noticed by thinking about the other factor. We can call that other factor a *scaling factor*. A scaling factor can change the size of the other factor. Let's look at the relationships in these expressions one more time. (Point to Problem 1(a).) When we multiplied 12 inches by a scaling factor equal to 1, what happened to the 12 inches?

S: 12 inches didn't change. → The product was the same size as 12 inches, even after we multiplied it.

T: (Point to Problem 1(b).) In this expression, $\frac{3}{4}$ was the scaling factor. Was this scaling factor more than or less than 1? How do you know?

S: Less than 1 because 4 fourths is 1.

T: What happened to the length of the string?

S: It became shorter.

T: (Point to Problem 1(c).) Also, in our last expression, what was the scaling factor?

S: 5 fourths.

T: Was 5 fourths more or less than 1?

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S: More than 1.

T: What happened to the length of the string?

S: It became longer. → The product was larger than 12 inches.

Problem 2: **a.** $\frac{4}{4} \times \frac{1}{3}$ **b.** $\frac{3}{4} \times \frac{1}{3}$ **c.** $\frac{5}{4} \times \frac{1}{3}$

T: (Post Problem 2 (a–c) on the board.) Considering the relationships that we've just noticed between our products and factors, evaluate these expressions.

S: (Work.)

T: Let's compare the products that you found to this factor. (Point to $\frac{1}{3}$.) What is the product of $\frac{1}{3}$ and $\frac{4}{4}$?

S: $\frac{4}{12}$.

T: Did the size of $\frac{1}{3}$ change when we multiplied it by a scaling factor equal to 1?

S: No.

T: (Write $\frac{4}{4} \times \frac{1}{3} = \frac{1}{3}$ under Problem 2(a).) Since we are comparing our product to 1 third, what is the scaling factor in the second expression? (Point to Problem 2(b).)

S: $\frac{3}{4}$.

T: Is this scaling factor more than or less than 1?

S: Less than 1.

T: What happened to the size of $\frac{1}{3}$ when we multiplied it by a scaling factor less than 1? Why? Turn and share.

S: The product was 3 twelfths. That is less than 1 third, which is 4 twelfths. → We only wanted part of 1 third this time, so the answer had to be smaller than 1 third. → When you multiply by less than 1, the product is smaller than what you started with.

T: (Write $\frac{3}{4} \times \frac{1}{3} < \frac{1}{3}$ under Problem 2(b).) In the last expression, $\frac{5}{4}$ is the scaling factor. Is the scaling factor more than or less than 1?

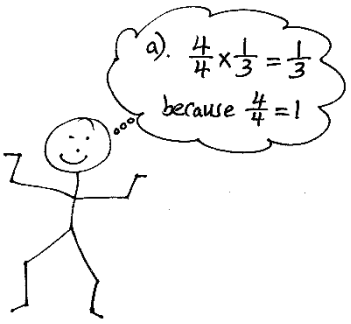
S: More than 1.

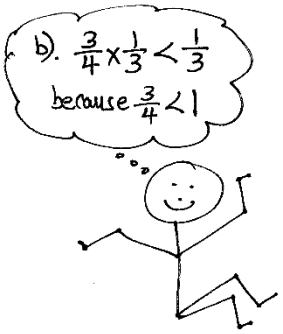
T: Say the product of $\frac{5}{4} \times \frac{1}{3}$.

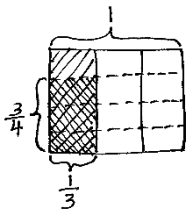
S: $\frac{5}{12}$.

T: Is 5 twelfths more than, less than, or equal to $\frac{1}{3}$?

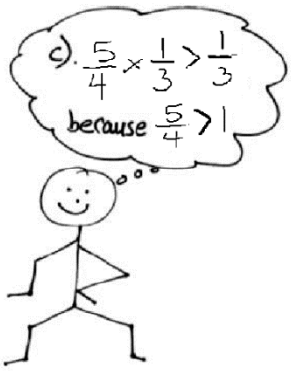
S: More than $\frac{1}{3}$.







$\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$



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T: (Write $\frac{5}{4} \times \frac{1}{3} > \frac{1}{3}$ under Problem 2(c).) Explain why the product of $\frac{1}{3}$ and $\frac{5}{4}$ is more than $\frac{1}{3}$.

S: (Share.)

Problem 3: **a.** $\frac{1}{2} \times \frac{5}{5}$ **b.** $\frac{1}{2} \times \frac{3}{5}$ **c.** $\frac{1}{2} \times \frac{9}{5}$

T: I'm going to show you some multiplication expressions where we start with $\frac{1}{2}$. The expressions will have different scaling factors. Think about what will happen to the size of 1 half when it is multiplied by the scaling factor. Tell whether the product will be equal to $\frac{1}{2}$, more than $\frac{1}{2}$, or less than $\frac{1}{2}$. Ready? (Show $\frac{1}{2} \times \frac{5}{5}$.)

S: Equal to $\frac{1}{2}$.

T: Tell a neighbor why.

S: The scaling factor is equal to 1.

T: (Show $\frac{1}{2} \times \frac{3}{5}$.)

S: Less than $\frac{1}{2}$.

T: Tell a neighbor why.

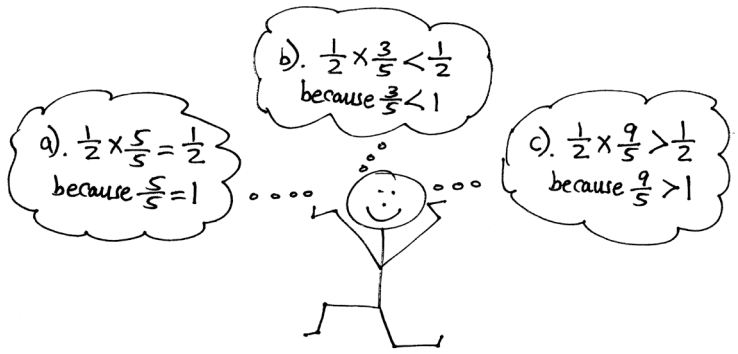
S: The scaling factor is less than 1.

T: (Show $\frac{1}{2} \times \frac{9}{5}$.)

S: More than $\frac{1}{2}$.

T: Tell a neighbor why.

S: The scaling factor is more than 1.



Repeat the questioning with the following possible problems: $\frac{1}{2} \times \frac{2}{3}$, $\frac{1}{2} \times \frac{1}{2}$, $\frac{1}{2} \times \frac{4}{3}$, and $\frac{1}{2} \times \frac{8}{8}$.

Problem 4:

At the book fair, Vald spends all of his money on new books. Pamela spends $\frac{2}{3}$ as much as Vald. Eli spends $\frac{4}{3}$ as much as Vald. Who spent the most? The least?

T: (Post Problem 4 on the board, and read it aloud with the students.) Read the first sentence again out loud.

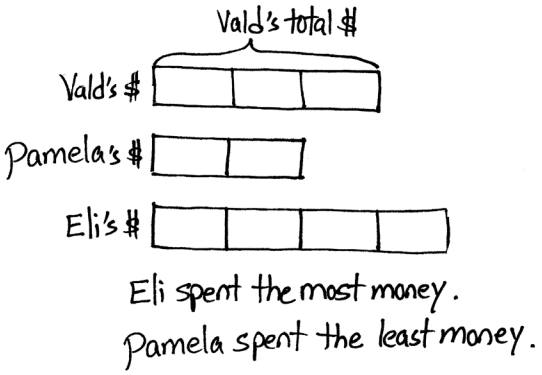
S: (Read.)

T: Before we begin drawing, to whose money will we make the comparisons?

S: Vald's money.

T: What can we draw from the first sentence?

S: We can make a tape diagram. → We should label a tape diagram *Vald's money*.



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T: Vald spent all of his money at the book fair. I'll draw a tape diagram and label it *Vald's money.* (Write *Vald's $*.) Read the next sentence aloud.

S: (Read.)

T: What can we draw from this sentence?

S: We can draw another tape that is shorter than Vald's.

T: Let me record that. (Draw a shorter tape representing Pamela's money.) How will we know how much shorter to draw it? Turn and talk.

S: We know she spent $\frac{2}{3}$ of the same amount. Since Pamela's units are thirds, we can split Vald's tape into 3 equal units, and then draw a tape below it that is 2 units long and label it *Pamela's money*. → I know Pamela's has 2 units, and those 2 units are 2 out of the 3 that Vald spent. I'll draw 2 units for Pam, and then make Vald's 1 unit longer than hers.

T: I'll record that. Thinking of $\frac{2}{3}$ as a scaling factor, did Pamela spend more or less than Vald? How do you know? Does our tape diagram show it?

S: Pamela spent less than Vald. If you think of $\frac{2}{3}$ as a scaling factor, it's less than 1, so she spent less than Vald. That's how we drew it. → She spent less than Vald. She only spent a part of the same amount as Vald. → Vald spent all his money, or $\frac{3}{3}$ of his money. Pamela only spent $\frac{2}{3}$ as much as Vald. You can see that in the diagram.

T: Read the third sentence and discuss what you can draw from this information.

S: (Read and discuss.)

T: Eli spent $\frac{4}{3}$ as much as Vald. If we think of $\frac{4}{3}$ as a scaling factor, what does that tell us about how much money Eli spent?

S: Eli spent more than Vald because $\frac{4}{3}$ is more than 1. → Again, Vald spent all of his money, or $\frac{3}{3}$ of it. $\frac{4}{3}$ is more than $\frac{3}{3}$, so Eli spent more than Vald. We have to draw a tape diagram that is one-third more than Vald's.

T: Since the scaling factor $\frac{4}{3}$ is more than 1, I'll draw a third tape diagram for Eli that is longer than Vald's money. What is the question we have to answer?

S: Who spent the most and least money at the book fair?

T: Does our tape diagram show enough information to answer this question?

S: Yes, it's very easy to see whose tape diagram is the longest and shortest. → Even though we don't know exactly how much Vald spent, we can still answer the question. Since the scaling factors are more than 1 and less than 1, we know who spent the most and least amount of money.

T: Answer the question in a complete sentence.

S: Eli spent the most money. Pamela spent the least money.

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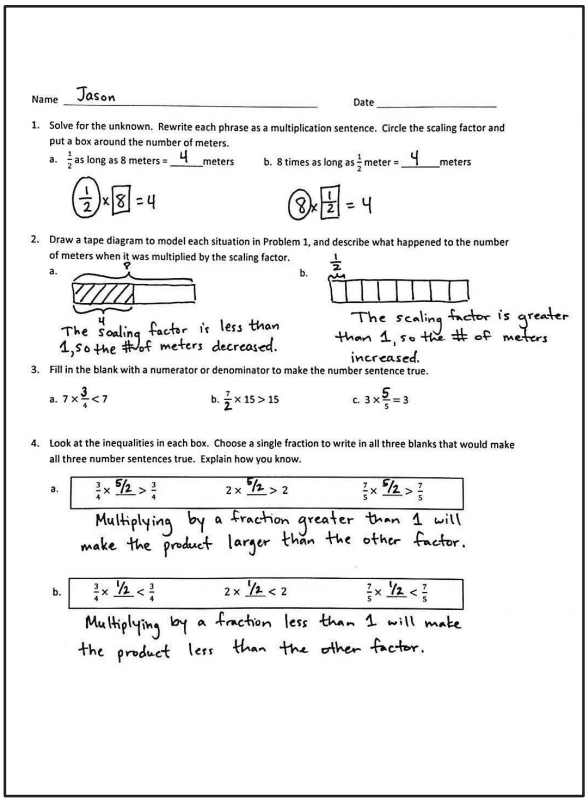
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Jason Date

1. Solve for the unknown. Rewrite each phrase as a multiplication sentence. Circle the scaling factor and put a box around the number of meters.
   a. $\frac{1}{2}$ as long as 8 meters = 4 meters b. 8 times as long as $\frac{1}{2}$ meter = 4 meters
   $(\frac{1}{2}) \times [8] = 4$ $(8) \times [\frac{1}{2}] = 4$
2. Draw a tape diagram to model each situation in Problem 1, and describe what happened to the number of meters when it was multiplied by the scaling factor.
   a. 8 / 4 — The scaling factor is less than 1, so the # of meters decreased.
   b. $\frac{1}{2}$ — The scaling factor is greater than 1, so the # of meters increased.
3. Fill in the blank with a numerator or denominator to make the number sentence true.
   a. $7 \times \frac{3}{4} < 7$ b. $\frac{7}{2} \times 15 > 15$ c. $3 \times \frac{5}{5} = 3$
4. Look at the inequalities in each box. Choose a single fraction to write in all three blanks that would make all three number sentences true. Explain how you know.
   a. $\frac{3}{4} \times \frac{5}{2} > \frac{3}{4}$ $2 \times \frac{5}{2} > 2$ $\frac{7}{5} \times \frac{5}{2} > \frac{7}{5}$
   Multiplying by a fraction greater than 1 will make the product larger than the other factor.
   b. $\frac{3}{4} \times \frac{1}{2} < \frac{3}{4}$ $2 \times \frac{1}{2} < 2$ $\frac{7}{5} \times \frac{1}{2} < \frac{7}{5}$
   Multiplying by a fraction less than 1 will make the product less than the other factor.

Student Debrief (10 minutes)

Lesson Objective: Compare the size of the product to the size of the factors.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what relationship did you notice between Parts (a) and (b)?
- For Problem 2, compare your tape diagrams with a partner. Are your drawings similar to or different from your partner's?
- Explain to a partner your thought process for solving Problem 3. How did you know what to put for the missing numerator or denominator?
- In Problem 4, did you notice a relationship between Parts (a) and (b)? How did you solve them?
- For Problem 5, did you and your partner use the same examples to support the solution? Can you also give some examples to support the idea that multiplication can make numbers larger?
- What's the scaling factor in Problem 6? What is an expression to solve this problem?
- How did you solve Problem 7? Share your solution and explain your strategy to a partner.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students may find it helpful to have a physical representation of the tape diagrams as they work to draw these models. Students can use square tiles or unifix cubes. For some students, arranging the manipulatives first and then drawing may be easier. Additionally, it may eliminate the need to redraw or erase.

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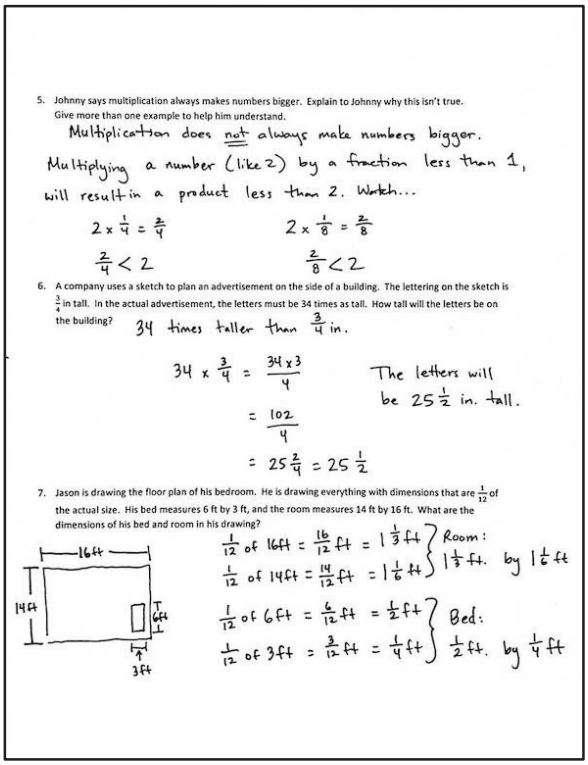
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



5. Johnny says multiplication always makes numbers bigger. Explain to Johnny why this isn't true. Give more than one example to help him understand.

Multiplication does not always make numbers bigger. Multiplying a number (like 2) by a fraction less than 1, will result in a product less than 2. Watch...

$2 \times \frac{1}{4} = \frac{2}{4}$ $\quad$ $2 \times \frac{1}{8} = \frac{2}{8}$

$\frac{2}{4} < 2$ $\quad$ $\frac{2}{8} < 2$

6. A company uses a sketch to plan an advertisement on the side of a building. The lettering on the sketch is $\frac{3}{4}$ in tall. In the actual advertisement, the letters must be 34 times as tall. How tall will the letters be on the building?

34 times taller than $\frac{3}{4}$ in.

$34 \times \frac{3}{4} = \frac{34 \times 3}{4}$

$= \frac{102}{4}$

$= 25\frac{2}{4} = 25\frac{1}{2}$

The letters will be $25\frac{1}{2}$ in. tall.

7. Jason is drawing the floor plan of his bedroom. He is drawing everything with dimensions that are $\frac{1}{12}$ of the actual size. His bed measures 6 ft by 3 ft, and the room measures 14 ft by 16 ft. What are the dimensions of his bed and room in his drawing?

$\frac{1}{12}$ of 16ft $= \frac{16}{12}$ ft $= 1\frac{1}{3}$ ft

$\frac{1}{12}$ of 14ft $= \frac{14}{12}$ ft $= 1\frac{1}{6}$ ft

Room: $1\frac{1}{3}$ ft. by $1\frac{1}{6}$ ft

$\frac{1}{12}$ of 6ft $= \frac{6}{12}$ ft $= \frac{1}{2}$ ft

$\frac{1}{12}$ of 3ft $= \frac{3}{12}$ ft $= \frac{1}{4}$ ft

Bed: $\frac{1}{2}$ ft. by $\frac{1}{4}$ ft

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Name _____ Date _____

1. Solve for the unknown. Rewrite each phrase as a multiplication sentence. Circle the scaling factor and put a box around the number of meters.

   a. $\frac{1}{2}$ as long as 8 meters = _____ meter(s)

   b. 8 times as long as $\frac{1}{2}$ meter = _____ meter(s)

2. Draw a tape diagram to model each situation in Problem 1, and describe what happened to the number of meters when it was multiplied by the scaling factor.

   a.

   b.

3. Fill in the blank with a numerator or denominator to make the number sentence true.

   a. $7 \times \frac{\quad}{4} < 7$

   b. $\frac{7}{\quad} \times 15 > 15$

   c. $3 \times \frac{\quad}{5} = 3$

4. Look at the inequalities in each box. Choose a single fraction to write in all three blanks that would make all three number sentences true. Explain how you know.

   a.

   $\frac{3}{4} \times ____ > \frac{3}{4}$ $\quad 2 \times ____ > 2$ $\quad \frac{7}{5} \times ____ > \frac{7}{5}$

   b.

   $\frac{3}{4} \times ____ < \frac{3}{4}$ $\quad 2 \times ____ < 2$ $\quad \frac{7}{5} \times ____ < \frac{7}{5}$

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5. Johnny says multiplication always makes numbers bigger. Explain to Johnny why this isn't true. Give more than one example to help him understand.

6. A company uses a sketch to plan an advertisement on the side of a building. The lettering on the sketch is $\frac{3}{4}$ inch tall. In the actual advertisement, the letters must be 34 times as tall. How tall will the letters be on the building?

7. Jason is drawing the floor plan of his bedroom. He is drawing everything with dimensions that are $\frac{1}{12}$ of the actual size. His bed measures 6 ft by 3 ft, and the room measures 14 ft by 16 ft. What are the dimensions of his bed and room in his drawing?

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Name _____ Date _____

Fill in the blank to make the number sentences true. Explain how you know.

a. $\frac{\quad}{3} \times 11 > 11$

b. $5 \times \frac{\quad}{8} < 5$

c. $6 \times \frac{2}{\quad} = 6$

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Name _____ Date _____

1. Solve for the unknown. Rewrite each phrase as a multiplication sentence. Circle the scaling factor and put a box around the number of meters.

   a. $\frac{1}{3}$ as long as 6 meters = _____ meter(s)

   b. 6 times as long as $\frac{1}{3}$ meter = _____ meter(s)

2. Draw a tape diagram to model each situation in Problem 1, and describe what happened to the number of meters when it was multiplied by the scaling factor.

   a.

   b.

3. Fill in the blank with a numerator or denominator to make the number sentence true.

   a. $5 \times \frac{\quad}{3} > 5$

   b. $\frac{6}{\quad} \times 12 < 12$

   c. $4 \times \frac{\quad}{5} = 4$

4. Look at the inequalities in each box. Choose a single fraction to write in all three blanks that would make all three number sentences true. Explain how you know.

   a.

$\frac{2}{3} \times ____ > \frac{2}{3}$	$4 \times ____ > 4$	$\frac{5}{3} \times ____ > \frac{5}{3}$

   b.

$\frac{2}{3} \times ____ < \frac{2}{3}$	$4 \times ____ < 4$	$\frac{5}{3} \times ____ < \frac{5}{3}$

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5. Write a number in the blank that will make the number sentence true.

   a. $3 \times _____ < 1$

   b. Explain how multiplying by a whole number can result in a product less than 1.

6. In a sketch, a fountain is drawn $\frac{1}{4}$ yard tall. The actual fountain will be 68 times as tall. How tall will the fountain be?

7. In blueprints, an architect's firm drew everything $\frac{1}{24}$ of the actual size. The windows will actually measure 4 ft by 6 ft and doors measure 12 ft by 8 ft. What are the dimensions of the windows and the doors in the drawing?

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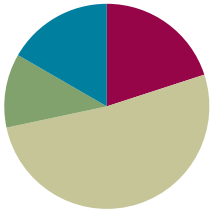
Lesson 23

Objective: Compare the size of the product to the size of the factors.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Compare the Size of a Product to the Size of One Factor **5.NF.5** (5 minutes)
- Compare Decimal Numbers **5.NBT.2** (2 minutes)
- Write Fractions as Decimals **5.NBT.2** (5 minutes)

Compare the Size of a Product to the Size of One Factor (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 21.

T: (Write $1 = \frac{\quad}{6}$.) On your personal white board, fill in the missing numerator.

S: (Write $1 = \frac{6}{6}$.)

T: (Write $9 \times ____ = 9$.) Say the missing whole number factor.

S: 1.

T: (Write $9 \times \frac{\quad}{2} = 9$.) Fill in the missing numerator to make a true number sentence.

S: (Write $9 \times \frac{2}{2} = 9$.)

T: (Write $9 \times \frac{\quad}{2} < 9$.) Fill in the missing numerator to make a true number sentence.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

It is very helpful for most learners to know and understand the objective of specific lessons. This knowledge helps them make connections to what they already know and what they need to learn. Each lesson in the modules has a stated objective. These objectives should be posted daily. Today's goal, comparing the size of the factors to the size of the product, includes math vocabulary that students should know. For a quick visual review, write an equation below the objective and draw lines showing which number is the factor and which number is the product. The objective is discussed and explored daily during the Debrief.

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S: (Write $9 \times \frac{1}{2} < 9$.)

T: (Write $9 \times \frac{\quad}{2} > 9$.) Fill in a missing numerator to make a true number sentence.

S: (Write the number sentence, filling in a numerator greater than 2.)

Continue this process with the following possible sequence: $\frac{3}{\quad} \times 7 = 7$, $\frac{3}{\quad} \times 6 < 6$, $\frac{3}{\quad} \times 6 > 6$, $\frac{4}{\quad} \times 8 < 8$, $\frac{\quad}{5} \times 9 = 9$, and $\frac{5}{\quad} \times 10 < 10$.

Compare Decimal Numbers (2 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: (Write 1 ____ 9.) Say the greater number.

S: 9.

T: On your personal white board, write the symbol to make the number sentence true.

S: (Write $1 < 9$.)

Continue this process with the following possible sequence: 1 ____ 0.9, 0.95 ____ 1, and 0.994 ____ 1.

Write Fractions as Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 22.

T: (Write $\frac{1}{50} = \frac{\quad}{100}$.) How many fifties are in 100?

S: 2.

T: (Write $\frac{1}{50} \times \frac{2}{2} = \frac{\quad}{100}$.) $\frac{1}{50}$ is the same as how many hundredths?

S: 2 hundredths.

T: (Write $\frac{1}{50} \times \frac{2}{2} = \frac{2}{100}$. Below it, write $\frac{1}{50}$ = ____.____.) On your personal white board, write $\frac{1}{50}$ as a decimal.

S: (Write $\frac{1}{50} = 0.02$.)

Continue this process with the following possible sequence: $\frac{3}{50}$, $\frac{7}{50}$, $3\frac{7}{50}$, $\frac{1}{20}$, $\frac{3}{20}$, $5\frac{3}{20}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{6}{5}$, $\frac{1}{25}$, $\frac{7}{25}$, $4\frac{7}{25}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{11}{4}$.

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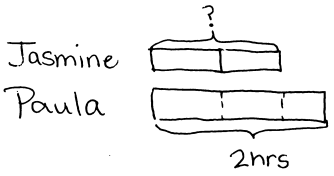
Application Problem (7 minutes)

Jasmine took $\frac{2}{3}$ as much time to take a math test as Paula. If Paula took 2 hours to take the test, how long did it take Jasmine to take the test? Express your answer in minutes.

Note: Scaling, as well as conversion, is required for today's Application Problem. This reviews Topic E and prepares students for continuing a study of scaling with decimals in today's lesson.



$\frac{2}{3} \times 2\text{hrs}$
$= \frac{4}{3}\text{ hrs}$
$= \frac{4}{3} \times 60\text{ min}$
$= \frac{240}{3}\text{ min}$
$= 80\text{ min}$

Jasmine finished the test in 80 minutes.

Concept Development (31 minutes)

Materials: (S) Personal white board

Problem 1: a. **2 meters $\times \frac{97}{100}$** **b.** **2 meters $\times \frac{101}{100}$** **c.** **2 meters $\times \frac{100}{100}$**

T: (Post Problem 1(a–c) on the board.) Let's compare products to the 2 meters in each expression. Let's notice what happens to 2 meters when we multiply, or scale, 2 meters by the other factors. Read the scaling factors out loud in the order they are written.

$2 \times \frac{97}{100}$ $2 \times \frac{101}{100}$ $2 \times \frac{100}{100}$

$2 \times 0.97 < 2$ $2 \times 1.01 > 2$ $2 \times 1.00 = 2$

S: 97 hundredths, 101 hundredths, 100 hundredths.

T: Without evaluating them, turn and talk with a neighbor about which expression is greater than, less than, and equal to 2 meters. Be sure to explain your thinking.

S: $2 \times \frac{100}{100}$ would be equal to 2 meters because it's being scaled by 1. → 2 meters $\times \frac{97}{100}$ would be less than 2 meters because it's being scaled by a fraction less than 1. → 2 meters $\times \frac{101}{100}$ would be more than 2 meters because it's being scaled by a fraction more than 1.

T: Rewrite the expressions using decimals to express the scaling factors.

S: (Work and show 2 meters × 0.97, 2 meters × 1.01, and 2 meters × 1.0.)

T: (Write decimal expressions below the fractional ones.) Which expression is greater than, less than, and equal to 2 meters? Turn and talk.

S: It's the same as before: 2 times 1 is equal to 1, 2 times 0.97 is less than 2, and 2 times 1.01 is more than 2. → Nothing has changed; we've just expressed the scaling factor as a decimal. We haven't changed the value.

T: (Write 2 × ____ < 2 on the board.) Write three decimal scaling factors that would make this number sentence true.

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S: (Work and show numbers less than 1.0.)

T: Finish my sentence. To get a product that is *less* than the number you started with, multiply by a scaling factor that is…

S: Less than 1.

T: (Write 2 × ____ > 2 on the board.) Show me some more decimal scaling factors that would make this number sentence true.

S: (Work and show numbers more than 1.0.)

T: Finish this sentence. To obtain a product that is *more* than the number you started with, multiply by a scaling factor that is….

S: More than 1.

Problem 2: a. 19.4 × 0.96 b. 19.4 × 0.02

T: (Post Problems 2(a) and (b) on the board.) Let's compare our product to the first factor—19.4. Let's consider the other factors the scaling factors. Read the scaling factors out loud in the order they are written.

$19.4 \times 0.96 < 19.4$ $19.4 \times 0.02 < 19.4$

S: 96 hundredths, 2 hundredths.

T: Look at the first expression. Will the product be more than, less than, or equal to 19.4? Tell a neighbor why.

S: The product will be less than 19.4 because the scaling factor is less than 1.

T: (Write < 19.4 next to Problem 2(a).) Look at the second expression. Will the product be more than, less than, or equal to 19.4? Tell a neighbor why.

S: It's also less than 19.4 because that scaling factor is also less than 1.

T: (Write < 19.4 next to Problem 2(b).) So, we know that both scaling factors will result in a product that is less than the number we started with. Which expression will give a greater product? Why? Turn and talk.

S: 19.4 times 96 hundredths will give a greater product. → Even though both scaling factors are less than 1, 96 hundredths is a much larger scaling factor than 2 hundredths. → 96 hundredths is close to 1. 2 hundredths is almost zero. The first expression will be really close to 19.4, and the second expression will be closer to zero.

T: (Point to Problem 2(a).) What is the scaling factor here?

S: 96 hundredths.

T: What would the scaling factor need to be for the product to be equal to 19.4?

S: 1.

T: Isn't 1 the same as 100 hundredths?

S: Yes.

T: So, this scaling factor, 96 hundredths, is *slightly less than* 1. True or false?

S: True.

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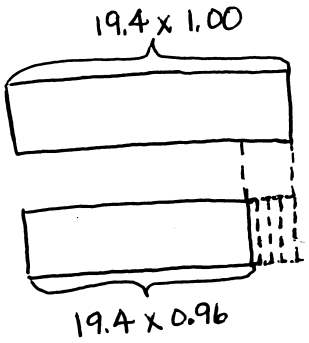
T: If this is true, what can we say about the product of 19.4 and 0.96? Turn and talk.

S: If we draw a tape diagram of 19.4 × 1, it would be 19.4 units long. Since 96 hundredths is just slightly less than 1, this means that 19.4 × 0.96 is slightly less than 19.4 × 1. The tape diagram should be slightly shorter than the first one we drew. → The expression 19.4 times 96 hundredths is just slightly less than 19.4.



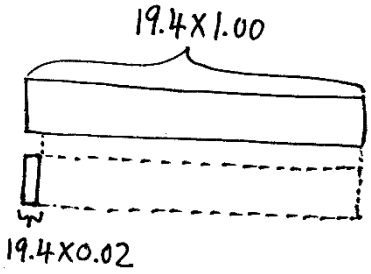
19.4 x 0.96 is slightly less than 19.4.

T: (Draw the first tape diagram as shown.) Imagine partitioning this tape into 100 equal parts. The tape for 19.4 times 96 hundredths should be as long as 96 of those hundredths, or just 4 hundredths less than this whole tape. (Draw a second tape diagram slightly shorter and label it 19.4 × 0.96.)

T: Make a statement about this expression. Is 19.4 times 96 hundredths slightly less than 19.4, or a lot less than 19.4?

S: It is slightly less than 19.4.

T: (Write *19.4 × 0.96 is slightly less than 19.4.*) Let's look at the other expression now. Is the scaling factor, 2 hundredths, slightly less than 1 or a lot less than 1? Turn and talk.



19.4X0.02 is a lot less than 19.4.

S: 1 is 100 hundredths; this is only 2 hundredths. It's a lot less than 1. → It's a lot less than 1. In fact, it's only slightly more than zero.

T: This scaling factor is a lot less than 1. Work with a partner to draw two tape diagrams. One should show 19.4, like we did before, and the other should show 19.4 times 2 hundredths.

S: (Work and share.)

T: Make a statement about this expression. Is 19.4 times 2 hundredths slightly less than 19.4, or a lot less than 19.4?

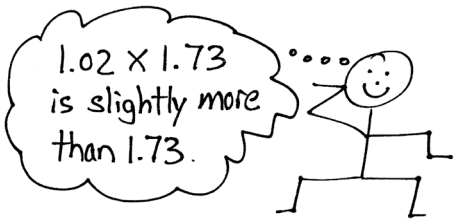
S: It is a lot less than 19.4.

T: (Write *19.4 × 0.02 is a lot less than 19.4.*)

Problem 3: a. 1.02 × 1.73 b. 29.01 × 1.73

T: (Post Problems 3(a) and (b) on the board.) Let's compare our products to the second factor in these expressions. (Point to 1.73 in both expressions.) We'll consider the first factors to be scaling factors. Read the scaling factors out loud in the order they are written.

S: 1 and 2 hundredths, 29 and 1 hundredth.



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T: Think about these expressions. Will the products be more than, less than, or equal to 1.73? Tell your neighbor why.

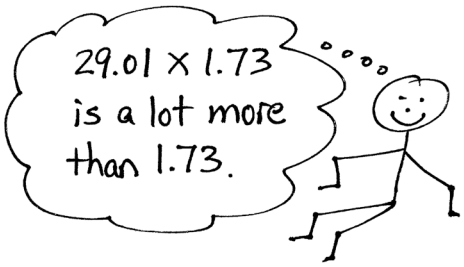
S: They'll both be more than 1.73 because both scaling factors are more than 1.

T: Let's be more specific. Look at the first expression. Will the product be slightly more than 1.73, or a lot more than 1.73? Tell a neighbor.

S: The product will be just slightly more than 1.73. The scaling factor is just 2 hundredths more than 1. → I can visualize two tape diagrams, and the one showing 1.73 times 1.02 is just a little bit longer, like 2 hundredths times longer than the tape showing 1.73. → The product will be slightly more than what we started with because the scaling factor is just slightly more than 1.

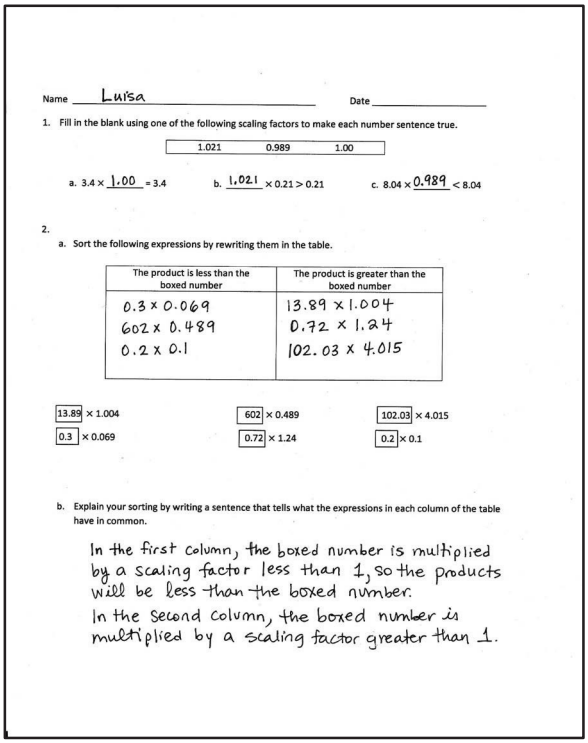
T: (Write *1.02 × 1.73 is slightly more than 1.73*.) Think about the second expression. Will its product be slightly more than 1.73, or a lot more than 1.73? Tell a neighbor.

S: The product will just be a lot more than 1.73. The scaling factor is almost 30 times more than 1, so the product will be almost 30 times more, too. → I can visualize two tape diagrams, and the one showing 1.73 times 29.01 is a lot longer, like 29 times longer than the tape showing just 1.73. → The product will be a lot more than what we started with because the scaling factor is a lot more than 1.



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Luisa Date

1. Fill in the blank using one of the following scaling factors to make each number sentence true.

1.021	0.989	1.00

a. 3.4 × 1.00 = 3.4 b. 1.02 × 0.21 > 0.21 c. 8.04 × 0.989 < 8.04

2. a. Sort the following expressions by rewriting them in the table.

The product is less than the boxed number	The product is greater than the boxed number
0.3 × 0.069 602 × 0.489 0.2 × 0.1	13.89 × 1.004 0.72 × 1.24 102.03 × 4.015

13.89 × 1.004 602 × 0.489 102.03 × 4.015
0.3 × 0.069 0.72 × 1.24 0.2 × 0.1

b. Explain your sorting by writing a sentence that tells what the expressions in each column of the table have in common.

In the first column, the boxed number is multiplied by a scaling factor less than 1, so the products will be less than the boxed number.
In the second column, the boxed number is multiplied by a scaling factor greater than 1.

Student Debrief (10 minutes)

Lesson Objective: Compare the size of the product to the size of the factors.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

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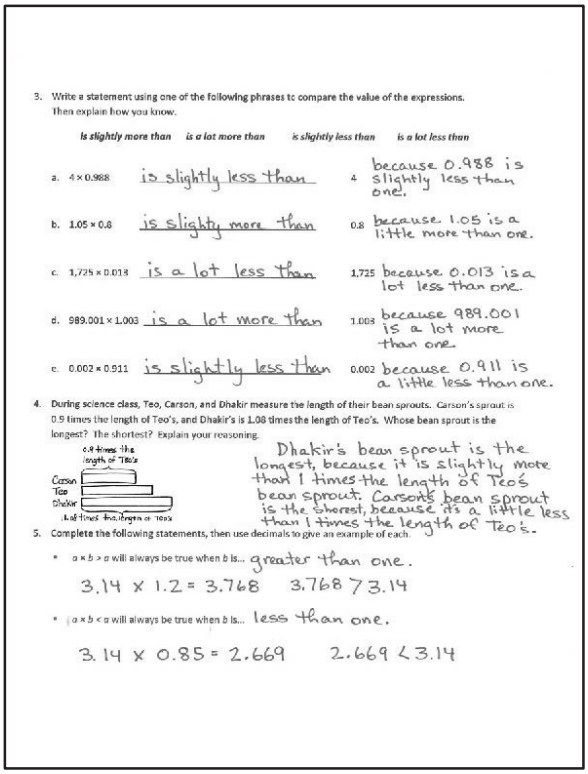
Any combination of the questions below may be used to lead the discussion.

- Share your solutions and explain your thought process for solving Problem 1 to a partner. How did you decide which number goes into which number sentence?
- Compare your solutions for Problem 2 with a partner. Did you have different answers? If so, explain your thinking behind each sorting.
- What was your strategy for solving Problem 3? Share it with a partner.
- How did you solve Problem 4? Did you make a drawing or tape diagram to compare the sprouts? Share it with and explain it to a partner.
- Share your decimal examples for Problem 5 with a partner. Did you have the same or different examples?



3. Write a statement using one of the following phrases to compare the value of the expressions. Then explain how you know.

is slightly more than *is a lot more than* *is slightly less than* *is a lot less than*

a. 4×0.988 is slightly less than 4 because 0.988 is slightly less than one.

b. 1.05×0.8 is slighty more than 0.8 because 1.05 is a little more than one.

c. $1,725 \times 0.013$ is a lot less than 1,725 because 0.013 is a lot less than one.

d. 989.001×1.003 is a lot more than 1.003 because 989.001 is a lot more than one.

e. 0.002×0.911 is slightly less than 0.002 because 0.911 is a little less than one.

4. During science class, Teo, Carson, and Dhakir measure the length of their bean sprouts. Carson's sprout is 0.9 times the length of Teo's, and Dhakir's is 1.08 times the length of Teo's. Whose bean sprout is the longest? The shortest? Explain your reasoning.

0.9 times the length of Teo's
Carson
Teo
Dhakir
1.08 times the length of Teo's

Dhakir's bean sprout is the longest, because it is slightly more that 1 times the length of Teo's bean sprout. Carson's bean sprout is the shortest, because it's a little less than 1 times the length of Teo's.

5. Complete the following statements, then use decimals to give an example of each.

* $a \times b > a$ will always be true when b is... greater than one.

  $3.14 \times 1.2 = 3.768$ $3.768 > 3.14$

* $a \times b < a$ will always be true when b is... less than one.

  $3.14 \times 0.85 = 2.669$ $2.669 < 3.14$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. Fill in the blank using one of the following scaling factors to make each number sentence true.

1.021	0.989	1.00

a. 3.4 × _____ = 3.4

b. _____ × 0.21 > 0.21

c. 8.04 × _____ < 8.04

2.

a. Sort the following expressions by rewriting them in the table.

The product is less than the boxed number:	The product is greater than the boxed number:

$\boxed{13.89} \times 1.004$ $\boxed{602} \times 0.489$ $\boxed{102.03} \times 4.015$

$\boxed{0.3} \times 0.069$ $\boxed{0.72} \times 1.24$ $\boxed{0.2} \times 0.1$

b. Explain your sorting by writing a sentence that tells what the expressions in each column of the table have in common.

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3. Write a statement using one of the following phrases to compare the value of the expressions. Then, explain how you know.

	is slightly more than	*is a lot more than*	*is slightly less than*	*is a lot less than*

a. 4×0.988 _____ 4

b. 1.05×0.8 _____ 0.8

c. $1{,}725 \times 0.013$ _____ 1,725

d. 989.001×1.003 _____ 1.003

e. 0.002×0.911 _____ 0.002

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4. During science class, Teo, Carson, and Dhakir measure the length of their bean sprouts. Carson's sprout is 0.9 times the length of Teo's, and Dhakir's is 1.08 times the length of Teo's. Whose bean sprout is the longest? The shortest? Explain your reasoning.

5. Complete the following statements; then use decimals to give an example of each.

   - $a \times b > a$ will always be true when b is…

   - $a \times b < a$ will always be true when b is…

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Name _____ Date _____

1. Fill in the blank using one of the following scaling factors to make each number sentence true.

1.009	1.00	0.898

a. 3.06 × _____ < 3.06

b. 5.2 × _____ = 5.2

c. _____ × 0.89 > 0.89

2. Will the product of 22.65 × 0.999 be greater than or less than 22.65? Without calculating, explain how you know.

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Name _____ Date _____

1.

a. Sort the following expressions by rewriting them in the table.

The product is less than the boxed number:	The product is greater than the boxed number:

$\boxed{12.5} \times 1.989$	$\boxed{828} \times 0.921$	$\boxed{321.46} \times 1.26$
$\boxed{0.007} \times 1.02$	$\boxed{2.16} \times 1.11$	$\boxed{0.05} \times 0.1$

b. What do the expressions in each column have in common?

2. Write a statement using one of the following phrases to compare the value of the expressions. Then, explain how you know.

is slightly more than ***is a lot more than*** ***is slightly less than*** ***is a lot less than***

a. 14×0.999 _____ 14

b. 1.01×2.06 _____ 2.06

c. $1{,}955 \times 0.019$ _____ 1,955

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d. Two thousand × 1.0001 _____ two thousand

e. Two-thousandths × 0.911 _____ two-thousandths

3. Rachel is 1.5 times as heavy as her cousin, Kayla. Another cousin, Jonathan, weighs 1.25 times as much as Kayla. List the cousins, from lightest to heaviest, and explain your thinking.

4. Circle your choice.

a. $a \times b > a$

For this statement to be true, b must be **greater than 1** **less than 1**

Write two expressions that support your answer. Be sure to include one decimal example.

b. $a \times b < a$

For this statement to be true, b must be **greater than 1** **less than 1**

Write two expressions that support your answer. Be sure to include one decimal example.

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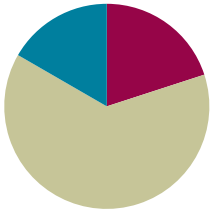
Lesson 24

Objective: Solve word problems using fraction and decimal multiplication.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(38 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Compare the Size of a Product to the Size of One Factor **5.NF.5** (4 minutes)
- Write Fractions as Decimals **5.NBT.2** (5 minutes)
- Write the Scaling Factor **5.NBT.3** (3 minutes)

Compare the Size of a Product to the Size of One Factor (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 21.

T: How many halves are in 1?
S: 2.
T: How many thirds are in 1?
S: 3.
T: How many fourths are in 1?
S: 4.
T: (Write $1 = \frac{\quad}{10}$.) On your personal white board, fill in the missing numerator.
S: (Write $1 = \frac{10}{10}$.)
T: (Write $6 \times ____ = 6$.) Say the missing factor.
S: 1.
T: (Write $6 \times \frac{\quad}{3} = 6$.) On your personal white board, write the equation, filling in the missing numerator.
S: (Write $6 \times \frac{3}{3} = 6$.)
T: (Write $6 \times \frac{\quad}{3} < 6$.) Fill in a numerator to make a true number sentence.

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S: (Write $6 \times \frac{1}{3} < 6$ or $6 \times \frac{2}{3} < 6$.)

T: (Write $9 \times \frac{\quad}{6} > 9$.) Fill in a numerator to make a true number sentence.

S: (Write a number sentence, filling in a numerator greater than 6.)

Continue this process with the following possible sequence: $\frac{2}{\quad} \times 5 = 5$, $\frac{2}{\quad} \times 5 < 5$, $\frac{2}{\quad} \times 5 > 5$, $\frac{4}{\quad} \times 9 < 9$, $\frac{\quad}{6} \times 8 = 8$, and $\frac{5}{\quad} \times 7 < 7$.

Write Fractions as Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 23.

T: (Write $\frac{1}{50} = \frac{\quad}{100}$.) How many fifties are in 100?

S: 2.

T: (Write $\frac{1}{50} \times \frac{2}{2} = \frac{\quad}{100}$.) $\frac{1}{50}$ is the same as how many hundredths?

S: 2 hundredths.

T: (Write $\frac{1}{50} \times \frac{2}{2} = \frac{2}{100}$. Below it, write $\frac{1}{50}$ = _____._____.) On your personal white board, write $\frac{1}{50}$ as a decimal.

S: (Write $\frac{1}{50} = 0.02$.)

Continue this process with the following possible sequence: $\frac{3}{50}$, $\frac{9}{50}$, $4\frac{9}{50}$, $\frac{1}{20}$, $\frac{3}{20}$, $4\frac{3}{20}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{8}{5}$, $\frac{1}{25}$, $\frac{9}{25}$, $5\frac{9}{25}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{15}{4}$.

Write the Scaling Factor (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 23.

T: (Write 3 × ____ = 3.) Say the unknown whole number factor.

S: 1.

T: (Write 3.5 × ____ = 3.5.) Say the unknown whole number factor.

S: 1.

T: (Write 4.2 × 1 = ____.) Say the product.

S: 4.2.

T: (Write ____ × 0.58 > 0.58.) Is the unknown factor greater or less than 1?

S: Greater than 1.

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T: Fill in a factor to make a true number sentence.
S: (Write a number sentence filling in a decimal number greater than 1.)
T: (Write 7.03 × ____ < 7.03.) Is the unknown factor greater or less than 1?
S: Less than 1.
T: Fill in a factor to make a true number sentence.

Continue this process with the following possible sequence: 6.07 × ____ < 6.07, ____ × 6.2 = 6.2, and 0.97 × ____ > 0.97.

Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The time normally allotted for the Application Problem has been included in the Concept Development portion of today's lesson.

Suggested Delivery of Instruction for Solving Lesson 24 Word Problems

1. Model the problem.

Have two pairs of student who can successfully model the problem work at the board while the other students work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give students two minutes to finish work on that question, sharing their work and thinking with a peer. All students should write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solution.

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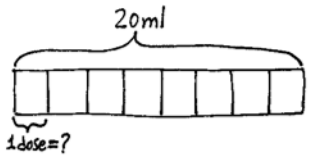
Problem 1

A vial contains 20 mL of medicine. If each dose is $\frac{1}{8}$ of the vial, how many mL is each dose? Express your answer as a decimal.



$\frac{1}{8}$ of 20ml
$= \frac{1}{8} \times 20$ ml
$= \frac{20}{8}$ ml
$= \frac{5}{2}$ ml
$= 2\frac{1}{2}$ ml

$2\frac{1}{2}$ ml $\times \frac{5}{5} = 2\frac{5}{10}$ ml $= 2.5$ml

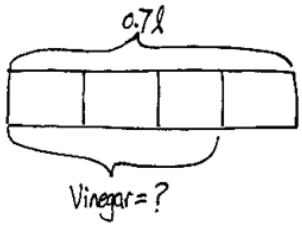
Each dose is $2\frac{1}{2}$ml or 2.5ml.

In this fraction of a set problem, students are asked to find one-eighth of 20 mL. Since the final answer must be expressed as a decimal, students again have some choices in how they solve. As illustrated, some students may choose to multiply $\frac{1}{8}$ by 20 to find the fractional mL in each dose. This method requires students to then simply express $2\frac{1}{2}$ as a decimal.

Other students may choose to first express $\frac{1}{8}$ as a decimal (0.125) and then multiply that by 20 to find 2.5 mL of medicine per dose. This method is perhaps more direct, but it does require that students recall that 8 is a factor of 1,000 to express $\frac{1}{8}$ as a decimal.

Problem 2

A container holds 0.7 liters of oil and vinegar. $\frac{3}{4}$ of the mixture is vinegar. How many liters of vinegar are in the container? Express your answer as both a fraction and a decimal.



$\frac{3}{4}$ of 0.7ℓ
$= \frac{3}{4} \times \frac{7}{10}$ ℓ
$= \frac{21}{40}$ ℓ

$\frac{21}{40}$ ℓ $\times \frac{25}{25} = \frac{525}{1000}$ ℓ $= 0.525$ℓ

There are $\frac{21}{40}$ℓ or 0.525ℓ of vinegar in the container.

In this fraction of a set problem, students are asked to find three-fourths of a set that is expressed using a decimal. Since the final answer needs to be expressed as both a fraction and decimal, students again have choices in their approaches. As illustrated, some students may choose to express 0.7 as a fraction and then multiply by three-fourths to find the fractional liters of vinegar in the container. This method requires the slightly complex step of converting a fraction with a denominator of 40 to a decimal. This process is not extremely challenging, but perhaps unfamiliar to some students.

Other students may choose to first express $\frac{3}{4}$ as a decimal (0.75) and then multiply it by 0.7 to find that 0.525 liters of vinegar are in the container. The decimal 0.525 is easily written as $\frac{525}{1,000}$ as a fraction. Students may simplify this fraction, but it is not required.

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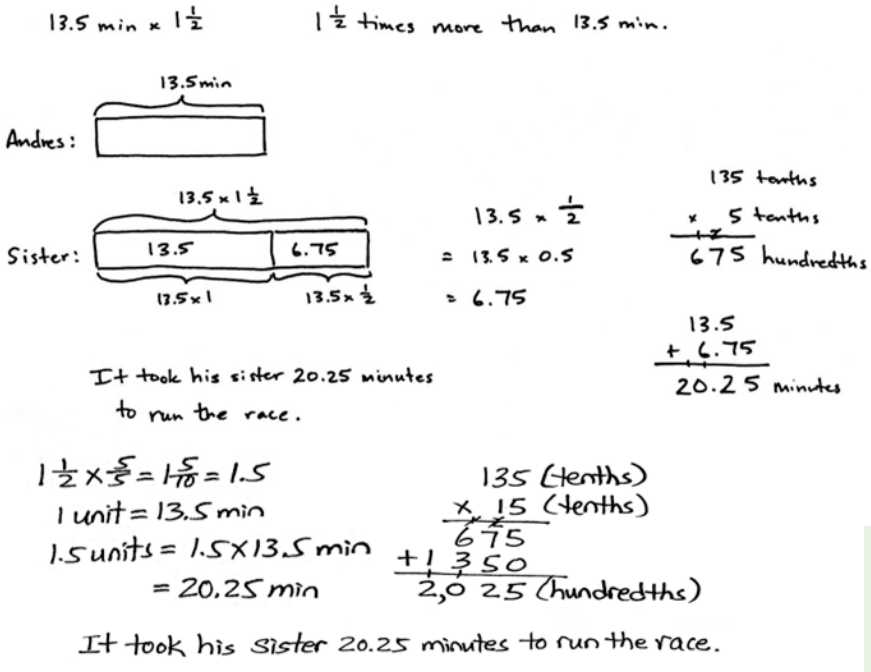
Problem 3

Andres completed a 5-km race in 13.5 minutes. His sister's time was $1\frac{1}{2}$ times longer than his time. How long, in minutes, did it take his sister to run the race?



In this problem, Andres' race time (13.5 minutes) is being multiplied by a scaling factor of $1\frac{1}{2}$. Students must interpret both a decimal and fractional factor, thus resulting in the expression of both factors as either decimals or fractions ($13\frac{5}{10} \times 1\frac{1}{2}$ or $\frac{135}{10} \times \frac{3}{2}$). Alternatively, students may have chosen to draw a tape diagram showing Andres' sister's time as 1 and a half times more than his. In this manner, students must multiply to find the value of the half-unit that represents the additional time his sister spent running and then add that sum to 13.5 minutes. The student's choice of approach provides an opportunity to discuss the efficiency of both approaches during the Student Debrief. In any case, students should find that Andres' sister completed the race in 20.25 (or $20\frac{1}{4}$) minutes.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Problems 3, 4, and 5 require that students compare quantities. For example, in Problem 3, students compare Andres' race time to his sister's time. Typically, when using tape diagrams to solve comparison word problems, at least two bars are used.

There is a strong connection between tape diagrams used with comparison story problems and bar graphs. The bars in bar graphs allow readers to compare quantities, which is similar to the bars used in comparison word problems. Although tape diagrams are typically drawn horizontally, they can be drawn vertically. Similarly, bar graphs can (and should) be drawn horizontally and vertically. In Problems 3, 4, and 5, it is easy to visualize additional data that would result in additional bars.

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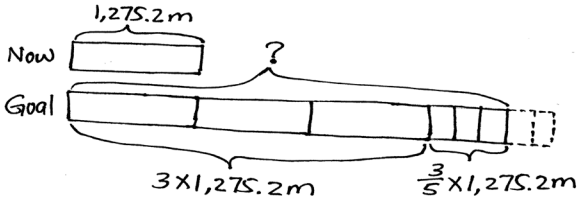
Problem 4

A clothing factory uses 1,275.2 meters of cloth a week to make shirts. How much cloth is needed to make $3\frac{3}{5}$ times as many shirts?



$3 \times 1{,}275.2 = 3{,}825.6$

$\frac{3}{5} \times \frac{1{,}275.2}{1} = 765.12$

$3{,}825.6 + 765.12 = 4{,}590.72$

$3\frac{3}{5} \times \frac{2}{2} = 3\frac{6}{10} = 3.6$

1 unit = 1,275.2 m

3.6 units = 3.6 × 1,275.2 m

= 4,590.72 m

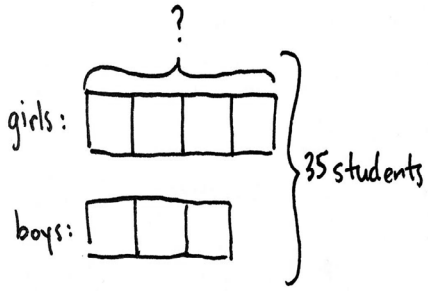
12,752 (tenths)
× 36 (tenths)
76512
+ 382560
459,072 (hundredths)

4,590.72 meters of cloth is needed to make $3\frac{3}{5}$ times as many shirts.

In this scaling problem, a length of cloth (1,275.2 m) is multiplied by a scaling factor of $3\frac{3}{5}$. Before students solve, ask them to identify the scaling factor and what comparison is being made (that of the initial amount of fabric and resulting amount). Though students do have the option of expressing both factors as fractions, the method of converting $3\frac{3}{5}$ to a decimal is far simpler. The efficiency of this approach can be a focus during the Student Debrief. Some students may also have chosen to draw a tape diagram showing 1,275.2 meters of cloth being scaled to $3\frac{3}{5}$ times its original length. In this manner, students could have tripled 1,275.2 first and then found three-fifths of it before combining those two totals. In either case, students should find that the factory would need 4,590.72 meters of cloth.

Problem 5

There are $\frac{3}{4}$ as many boys as girls in a class of fifth-graders. If there are 35 students in the class, how many are girls?



7 units = 35

1 unit = 35 ÷ 7 = 5

4 units = 4 × 5 = 20

20 students are girls.

What may seem like a simple problem is actually rather challenging because students are required to work backward as they solve. The word problem states that there are $\frac{3}{4}$ as many boys as girls in the class, yet the number of girls is unknown. Students should first reason that, since the number of boys is a scaled multiple

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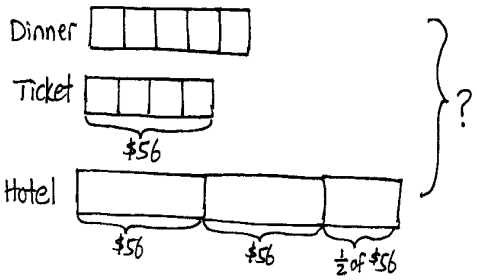
of the number of girls, a tape should first be drawn to represent the girls. From that tape, students can draw a smaller tape (one that is three-fourths the size of the tape representing the girls) to represent the boys in the class. In this way, students can see that 3 units are boys, and 4 units are girls. Since there are 35 students in the class and 7 total units, each unit represents 5 students. Four of those units are girls, so there are 20 girls in the class.

Problem 6

Ciro purchased a concert ticket for \$56. The cost of the ticket was $\frac{4}{5}$ the cost of his dinner. The cost of his hotel was $2\frac{1}{2}$ times as much as his ticket. How much did Ciro spend altogether for the concert ticket, hotel, and dinner?



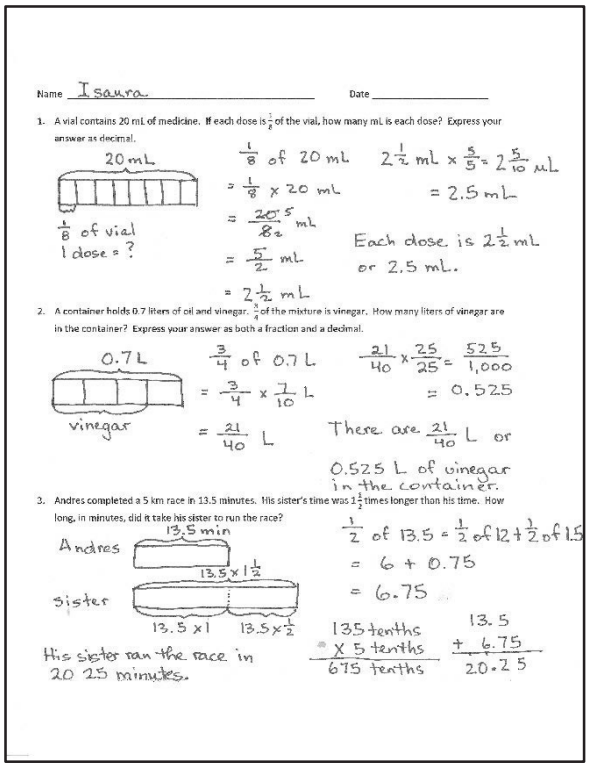
4 units = \$56
1 unit = \$56 ÷ 4 = \$14
Dinner: 5 units = 5 × \$14 = \$70
Hotel: \$56 × 2.5 = \$140

Ciro spent \$266 altogether.

\$70
\$56
+\$140
\$266

In this problem, students must read and work carefully to identify that the cost of the concert ticket plays two roles. In relation to the cost of the dinner, the ticket cost can be considered the scaling factor because it represents $\frac{4}{5}$ the cost of dinner. However, in relation to the cost of the hotel, the ticket cost should be considered the factor being scaled (because the hotel cost is $2\frac{1}{2}$ times greater). This understanding is crucial for drawing an accurate model and should be discussed thoroughly as students draw, as well as repeated during the Student Debrief.

Once the modeling is complete, the steps toward finding a solution are relatively simple. Since the ticket cost represents $\frac{4}{5}$ the cost of dinner, division shows that each unit (or fifth) is equal to \$14. Therefore, 5 units (5 fifths), or the cost of dinner, is equal to \$70. The model representing the cost of the hotel very clearly shows 2 units of \$56 and a half unit of \$56, which, in total, equals \$140. Students must use addition to find the total cost of Ciro's spending.


Name Isaura Date

1. A vial contains 20 mL of medicine. If each dose is $\frac{1}{8}$ of the vial, how many mL is each dose? Express your answer as decimal.

20 mL
$\frac{1}{8}$ of vial
1 dose = ?

$\frac{1}{8}$ of 20 mL
$= \frac{1}{8} \times 20$ mL
$= \frac{20}{8}$ mL
$= \frac{5}{2}$ mL
$= 2\frac{1}{2}$ mL

$2\frac{1}{2}$ mL × $\frac{5}{5}$ = $2\frac{5}{10}$ mL
= 2.5 mL

Each dose is $2\frac{1}{2}$ mL or 2.5 mL.

2. A container holds 0.7 liters of oil and vinegar. $\frac{3}{4}$ of the mixture is vinegar. How many liters of vinegar are in the container? Express your answer as both a fraction and a decimal.

0.7 L
vinegar

$\frac{3}{4}$ of 0.7 L
$= \frac{3}{4} \times \frac{7}{10}$ L
$= \frac{21}{40}$ L

$\frac{21}{40} \times \frac{25}{25} = \frac{525}{1,000}$
= 0.525

There are $\frac{21}{40}$ L or 0.525 L of vinegar in the container.

3. Andres completed a 5 km race in 13.5 minutes. His sister's time was $1\frac{1}{2}$ times longer than his time. How long, in minutes, did it take his sister to run the race?

13.5 min
Andres
sister
13.5 × 1
13.5 × $\frac{1}{2}$
13.5 × $1\frac{1}{2}$

His sister ran the race in 20.25 minutes.

$\frac{1}{2}$ of 13.5 = $\frac{1}{2}$ of 12 + $\frac{1}{2}$ of 1.5
= 6 + 0.75
= 6.75

135 tenths × 5 tenths = 675 tenths

13.5 + 6.75 = 20.25

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Student Debrief (10 minutes)

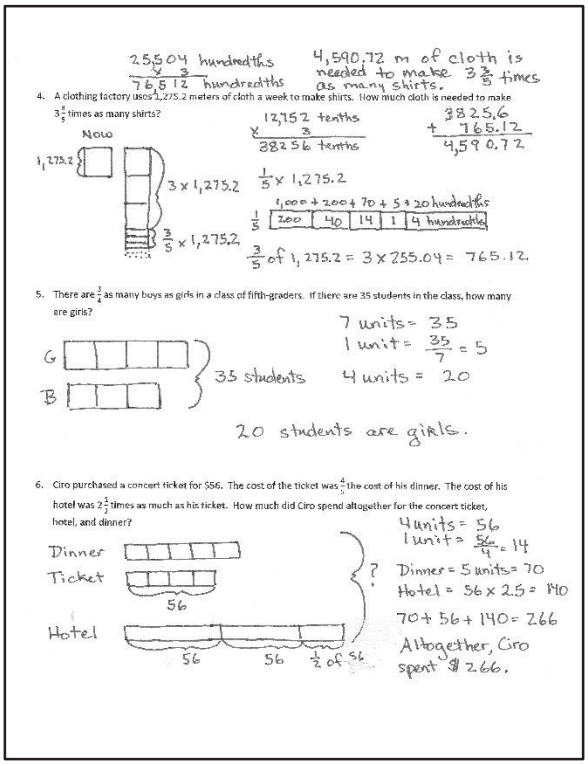
Lesson Objective: Solve word problems using fraction and decimal multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For all the problems in this Problem Set, there are several ways to solve for the solutions. With a partner, compare and share your strategies for each step in Problem 4.
- How did you solve Problem 5? Explain your strategy to a partner. Can you find how many boys there are in the classroom? How many more girls than boys are in the classroom?
- Did you make any drawings or tape diagrams for Problem 6? Share and compare with a partner. Does drawing a tape diagram help you solve this problem? Explain.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

A daily goal is to have students discuss their thinking. One possible strategy to achieve this goal is to partner each student with a peer who has a different perspective.

Ask students to separate themselves into groups that solved a specific problem in a similar way. Once these groups are formed, ask each student to partner with a peer in another group. Let these partners describe and discuss their strategies and solutions with each other. It is more difficult for students to explain different approaches than similar approaches.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. A vial contains 20 mL of medicine. If each dose is $\frac{1}{8}$ of the vial, how many mL is each dose? Express your answer as a decimal.

2. A container holds 0.7 liters of oil and vinegar. $\frac{3}{4}$ of the mixture is vinegar. How many liters of vinegar are in the container? Express your answer as both a fraction and a decimal.

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3. Andres completed a 5-km race in 13.5 minutes. His sister's time was $1\frac{1}{2}$ times longer than his time. How long, in minutes, did it take his sister to run the race?

4. A clothing factory uses 1,275.2 meters of cloth a week to make shirts. How much cloth is needed to make $3\frac{3}{5}$ times as many shirts?

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5. There are $\frac{3}{4}$ as many boys as girls in a class of fifth-graders. If there are 35 students in the class, how many are girls?

6. Ciro purchased a concert ticket for $56. The cost of the ticket was $\frac{4}{5}$ the cost of his dinner. The cost of his hotel was $2\frac{1}{2}$ times as much as his ticket. How much did Ciro spend altogether for the concert ticket, hotel, and dinner?

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Name _____ Date _____

1. An artist builds a sculpture out of metal and wood that weighs 14.9 kilograms. $\frac{3}{4}$ of this weight is metal, and the rest is wood. How much does the wood part of the sculpture weigh?

2. On a boat tour, there are half as many children as there are adults. There are 30 people on the tour. How many children are there?

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Name _____ Date _____

1. Jesse takes his dog and cat for their annual vet visit. Jesse's dog weighs 23 pounds. The vet tells him his cat's weight is $\frac{5}{8}$ as much as his dog's weight. How much does his cat weigh?

2. An image of a snowflake is 1.8 centimeters wide. If the actual snowflake is $\frac{1}{8}$ the size of the image, what is the width of the actual snowflake? Express your answer as a decimal.

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3. A community bike ride offers a short 5.7-mile ride for children and families. The short ride is followed by a long ride, $5\frac{2}{3}$ times as long as the short ride, for adults. If a woman bikes the short ride with her children and then the long ride with her friends, how many miles does she ride altogether?

4. Sal bought a house for $78,524.60. Twelve years later he sold the house for $2\frac{3}{4}$ times as much. What was the sale price of the house?

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5. In the fifth grade at Lenape Elementary School, there are $\frac{4}{5}$ as many students who do not wear glasses as those who do wear glasses. If there are 60 students who wear glasses, how many students are in the fifth grade?

6. At a factory, a mechanic earns $17.25 an hour. The president of the company earns $6\frac{2}{3}$ times as much for each hour he works. The janitor at the same company earns $\frac{3}{5}$ as much as the mechanic. How much does the company pay for all three employees' wages for one hour of work?

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Topic G

Division of Fractions and Decimal Fractions

5.OA.1, 5.NBT.7, 5.NF.7

Focus Standards:	5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
	5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
	5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students capable of multiplying fractions can generally develop strategies to divide fractions by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade level.) a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*
Instructional Days:	7	
Coherence -Links from:	G4–M5	Fraction Equivalence, Ordering, and Operations
	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions
	G6–M4	Expressions and Equations

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Topic G begins the work of division with fractions—both fractions and decimal fractions. Students use tape diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number (**5.NF.7**). Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering cases such as $5 \div \frac{1}{4}$. They also reason about the size of the unit when $\frac{1}{4}$ is partitioned into 5 equal parts: $\frac{1}{4} \div 5$. Using this thinking as a backdrop, students are introduced to decimal fraction divisors and use equivalent fraction and place value thinking to reason about the size of quotients, calculate quotients, and sensibly place the decimal in quotients (**5.NBT.7**).

A Teaching Sequence Toward Mastery of Division of Fractions and Decimal Fractions	
Objective 1:	**Divide a whole number by a unit fraction. (Lesson 25)**
Objective 2:	**Divide a unit fraction by a whole number. (Lesson 26)**
Objective 3:	**Solve problems involving fraction division. (Lesson 27)**
Objective 4:	**Write equations and word problems corresponding to tape and number line diagrams. (Lesson 28)**
Objective 5:	**Connect division by a unit fraction to division by 1 tenth and 1 hundredth. (Lesson 29)**
Objective 6:	**Divide decimal dividends by non-unit decimal divisors. (Lessons 30–31)**

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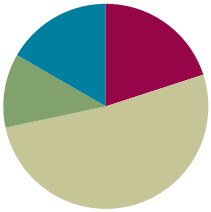
Lesson 25

Objective: Divide a whole number by a unit fraction.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Write Fractions as Decimals **5.NBT.2** (7 minutes)
- Multiply Fractions by Decimals **5.NBT.7** (5 minutes)

Write Fractions as Decimals (7 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 23.

T: (Write $\frac{1}{2} = \frac{\quad}{100}$.) $\frac{1}{2}$ is how many hundredths?

T: (Write $\frac{1}{2} = \frac{50}{100}$.) Write $\frac{1}{2}$ as a decimal.

S: (Write $\frac{1}{2} = 0.5$ or $\frac{1}{2} = 0.50$.)

T: (Write $\frac{1}{4} = \frac{\quad}{100}$.) $\frac{1}{4}$ is how many hundredths?

S: 25 hundredths.

T: (Write $\frac{1}{4} = \frac{25}{100}$.) Write $\frac{1}{4}$ as a decimal.

S: (Write $\frac{1}{4} = 0.25$.)

T: (Write $\frac{3}{4} = \frac{\quad}{100}$.) $\frac{3}{4}$ is how many hundredths?

S: 75 hundredths.

T: (Write $\frac{3}{4} = \frac{75}{100}$.) Write $\frac{3}{4}$ as a decimal.

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S: (Write $\frac{3}{4} = 0.75$.)

T: (Write $1\frac{3}{4} =$ __.__.) Write $1\frac{3}{4}$ as a decimal.

S: (Write $1\frac{3}{4} = 1.75$.)

Continue with the following possible sequence: $\frac{1}{25}$, $\frac{8}{25}$, $3\frac{8}{25}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{7}{5}$, $\frac{1}{20}$, $\frac{9}{20}$, $4\frac{9}{20}$, $\frac{1}{50}$, $\frac{7}{50}$, $\frac{11}{50}$, and $3\frac{11}{50}$.

Multiply Fractions by Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 24.

T: (Write $\frac{1}{2} \times \frac{1}{2} =$ ____.) On your personal white board, write the multiplication sentence with the answer.

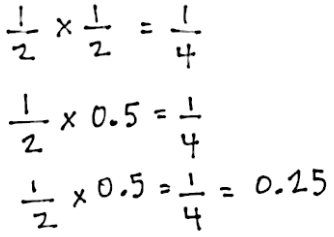
S: (Write $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.)

T: (Write $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Beneath it, write $\frac{1}{2} \times 0.$____ $= \frac{1}{4}$.) Fill in the missing digit.

S: (Write $\frac{1}{2} \times 0.5 = \frac{1}{4}$.)

T: (Write $\frac{1}{2} \times 0.5 = \frac{1}{4} =$ __.__.) Complete the equation.

S: (Write $\frac{1}{2} \times 0.5 = \frac{1}{4} = 0.25$.)



T: (Write $\frac{1}{2} \times \frac{1}{50} =$ ___ = __.__.) On your personal white board, write the multiplication sentence with the answer as a fraction and decimal.

S: (Write $\frac{1}{2} \times \frac{1}{50} = \frac{1}{100} = 0.01$.)

T: (Write $0.5 \times \frac{1}{50} =$ __.__.) Complete the equation.

S: (Write $0.5 \times \frac{1}{50} = 0.01$.)

T: (Write $\frac{3}{5} \times 0.7 =$ __.__.) Rewrite the multiplication sentence as a fraction times a fraction.

S: (Write $\frac{6}{10} \times \frac{7}{10} = 0.42$.)

T: (Write $0.8 \times \frac{2}{5} =$ __.__ × __.__ = — = __.__.) Rewrite the multiplication sentence, filling in the blanks.

S: (Write $0.8 \times \frac{2}{5} = 0.8 \times 0.4 = \frac{32}{100} = 0.32$.)

T: (Write $\frac{4}{5} \times 0.9 =$ __.__ × __.__ = __.__.) Rewrite the multiplication sentence, filling in the blanks.

S: (Write $\frac{4}{5} \times 0.9 = 0.8 \times 0.9 = 0.72$.)

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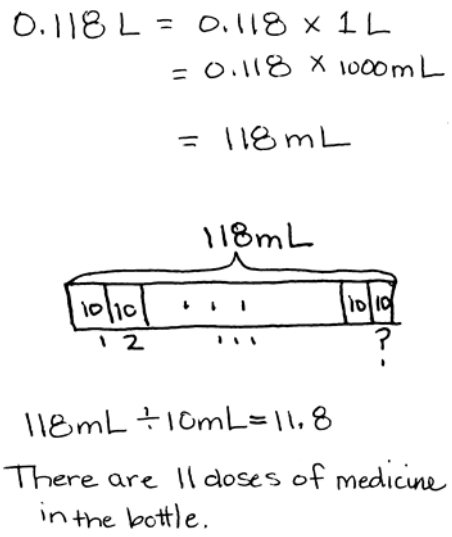
Application Problem (7 minutes)

The label on a 0.118 L bottle of cough syrup recommends a dose of 10 mL for children aged 6 to 10 years. How many 10 mL doses are in the bottle?

Note: This problem requires students to access their knowledge of converting among different size measurement units—a reference to Modules 1 and 2. Students may disagree on whether the final answer should be a whole number or decimal. There are only 11 complete 10 mL doses in the bottle, but many students will divide 118 by 10 and give 11.8 doses as their final answer. This invites interpretation of the remainder since both answers are correct.



Concept Development (31 minutes)

Materials: (S) Personal white board, 4" × 2" rectangular paper (several pieces per student), scissors

Problem 1

Jenny buys 2 pounds of pecans.

a. If Jenny puts 2 pounds in each bag, how many bags can she make?

b. If she puts 1 pound in each bag, how many bags can she make?

c. If she puts $\frac{1}{2}$ pound in each bag, how many bags can she make?

d. If she puts $\frac{1}{3}$ pound in each bag, how many bags can she make?

e. If she puts $\frac{1}{4}$ pound in each bag, how many bags can she make?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

In addition to tape diagrams and area models, students can also use region models to represent the information in these problems. For example, students can draw circles to represent the pounds of pecans and divide the circles in half to represent halves.

Note: Continue this questioning sequence to include fifths and sixths.

T: (Post Problem 1(a) on the board, and read it aloud with the students.) Work with your partner to write a division sentence that explains your thinking. Be prepared to share.

S: (Work.)

T: Say the division sentence to solve this problem.

S: 2 ÷ 2 = 1.

T: (Record on the board.) How many bags of pecans can she make?

S: 1 bag.

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T: (Post Problem 1(b).) Write a division sentence for this situation and solve.

S: (Solve.)

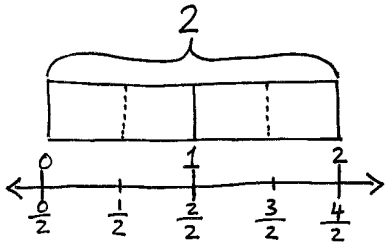
T: Say the division sentence to solve this problem.

S: $2 \div 1 = 2$.

T: (Record directly beneath the first division sentence.) Answer the question in a complete sentence.

S: She can make 2 bags.

T: (Post Problem 1(c).) If Jenny puts 1 half pound in each of the bags, how many bags can she make? What would that division sentence look like? Turn and talk.



$2 \div \frac{1}{2} = 4$

She can make 4 bags.

S: We still have 2 as the amount that's divided up, so it should still be $2 \div \frac{1}{2}$. → We are sort of putting pecans in half-pound groups, so 1 half will be our divisor, the size of the group. → It's like asking how many halves are in 2.

T: (Write $2 \div \frac{1}{2}$ directly beneath the other division sentences.) Will the answer be more or less than 2? Talk to your partner.

S: I looked at the other problems and saw a pattern. $2 \div 2 = 1$, $2 \div 1 = 2$, and now I think $2 \div \frac{1}{2}$ will be more than 2. → It should be more because we're cutting each pound into halves so that will make more groups. → I can visualize that each whole pound would have 2 halves, so there should be 4 half pounds in 2 pounds.

T: Let's use a piece of rectangular paper to represent 2 pounds of pecans. Cut it into 2 equal pieces. What does each piece represent?

S: 1 pound of pecans.

T: Fold each pound into halves and cut.

S: (Fold and cut.)

T: How many halves were in 2 wholes?

S: 4 halves.

T: Let me model what you just did using a tape diagram. The tape represents 2 wholes. (Label 2 on top.) Each unit is one whole. (Partition the tape with one line down the middle.) The dotted lines cut each whole into halves. (Partition each whole with a dotted line.) How many halves are in 1 whole?

S: 2 halves.

MP.4

T: How many halves are in 2 wholes?

S: 4 halves.

T: Yes. I'll draw a number line underneath the tape diagram and label the wholes. (Label 0, 1, and 2 on the number line.) Now, I can put a tick mark for each half. Count the halves with me as I label. (Label $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$.)

S: $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$.

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T: There are 4 halves in 2 wholes. (Write $2 \div \frac{1}{2} = 4$.) She can make 4 bags. But how can we be sure 4 halves is correct? How do we check a division problem? By what do we need to multiply the quotient?

S: The divisor.

T: What is the quotient?

S: 4.

T: The divisor?

S: 1 half.

T: What would our checking expression be? Write it with your partner.

MP.4

S: $4 \times \frac{1}{2}$.

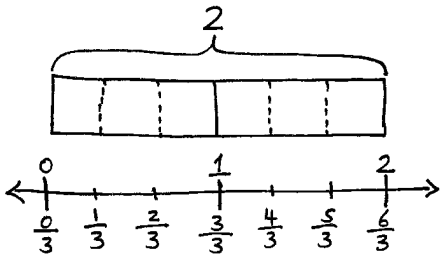
T: Complete the number sentence. (Pause.) Read the complete sentence.

S: $4 \times \frac{1}{2} = 2$ or $\frac{1}{2} \times 4 = \frac{1 \times 4}{2} = \frac{4}{2} = 2$.

T: Were we correct?

S: Yes.

T: Let's remember this thinking as we continue.



$2 \div \frac{1}{3} = 6$

She can make 6 bags.

Repeat the modeling process with Problems 1(d) and (e), divisors of 1 third and 1 fourth.

Extend the dialogue when dividing by 1 fourth to search for patterns:

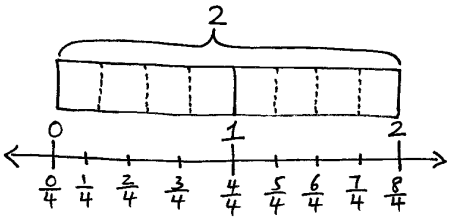
T: (Point to all the number sentences in the previous problems: $2 \div 2 = 1$, $2 \div 1 = 2$, $2 \div \frac{1}{2} = 4$, $2 \div \frac{1}{3} = 6$, and $2 \div \frac{1}{4} = 8$.) Take a look at these problems. What patterns do you notice? Turn and share.

S: The 2 pounds are the same, but each time, the answer is being divided into a smaller and smaller unit. → The answer is getting larger and larger. → When the 2 pounds is divided into smaller units, then the answer is bigger.

T: Explain to your partner why the quotient is becoming larger as it is divided by smaller units.

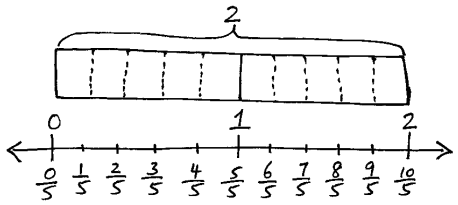
S: When we cut a whole into smaller parts, then we'll get more parts. → The more units we split from one whole, then the more parts we'll have. That's why the quotient is becoming larger.

T: Based on the patterns, solve how many bags Jenny can make if she puts $\frac{1}{5}$ pound into each bag. Draw a tape diagram and a number line on your personal white board to explain your thinking.



$2 \div \frac{1}{4} = 8$

She can make 8 bags.



$2 \div \frac{1}{5} = 10$

She can make 10 bags.

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S: (Solve.)
T: Say the division sentence.
S: $2 \div \frac{1}{5} = 10$.
T: Answer the question in a complete sentence.
S: She can make 10 bags.

Problem 2

Jenny buys 2 pounds of pecans.

a. If this is $\frac{1}{2}$ the number she needs to make pecan pies, how many pounds will she need?

b. If this is $\frac{1}{3}$ the number she needs to make pecan pies, how many pounds will she need?

c. If this is $\frac{1}{4}$ the number she needs to make pecan pies, how many pounds will she need?

T: We can also ask different questions about Jenny and her two pounds of pecans. (Post Problem 2(a).) Two is half of what number?
S: 4.
T: Give me the division sentence.
S: It's not division! It's multiplication. → It's 2 twos. That's four.
T: Give me the multiplication number sentence.
S: $2 \times 2 = 4$. → $\frac{1}{2} \times 4 = 2$.
T: Hold on. Stop. Let's try to write a division expression for this whole number situation. (Write $4 \times$ ____ = 8.)
S: What would the division expression be?
S: $8 \div 4$.
T: Tell me the complete number sentence.
S: $8 \div 4 = 2$.
T: (Write $\frac{1}{2} \times$ ____ = 2.) Now, try the same process for this problem. Give me the division expression.
S: $2 \div \frac{1}{2}$.
T: Tell me the complete number sentence.
S: $2 \div \frac{1}{2} = 4$.

NOTES ON TABLE 2 OF THE COMMON CORE LEARNING STANDARDS:

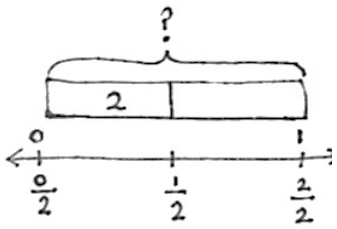
It is important to distinguish between interpretations of division. It may be easier to understand the distinction by using the word *unit* rather than *group*.

Measurement division*:* In this case, the divisor represents the *size of the unit*, and the quotient tells the *number of units*. When solving $12 \div 4$, think, "How many 4's in 12?" Likewise, when solving $3 \div \frac{1}{2}$, think, "How many halves in 3?"

- 12 cards are put in packs of 4. How many packs are there?
- 3 meters of cloth are cut into $\frac{1}{2}$ meter strips. How many strips are cut?

Partitive division: In this case, the divisor represents the *number of units*, and the quotient represents the *size of the unit.* When solving $12 \div 4$, think, "12 is 4 of what number?" Likewise, when solving $3 \div \frac{1}{2}$, think, "3 is half of what number?"

- 12 cards are shared equally with 4 people. How many cards does each person receive?
- 3 miles is $\frac{1}{2}$ the trip. How far is the whole trip?



$2 \div \frac{1}{2} = 2 \times 2 = 4$

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T: Yes. We are finding how much is in one unit just like we did with 8 ÷ 2 = 4. In this case, the whole is the unit.

T: What is the whole unit in this story?

S: The whole number she needs for pecan pies.

T: Let's go back and answer our question. Jenny buys 2 pounds of pecans. If this is $\frac{1}{2}$ the number she needs to make pecan pies, how many pounds will she need?

S: She will need 4 pounds of pecans.

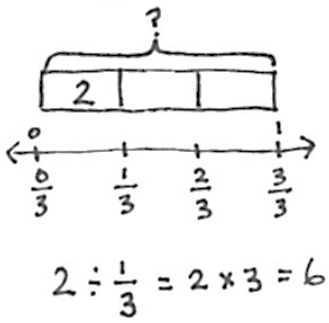
T: Yes.

T: (Post Problem 2(b) on the board.) What's the answer?

S: 6.

T: Give me the division sentence.

S: $2 \div \frac{1}{3} = 6$.



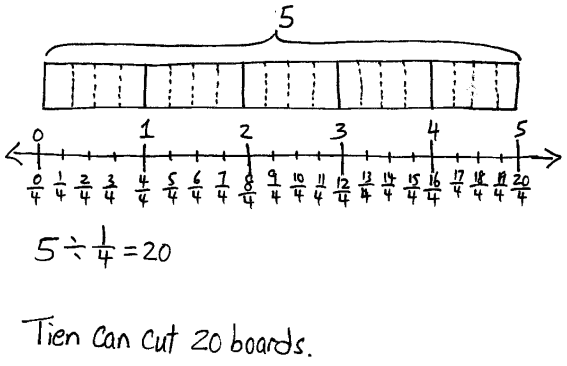
T: Explain to your partner why that is true.

S: We are looking for the whole number of pounds. Two is a third, so we divide it by a third. → I still think of it as multiplication though—2 times 3 equals 6. → However, the problem doesn't mention 3, it says a third, so $2 \div \frac{1}{3} = 2 \times 3$. → So, dividing by a third is the same as multiplying by 3.

T: We can see in our tape diagram that this is true. (Write $2 \div \frac{1}{3} = 2 \times 3$.) Explain to your partner why. If you like, use the story of the pecans.

Problem 3

Tien wants to cut $\frac{1}{4}$ foot lengths from a board that is 5 feet long. How many boards can he cut?



T: (Post Problem 3 on the board, and read it together with the class.) What is the length of the board Tien has to cut?

S: 5 feet.

T: How can we find the number of boards 1 fourth of a foot long? Turn and talk.

S: We have to divide. → The division expression is $5 \div \frac{1}{4}$. → I can draw 5 wholes and cut each whole into fourths. Then, I can count how many fourths are in 5 wholes.

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T: On your personal white board, draw and solve this problem independently.

S: (Work.)

T: How many quarter feet are in 1 foot?

S: 4.

T: How many quarter feet are in 5 feet?

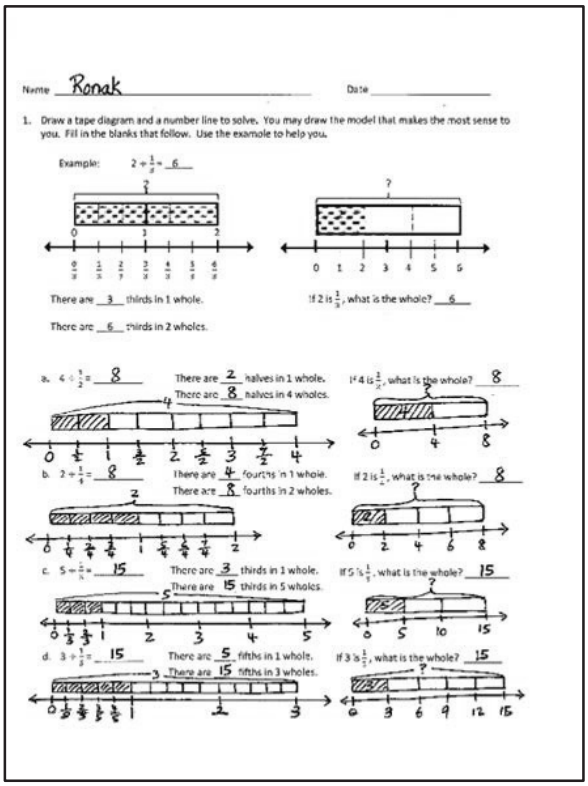
S: 20.

T: Say the division sentence.

S: $5 \div \frac{1}{4} = 20$.

T: Check your work, and then answer the question in a complete sentence.

S: Tien can cut 20 boards.


Name Ronak Date

1. Draw a tape diagram and a number line to solve. You may draw the model that makes the most sense to you. Fill in the blanks that follow. Use the example to help you.

Example: $2 \div \frac{1}{3} = 6$

There are 3 thirds in 1 whole. If 2 is $\frac{2}{3}$, what is the whole? 6

There are 6 thirds in 2 wholes.

a. $4 \div \frac{1}{2} = 8$ There are 2 halves in 1 whole. There are 8 halves in 4 wholes. If 4 is $\frac{4}{1}$, what is the whole? 8

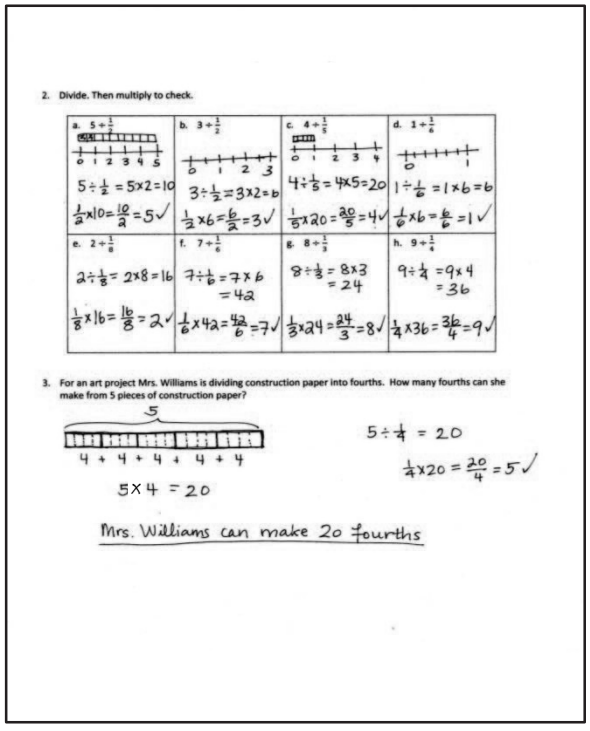
b. $2 \div \frac{1}{4} = 8$ There are 4 fourths in 1 whole. There are 8 fourths in 2 wholes. If 2 is $\frac{2}{1}$, what is the whole? 8

c. $5 \div \frac{1}{3} = 15$ There are 3 thirds in 1 whole. There are 15 thirds in 5 wholes. If 5 is $\frac{5}{1}$, what is the whole? 15

d. $3 \div \frac{1}{5} = 15$ There are 5 fifths in 1 whole. There are 15 fifths in 3 wholes. If 3 is $\frac{3}{1}$, what is the whole? 15

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


2. Divide. Then multiply to check.

a. $5 \div \frac{1}{2} = 5 \times 2 = 10$; $\frac{1}{2} \times 10 = \frac{10}{2} = 5$ ✓

b. $3 \div \frac{1}{2} = 3 \times 2 = 6$; $\frac{1}{2} \times 6 = \frac{6}{2} = 3$ ✓

c. $4 \div \frac{1}{5} = 4 \times 5 = 20$; $\frac{1}{5} \times 20 = \frac{20}{5} = 4$ ✓

d. $1 \div \frac{1}{6} = 1 \times 6 = 6$; $\frac{1}{6} \times 6 = \frac{6}{6} = 1$ ✓

e. $2 \div \frac{1}{8} = 2 \times 8 = 16$; $\frac{1}{8} \times 16 = \frac{16}{8} = 2$ ✓

f. $7 \div \frac{1}{6} = 7 \times 6 = 42$; $\frac{1}{6} \times 42 = \frac{42}{6} = 7$ ✓

g. $8 \div \frac{1}{3} = 8 \times 3 = 24$; $\frac{1}{3} \times 24 = \frac{24}{3} = 8$ ✓

h. $9 \div \frac{1}{4} = 9 \times 4 = 36$; $\frac{1}{4} \times 36 = \frac{36}{4} = 9$ ✓

3. For an art project Mrs. Williams is dividing construction paper into fourths. How many fourths can she make from 5 pieces of construction paper?

5

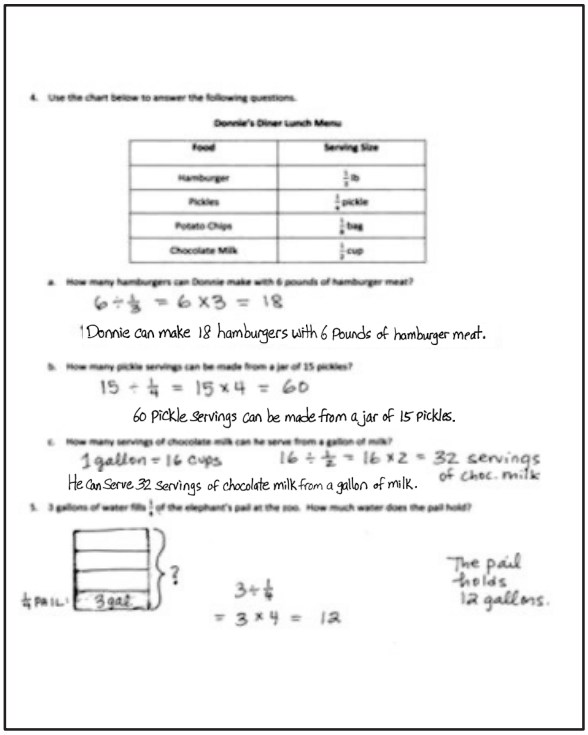
4 + 4 + 4 + 4 + 4

5 × 4 = 20

$5 \div \frac{1}{4} = 20$

$\frac{1}{4} \times 20 = \frac{20}{4} = 5$ ✓

Mrs. Williams can make 20 fourths


4. Use the chart below to answer the following questions.

Donnie's Diner Lunch Menu

Food	Serving Size
Hamburger	$\frac{1}{3}$ lb
Pickles	$\frac{1}{4}$ pickle
Potato Chips	$\frac{1}{8}$ bag
Chocolate Milk	$\frac{1}{2}$ cup

a. How many hamburgers can Donnie make with 6 pounds of hamburger meat?

$6 \div \frac{1}{3} = 6 \times 3 = 18$

Donnie can make 18 hamburgers with 6 pounds of hamburger meat.

b. How many pickle servings can be made from a jar of 15 pickles?

$15 \div \frac{1}{4} = 15 \times 4 = 60$

60 pickle servings can be made from a jar of 15 pickles.

c. How many servings of chocolate milk can he serve from a gallon of milk?

1 gallon = 16 cups $16 \div \frac{1}{2} = 16 \times 2 = 32$ servings of choc. milk

He can serve 32 servings of chocolate milk from a gallon of milk.

5. 3 gallons of water fills $\frac{1}{4}$ of the elephant's pail at the zoo. How much water does the pail hold?

$\frac{1}{4}$ PAIL 3 gal ?

$3 \div \frac{1}{4}$

$= 3 \times 4 = 12$

The pail holds 12 gallons.

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Student Debrief (10 minutes)

Lesson Objective: Divide a whole number by a unit fraction.

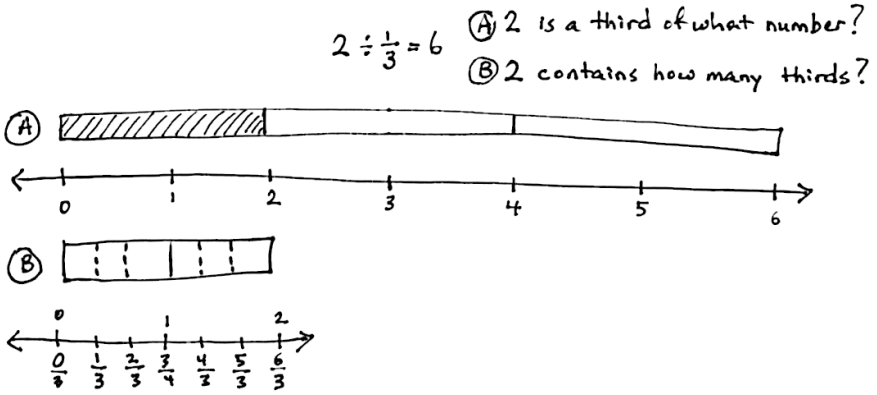
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what do you notice about (a) and (b), and (c) and (d)? What are the whole and the divisor in the problems?
- Share your solution and compare your strategy for solving Problem 2 with a partner.
- Explain your strategy for solving Problems 3 and 4 with a partner.
- Problem 5 on the Problem Set is a partitive division problem. Students are not likely to interpret the problem as division and will more likely use a missing factor strategy to solve (which is certainly appropriate).

  Problem 5 can be expressed as $3 \div \frac{1}{4}$. This could be thought of as *3 gallons is 1 out of 4 parts needed to fill the pail* or *3 is 1 fourth of what number*? Asking students to consider this interpretation will be beneficial in future encounters with fraction division. (See the UDL box. The model to the right puts the two interpretations next to each other.)



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The second to last bullet in today's Debrief brings out an interpretation of fraction division in a context that is particularly useful for Grade 6's encounters with non-unit fraction division. In Grade 6, Problem 5 might read as follows:

> $\frac{2}{3}$ *of a gallon of water fills the pail to* $\frac{3}{4}$ *of its capacity. How much water does the pail hold?*

This could be expressed as $\frac{2}{3} \div \frac{3}{4}$. That is, $\frac{2}{3}$ is 3 of the 4 *groups* needed to completely fill the pail. This type of problem can be thought of partitively as *2 thirds is 3 fourths of what number* or $\frac{2}{3} = \frac{3}{4} \times$ ___. This leads to explaining the *invert and multiply* strategy. Working from a tape diagram, this problem would be stated as follows:

3 units = $\frac{2}{3}$

1 unit = $(\frac{2}{3} \div 3)$

We need 4 units to fill the pail:

4 units = $(\frac{2}{3} \div 3) \times 4$

$= \frac{2}{3} \times \frac{4}{3}$

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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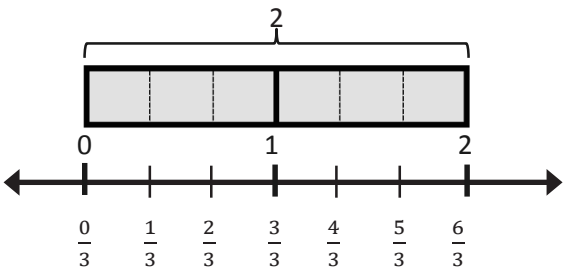
Name _____ Date _____

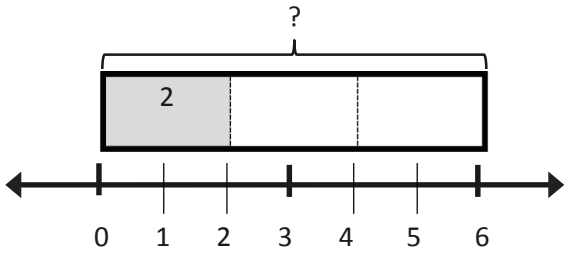
1. Draw a tape diagram and a number line to solve. You may draw the model that makes the most sense to you. Fill in the blanks that follow. Use the example to help you.

Example: $2 \div \frac{1}{3} =$ 6



There are 3 thirds in 1 whole.

There are 6 thirds in 2 wholes.



If 2 is $\frac{1}{3}$, what is the whole? 6

a. $4 \div \frac{1}{2} =$ _____

There are ____ halves in 1 whole.

There are ____ halves in 4 wholes.

If 4 is $\frac{1}{2}$, what is the whole? _____

b. $2 \div \frac{1}{4} =$ _____

There are ____ fourths in 1 whole.

There are ____ fourths in 2 wholes.

If 2 is $\frac{1}{4}$, what is the whole? _____

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c. $5 \div \frac{1}{3} =$ _____ There are ____ thirds in 1 whole. There are ____ thirds in 5 wholes. If 5 is $\frac{1}{3}$, what is the whole? _____

d. $3 \div \frac{1}{5} =$ _____ There are ____ fifths in 1 whole. There are ____ fifths in 3 wholes. If 3 is $\frac{1}{5}$, what is the whole? _____

2. Divide. Then, multiply to check.

a. $5 \div \frac{1}{2}$	b. $3 \div \frac{1}{2}$	c. $4 \div \frac{1}{5}$	d. $1 \div \frac{1}{6}$
e. $2 \div \frac{1}{8}$	f. $7 \div \frac{1}{6}$	g. $8 \div \frac{1}{3}$	h. $9 \div \frac{1}{4}$

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3. For an art project, Mrs. Williams is dividing construction paper into fourths. How many fourths can she make from 5 pieces of construction paper?

4. Use the chart below to answer the following questions.

Donnie's Diner Lunch Menu

Food	Serving Size
Hamburger	$\frac{1}{3}$ lb
Pickles	$\frac{1}{4}$ pickle
Potato chips	$\frac{1}{8}$ bag
Chocolate milk	$\frac{1}{2}$ cup

a. How many hamburgers can Donnie make with 6 pounds of hamburger meat?

b. How many pickle servings can be made from a jar of 15 pickles?

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c. How many servings of chocolate milk can he serve from a gallon of milk?

5. Three gallons of water fill $\frac{1}{4}$ of the elephant's pail at the zoo. How much water does the pail hold?

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Name _____ Date _____

1. Draw a tape diagram and a number line to solve. Fill in the blanks that follow.

a. $5 \div \frac{1}{2} =$ _____

There are ____ halves in 1 whole.

There are ____ halves in 5 wholes.

5 is $\frac{1}{2}$ of what number? _____

b. $4 \div \frac{1}{4} =$ _____

There are ____ fourths in 1 whole.

There are ____ fourths in ____ wholes.

4 is $\frac{1}{4}$ of what number? _____

2. Ms. Leverenz is doing an art project with her class. She has a 3 foot piece of ribbon. If she gives each student an eighth of a foot of ribbon, will she have enough for her class of 22 students?

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Name _____ Date _____

1. Draw a tape diagram and a number line to solve. Fill in the blanks that follow.

a. $3 \div \frac{1}{3} =$ _____

There are ____ thirds in 1 whole.

There are ____ thirds in 3 wholes.

If 3 is $\frac{1}{3}$, what is the whole? _____

b. $3 \div \frac{1}{4} =$ _____

There are ____ fourths in 1 whole.

There are ____ fourths in __ wholes.

If 3 is $\frac{1}{4}$, what is the whole? _____

c. $4 \div \frac{1}{3} =$ _____

There are ____ thirds in 1 whole.

There are ____ thirds in __ wholes.

If 4 is $\frac{1}{3}$, what is the whole? _____

d. $5 \div \frac{1}{4} =$ _____

There are ____ fourths in 1 whole.

There are ____ fourths in __ wholes.

If 5 is $\frac{1}{4}$, what is the whole? _____

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2. Divide. Then, multiply to check.

a. $2 \div \frac{1}{4}$	b. $6 \div \frac{1}{2}$	c. $5 \div \frac{1}{4}$	d. $5 \div \frac{1}{8}$
e. $6 \div \frac{1}{3}$	f. $3 \div \frac{1}{6}$	g. $6 \div \frac{1}{5}$	h. $6 \div \frac{1}{10}$

3. A principal orders 8 sub sandwiches for a teachers' meeting. She cuts the subs into thirds and puts the mini-subs onto a tray. How many mini-subs are on the tray?

4. Some students prepare 3 different snacks. They make $\frac{1}{8}$ pound bags of nut mix, $\frac{1}{4}$ pound bags of cherries, and $\frac{1}{6}$ pound bags of dried fruit. If they buy 3 pounds of nut mix, 5 pounds of cherries, and 4 pounds of dried fruit, how many of each type of snack bag will they be able to make?

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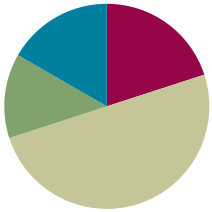
Lesson 26

Objective: Divide a unit fraction by a whole number.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Count by Fractions **5.NF.7** (5 minutes)
- Divide Whole Numbers by Fractions **5.NF.7** (4 minutes)
- Multiply Fractions **5.NF.4** (3 minutes)

Count by Fractions (5 minutes)

Note: This fluency activity reviews Lesson 21.

$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	$\frac{8}{4}$	$\frac{9}{4}$	$\frac{10}{4}$	$\frac{11}{4}$	$\frac{12}{4}$
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	2	$\frac{9}{4}$	$\frac{10}{4}$	$\frac{11}{4}$	3
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{2}{4}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{2}{4}$	$2\frac{3}{4}$	3
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3

T: Count by fourths to 12 fourths. (Write as students count.)

S: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, 6 fourths, 7 fourths, 8 fourths, 9 fourths, 10 fourths, 11 fourths, 12 fourths.

T: Let's count by fourths again. This time, when we arrive at a whole number, say the whole number. (Write as students count.)

S: 1 fourth, 2 fourths, 3 fourths, 1 whole, 5 fourths, 6 fourths, 7 fourths, 2 wholes, 9 fourths, 10 fourths, 11 fourths, 3 wholes.

T: Let's count by fourths again. This time, change improper fractions to mixed numbers.

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S: 1 fourth, 2 fourths, 3 fourths, 1 whole, 1 and 1 fourth, 1 and 2 fourths, 1 and 3 fourths, 2 wholes, 2 and 1 fourth, 2 and 2 fourths, 2 and 3 fourths, 3 wholes.

T: Let's count by fourths again. This time, simplify 2 fourths to 1 half. (Write as students count.)

S: 1 fourth, 1 half, 3 fourths, 1 whole, 1 and 1 fourth, 1 and 1 half, 1 and 3 fourths, 2 wholes, 2 and 1 fourth, 2 and 1 half, 2 and 3 fourths, 3 wholes.

Continue the process, counting by fifths to 15 fifths.

Divide Whole Numbers by Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 25.

T: (Write $1 \div \frac{1}{2} =$____.) Say the division problem.

S: $1 \div \frac{1}{2}$.

T: How many halves are in 1 whole?

S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2}$.) How many halves are in 2 wholes?

S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2}$.) How many halves are in 3 wholes?

S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $8 \div \frac{1}{2}$.) On your personal white board, write the complete number sentence.

S: (Write $8 \div \frac{1}{2} = 16$.)

Continue with the following possible sequence: $1 \div \frac{1}{3}$, $2 \div \frac{1}{3}$, $5 \div \frac{1}{3}$, $1 \div \frac{1}{4}$, $2 \div \frac{1}{4}$, $7 \div \frac{1}{4}$, $3 \div \frac{1}{5}$, $4 \div \frac{1}{6}$, and $7 \div \frac{1}{8}$.

Multiply Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 13–16.

T: (Write $\frac{1}{2} \times \frac{1}{3} =$ ____.) Say the multiplication number sentence with the answer.

S: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Continue this process with the following possible problems: $\frac{1}{2} \times \frac{1}{4}$ and $\frac{1}{2} \times \frac{1}{5}$.

T: (Write $\frac{1}{2} \times \frac{1}{9} =$ ____.) On your personal white board, write the number sentence and the answer.

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S: (Write $\frac{1}{2} \times \frac{1}{9} = \frac{1}{18}$.)

T: (Write $\frac{1}{2} \times \frac{5}{9} =$ ____.) Say the multiplication sentence with the answer.

S: $\frac{1}{2} \times \frac{5}{9} = \frac{5}{18}$.

Repeat this process with the following possible problems: $\frac{1}{4} \times \frac{3}{3}$, $\frac{1}{3} \times \frac{3}{2}$, and $\frac{3}{4} \times \frac{1}{5}$.

T: (Write $\frac{2}{3} \times \frac{2}{5} =$____.) Write the multiplication sentence with the answer.

S: (Write $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.)

Continue this process with the following possible problem: $\frac{3}{4} \times \frac{3}{5}$.

T: (Write $\frac{1}{4} \times \frac{3}{5} =$ ____.) On your personal white board, write the number sentence and the answer.

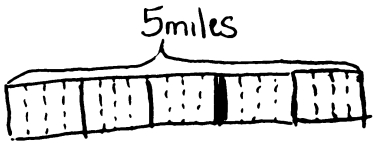
S: (Write $\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$.)

T: (Write $\frac{3}{4} \times \frac{4}{3} =$____.) Try this problem.

S: (Write $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$.)

Application Problem (8 minutes)

A race begins with $2\frac{1}{2}$ miles through town, continues through the park for $2\frac{1}{3}$ miles, and finishes at the track after the last $\frac{1}{6}$ mile. A volunteer is stationed every quarter mile and at the finish line to pass out cups of water and cheer on the runners. How many volunteers are needed?



Race: $2\frac{1}{2} + 2\frac{1}{3} + \frac{1}{6} = 2\frac{3}{6} + 2\frac{2}{6} + \frac{1}{6}$

$= 4 + \frac{6}{6}$

$= 5$

Volunteers: $5 \div \frac{1}{4} = 20$

20 volunteers are needed.

Note: This multi-step problem requires that students first add three fractions and then divide the sum by a fraction, which reinforces yesterday's division of a whole number by a unit fraction. (How many $\frac{1}{4}$ miles are in 5 miles?) It also reviews adding fractions with different denominators (Module 3).

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Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1

Nolan gives some pans of brownies to his 3 friends to share equally.

a. If he has 3 pans of brownies, how many pans of brownies will each friend receive?

b. If he has 1 pan of brownies, how many pans of brownies will each friend receive?

c. If he has $\frac{1}{2}$ pan of brownies, how many pans of brownies will each friend receive?

d. If he has $\frac{1}{3}$ pan of brownies, how many pans of brownies will each friend receive?

T: (Post Problem 1(a) on the board, and read it aloud with the students.) Work on your personal white board, and write a division sentence to solve this problem. Be prepared to share.

S: (Work.)

T: How many pans of brownies does Nolan have?

S: 3 pans.

T: The 3 pans of brownies are divided equally with how many friends?

S: 3 friends.

T: Say the division sentence with the answer.

S: $3 \div 3 = 1$.

T: Answer the question in a complete sentence.

S: Each friend will receive 1 pan of brownies.

T: (In the problem, erase *3 pans* and replace it with *1 pan*.) Imagine that Nolan has 1 pan of brownies. If he gave it to his 3 friends to share equally, what portion of the brownies will each friend receive? Write a division sentence to show how you know.

S: (Write $1 \div 3 = \frac{1}{3}$ pan.)

T: Nolan starts out with how many pans of brownies?

S: 1 pan.

T: The 1 pan of brownies is divided equally by how many friends?

S: 3 friends.

T: Say the division sentence with the answer.

S: $1 \div 3 = \frac{1}{3}$.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

While the tape diagramming in the beginning of this lesson is presented as teacher-directed, it is equally acceptable to elicit each step of the diagram from the students through questioning. Many students benefit from verbalizing the next step in a diagram.

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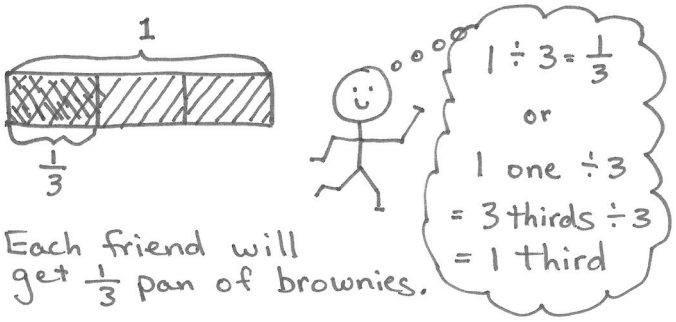
T: Let's model that thinking with a tape diagram. I'll draw a bar and shade it in representing 1 pan of brownies. Next, I'll partition it equally with dotted lines into 3 units, and each unit is $\frac{1}{3}$. (Draw a bar and cut it equally into three parts.) How many pans of brownies did each friend receive this time? Answer the question in a complete sentence.



S: Each friend will receive $\frac{1}{3}$ pan of brownies. (Label $\frac{1}{3}$ underneath one part.)

T: Let's rewrite the problem as thirds. How many thirds are in 1 one?

S: 3 thirds.

T: (Write 3 thirds ÷ 3 = ___.) What is 3 thirds divided by 3?

S: 1 third. (Write 3 thirds ÷ 3 = 1 third.)

T: Another way to interpret this division expression would be to ask, "1 is 3 of what number?" And of course, we know that 3 thirds makes 1.

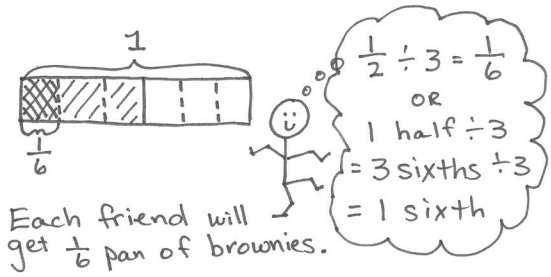
T: But just to be sure, let's check our work. How do we check a division problem?

S: Multiply the answer and the divisor.

T: Check it now.

S: (Work and show $\frac{1}{3} \times 3 = \frac{3}{3} = 1$.)

T: (Replace *1 pan* in the problem with $\frac{1}{2}$ *pan*.) Now, imagine that he only has $\frac{1}{2}$ pan. Still sharing them with 3 friends equally, how many pans of brownies will each friend receive?



T: Now that we have half of a pan instead of 1 whole pan to share, will each friend receive more or less than $\frac{1}{3}$ pan? Turn and discuss.

S: Less than $\frac{1}{3}$ pan. → We have less to share, but we are sharing with the same number of people. They will receive less. → Since we're starting out with $\frac{1}{2}$ pan, which is less than 1 whole pan, the answer should be less than $\frac{1}{3}$ pan.

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T: (Draw a bar and cut it into 2 parts. Shade in 1 part.) How can we show how many people are sharing this $\frac{1}{2}$ pan of brownies? Turn and talk.

S: We can draw dotted lines to show the 3 equal parts that he cuts the half into. → We have to show the same size units, so I'll cut the half that's shaded into 3 parts and the other half into 3 parts, too.

T: (Partition the whole into 6 parts.) What fraction of the pan will each friend receive?

S: $\frac{1}{6}$. (Label $\frac{1}{6}$ underneath one part.)

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$.) Let's think again. 1 half is equal to how many sixths? Look at the tape diagram to help you.

S: 3 sixths.

T: So, what is 3 sixths divided by 3? (Write 3 sixths ÷ 3 =____.)

S: 1 sixth. (Write 3 sixths ÷ 3 = 1 sixth.)

T: What other question could we ask from this division sentence?

S: $\frac{1}{2}$ is 3 of what number?

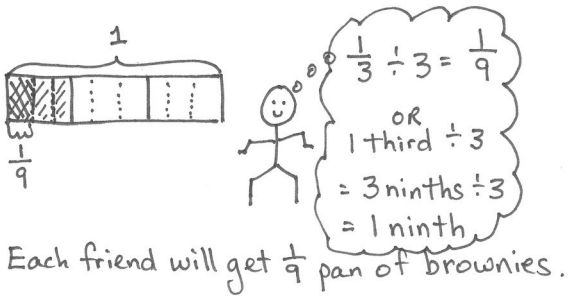
T: Also, 3 of what number makes half?

S: Three 1 sixths makes half.

T: Check your work, and then answer the question in a complete sentence.

S: Each friend will receive $\frac{1}{6}$ pan of brownies.

T: (Erase the $\frac{1}{2}$ in the problem, and replace it with $\frac{1}{3}$.) What if Nolan only has a third of a pan of brownies that he lets 3 friends share equally? How much of a pan of brownies will each friend get? Work with a partner to solve it.



S: (Work.)

T: Answer the question in a complete sentence.

S: Each friend will receive $\frac{1}{9}$ pan of brownies.

T: (Point to all the previous division sentences: $3 \div 3 = 1$, $1 \div 3 = \frac{1}{3}$, $\frac{1}{2} \div 3 = \frac{1}{6}$, and $\frac{1}{3} \div 3 = \frac{1}{9}$.) Compare our division sentences. What do you notice about the quotients? Turn and talk.

S: The answer is becoming smaller and smaller because Nolan kept giving his friends a smaller and smaller part of a pan to share. → The original whole is becoming smaller from 3 to 1, to $\frac{1}{2}$, to $\frac{1}{3}$, and the 3 people sharing the brownies stayed the same. That's why the answer is becoming smaller.

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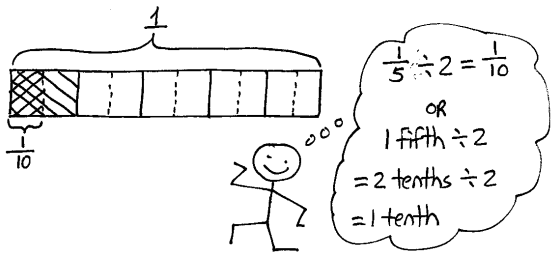
Problem 2

$\frac{1}{5} \div 2$

T: (Post Problem 2 on the board.) Work independently to solve this problem on your personal white board. Draw a tape diagram to show your thinking.



S: (Work.)

T: What's the answer?

S: $\frac{1}{10}$.

T: How many tenths are in 1 fifth?

S: 2 tenths.

T: (Write 2 tenths ÷ 2 = ___.) What's 2 tenths divided by 2?

S: 1 tenth. (Write 2 tenths ÷ 2 = 1 tenth.)

T: Let's try another way: (Write $\frac{1}{5}$ = 2 × _____.) Fill in the missing factor.

S: 1 tenth.

T: Let's check our work aloud together. What is the quotient?

S: 1 tenth.

T: The divisor?

S: 2.

T: Let's multiply the quotient by the divisor. What is 1 tenth times 2?

S: 2 tenths.

T: Is 2 tenths the same unit as our original whole?

S: No.

T: Did we make a mistake?

S: No, 2 tenths is just another way to say 1 fifth.

T: Say 2 tenths in its simplest form.

S: 1 fifth.

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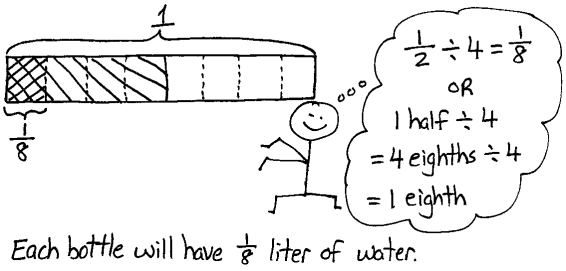
Problem 3

If Melanie pours $\frac{1}{2}$ liter of water into 4 bottles, putting an equal amount in each, how many liters of water will be in each bottle?

T: (Post Problem 3 on the board and read it with the class.) How many liters of water does Melanie have?



S: Half a liter.

T: Half a liter is being poured into how many bottles?

S: 4 bottles.

T: How do you solve this problem? Turn and discuss.

S: We have to divide. → The division expression is $\frac{1}{2} \div 4$. → I need to divide the dividend 1 half by the divisor 4. → I can draw 1 half and cut it into 4 equal parts. → I can think of this as $\frac{1}{2} = 4 \times$ ___.

T: On your personal white board, draw a tape diagram and solve this problem independently.

S: (Work.)

T: Say the division sentence and the answer.

S: $\frac{1}{2} \div 4 = \frac{1}{8}$. (Write $\frac{1}{2} \div 4 = \frac{1}{8}$.)

T: Now, say the division sentence using eighths and unit form.

S: 4 eighths ÷ 4 = 1 eighth.

T: Show me your checking solution.

S: (Work and show $\frac{1}{8} \times 4 = \frac{4}{8} = \frac{1}{2}$.)

T: If you used a multiplication sentence with a missing factor, say it now.

S: $\frac{1}{2} = 4 \times \frac{1}{8}$.

T: No matter your strategy, we all got the same result. Answer the question in a complete sentence.

S: Each bottle will have $\frac{1}{8}$ liter of water.

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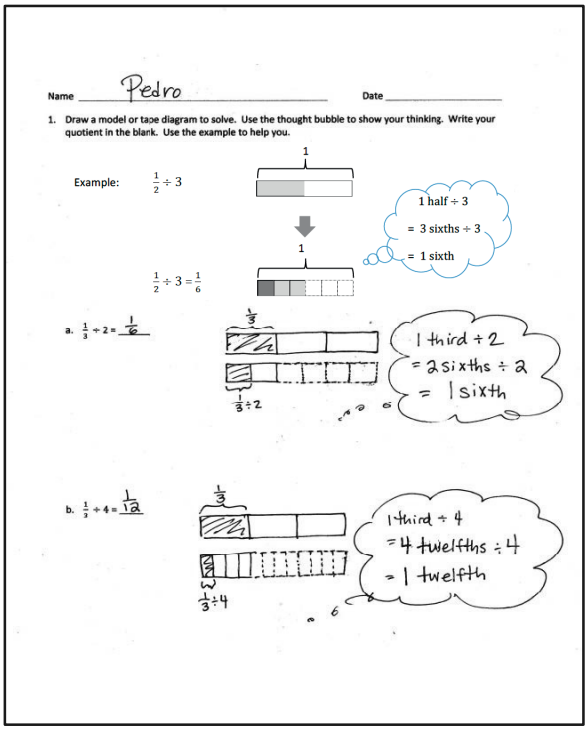
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Pedro Date

1. Draw a model or tape diagram to solve. Use the thought bubble to show your thinking. Write your quotient in the blank. Use the example to help you.

Example: $\frac{1}{2} \div 3$

1 half ÷ 3
= 3 sixths ÷ 3
= 1 sixth

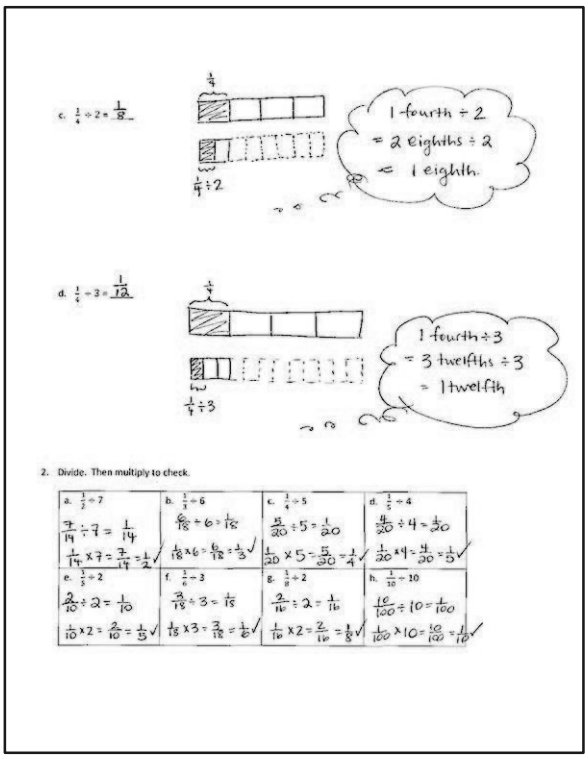
$\frac{1}{2} \div 3 = \frac{1}{6}$

a. $\frac{1}{3} \div 2 = \frac{1}{6}$

1 third ÷ 2
= 2 sixths ÷ 2
= 1 sixth

b. $\frac{1}{3} \div 4 = \frac{1}{12}$

1 third ÷ 4
= 4 twelfths ÷ 4
= 1 twelfth


c. $\frac{1}{4} \div 2 = \frac{1}{8}$

1 fourth ÷ 2
= 2 eighths ÷ 2
= 1 eighth

d. $\frac{1}{4} \div 3 = \frac{1}{12}$

1 fourth ÷ 3
= 3 twelfths ÷ 3
= 1 twelfth

2. Divide. Then multiply to check.

a. $\frac{1}{2} \div 7$ $\frac{7}{14} \div 7 = \frac{1}{14}$ $\frac{1}{14} \times 7 = \frac{7}{14} = \frac{1}{2}$ ✓	b. $\frac{1}{3} \div 6$ $\frac{6}{18} \div 6 = \frac{1}{18}$ $\frac{1}{18} \times 6 = \frac{6}{18} = \frac{1}{3}$ ✓	c. $\frac{1}{4} \div 5$ $\frac{5}{20} \div 5 = \frac{1}{20}$ $\frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$ ✓	d. $\frac{1}{5} \div 4$ $\frac{4}{20} \div 4 = \frac{1}{20}$ $\frac{1}{20} \times 4 = \frac{4}{20} = \frac{1}{5}$ ✓
e. $\frac{1}{5} \div 2$ $\frac{2}{10} \div 2 = \frac{1}{10}$ $\frac{1}{10} \times 2 = \frac{2}{10} = \frac{1}{5}$ ✓	f. $\frac{1}{6} \div 3$ $\frac{3}{18} \div 3 = \frac{1}{18}$ $\frac{1}{18} \times 3 = \frac{3}{18} = \frac{1}{6}$ ✓	g. $\frac{1}{8} \div 2$ $\frac{2}{16} \div 2 = \frac{1}{16}$ $\frac{1}{16} \times 2 = \frac{2}{16} = \frac{1}{8}$ ✓	h. $\frac{1}{10} \div 10$ $\frac{10}{100} \div 10 = \frac{1}{100}$ $\frac{1}{100} \times 10 = \frac{10}{100} = \frac{1}{10}$ ✓

Student Debrief (10 minutes)

Lesson Objective: Divide a unit fraction by a whole number.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what is the relationship between (a) and (b), (c) and (d), and (b) and (d)?
- Why is the quotient of Problem 1(c) greater than that of Problem 1(d)? Is it reasonable? Explain your thinking to your partner.
- In Problem 2, what is the relationship between (c) and (d) and (b) and (f)?
- Compare your drawing for Problem 3 with that of a partner. How is it the same as or different from your partner's?
- How did you solve Problem 5? Share your solution with and explain your strategy to a partner.

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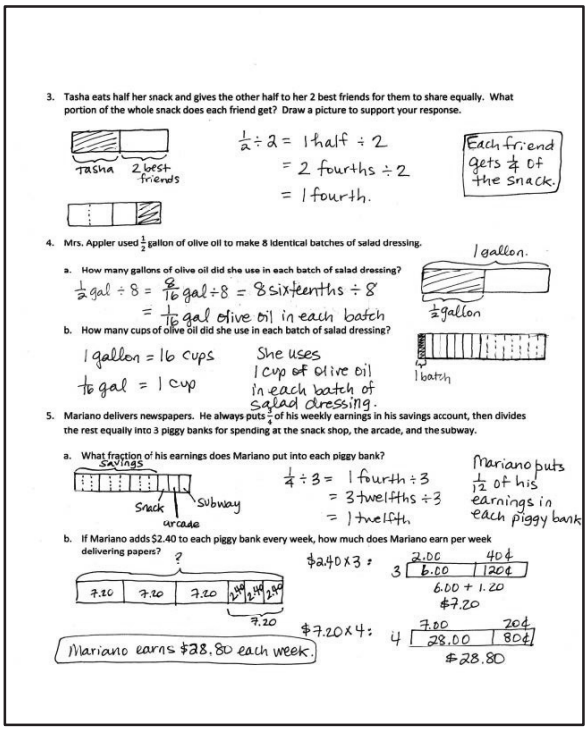
- While the invert and multiply strategy is not explicitly taught (nor should it be while students grapple with these abstract concepts of division), discussing various ways of thinking about division in general can be fruitful. A discussion might proceed as follows:

T: Is dividing something by 2 the same as taking 1 half of it? For example, is $4 \div 2 = 4 \times \frac{1}{2}$? (Write this on the board and allow some quiet time for thinking.) Can you think of some examples?

S: Yes. → If 4 cookies are divided between 2 people, each person receives half of the cookies.T: So, if that's true, would this also be true: $\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$? (Write and allow quiet time.) Can you think of some examples?

S: Yes. → If there is only 1 fourth of a candy bar and 2 people share it, they would each receive half of the fourth. That would be 1 eighth of the whole candy bar.



Once this idea is introduced, search for opportunities in visual models to highlight it. For example, in today's lesson, Problem 3's tape diagram was drawn to show $\frac{1}{2}$ divided into 4 equal parts. The model highlights that the answer is $\frac{1}{4}$ of that $\frac{1}{2}$, an answer that can be obtained by multiplying $\frac{1}{4} \times \frac{1}{2}$.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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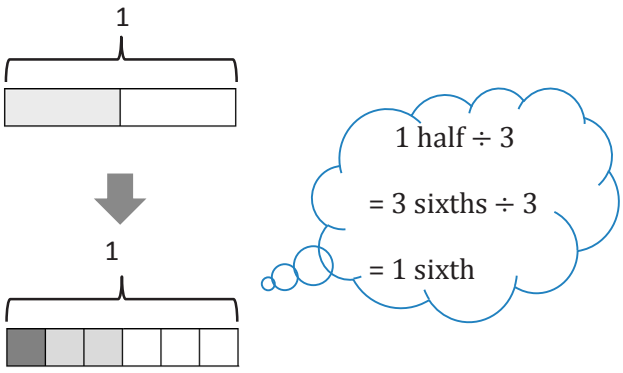
Name _____ Date _____

1. Draw a model or tape diagram to solve. Use the thought bubble to show your thinking. Write your quotient in the blank. Use the example to help you.

Example: $\frac{1}{2} \div 3$



$\frac{1}{2} \div 3 = \frac{1}{6}$

a. $\frac{1}{3} \div 2 =$ _____

b. $\frac{1}{3} \div 4 =$ _____

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c. $\frac{1}{4} \div 2 =$ _____

d. $\frac{1}{4} \div 3 =$ _____

2. Divide. Then, multiply to check.

a. $\frac{1}{2} \div 7$	b. $\frac{1}{3} \div 6$	c. $\frac{1}{4} \div 5$	d. $\frac{1}{5} \div 4$
e. $\frac{1}{5} \div 2$	f. $\frac{1}{6} \div 3$	g. $\frac{1}{8} \div 2$	h. $\frac{1}{10} \div 10$

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3. Tasha eats half her snack and gives the other half to her two best friends for them to share equally. What portion of the whole snack does each friend get? Draw a picture to support your response.

4. Mrs. Appler used $\frac{1}{2}$ gallon of olive oil to make 8 identical batches of salad dressing.

   a. How many gallons of olive oil did she use in each batch of salad dressing?

   b. How many cups of olive oil did she use in each batch of salad dressing?

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5. Mariano delivers newspapers. He always puts $\frac{3}{4}$ of his weekly earnings in his savings account and then divides the rest equally into 3 piggy banks for spending at the snack shop, the arcade, and the subway.

   a. What fraction of his earnings does Mariano put into each piggy bank?

   b. If Mariano adds $2.40 to each piggy bank every week, how much does Mariano earn per week delivering papers?

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Name _____ Date _____

1. Solve. Support at least one of your answers with a model or tape diagram.

   a. $\frac{1}{2} \div 4 =$ _____

   b. $\frac{1}{8} \div 5 =$ _____

2. Larry spends half of his workday teaching piano lessons. If he sees 6 students, each for the same amount of time, what fraction of his workday is spent with each student?

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Name _____ Date _____

1. Solve and support your answer with a model or tape diagram. Write your quotient in the blank.

a. $\frac{1}{2} \div 4 =$ _____

b. $\frac{1}{3} \div 6 =$ _____

c. $\frac{1}{4} \div 3 =$ _____

d. $\frac{1}{5} \div 2 =$ _____

2. Divide. Then, multiply to check.

a. $\frac{1}{2} \div 10$	b. $\frac{1}{4} \div 10$	c. $\frac{1}{3} \div 5$	d. $\frac{1}{5} \div 3$
e. $\frac{1}{8} \div 4$	f. $\frac{1}{7} \div 3$	g. $\frac{1}{10} \div 5$	h. $\frac{1}{5} \div 20$

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3. Teams of four are competing in a quarter-mile relay race. Each runner must run the same exact distance. What is the distance each teammate runs?

4. Solomon has read $\frac{1}{3}$ of his book. He finishes the book by reading the same amount each night for 5 nights.

   a. What fraction of the book does he read each of the 5 nights?

   b. If he reads 14 pages on each of the 5 nights, how long is the book?

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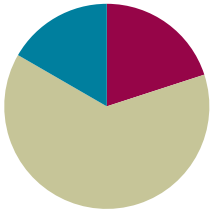
Lesson 27

Objective: Solve problems involving fraction division.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(38 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Count by Fractions **5.NF.7** (6 minutes)
- Divide Whole Numbers by Unit Fractions **5.NF.7** (3 minutes)
- Divide Unit Fractions by Whole Numbers **5.NF.7** (3 minutes)

Count by Fractions (6 minutes)

Note: This fluency activity reviews Lesson 25.

T: Count by sixths to 12 sixths. (Write as students count.)

S: 1 sixth, 2 sixths, 3 sixths, 4 sixths, 5 sixths, 6 sixths, 7 sixths, 8 sixths, 9 sixths, 10 sixths, 11 sixths, 12 sixths.

T: Let's count by sixths again. This time, when we arrive at a whole number, say the whole number. (Write as students count.)

S: 1 sixth, 2 sixths, 3 sixths, 4 sixths, 5 sixths, 1 whole, 7 sixths, 8 sixths, 9 sixths, 10 sixths, 11 sixths, 2 wholes.

T: Let's count by sixths again. This time, change improper fractions to mixed numbers. (Write as students count.)

S: 1 sixth, 2 sixths, 3 sixths, 4 sixths, 5 sixths, 1 whole, 1 and 1 sixth, 1 and 2 sixths, 1 and 3 sixths, 1 and 4 sixths, 1 and 5 sixths, 2 wholes.

T: Let's count by sixths again. This time, simplify 3 sixths to 1 half. (Write as students count.)

S: 1 sixth, 2 sixths, 1 half, 4 sixths, 5 sixths, 1 whole, 1 and 1 sixth, 1 and 2 sixths, 1 and 1 half, 1 and 4 sixths, 1 and 5 sixths, 2 wholes.

T: Let's count by sixths again. This time, simplify 2 sixths to 1 third and 4 sixths to 2 thirds. (Write as students count.)

S: 1 sixth, 1 third, 1 half, 2 thirds, 5 sixths, 1 whole, 1 and 1 sixth, 1 and 1 third, 1 and 1 half, 1 and 2 thirds, 1 and 5 sixths, 2 wholes.

Continue the process counting by eighths to 8 eighths or, if time allows, 16 eighths.

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Divide Whole Numbers by Unit Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 25.

T: (Write $1 \div \frac{1}{2}$.) Say the division expression.

S: $1 \div \frac{1}{2}$.

T: How many halves are in 1 whole?

S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2} =$ ____.) How many halves are in 2 wholes?

S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2} =$ ____.) How many halves are in 3 wholes?

S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $6 \div \frac{1}{2} =$ ____.) On your personal white board, write the division sentence with the answer.

S: (Write $6 \div \frac{1}{2} = 12$.)

Continue with the following possible suggestions: $1 \div \frac{1}{3}$, $2 \div \frac{1}{3}$, $7 \div \frac{1}{3}$, $1 \div \frac{1}{4}$, $2 \div \frac{1}{4}$, $9 \div \frac{1}{4}$, $5 \div \frac{1}{5}$, $6 \div \frac{1}{6}$, and $8 \div \frac{1}{8}$.

Divide Unit Fractions by Whole Numbers (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 26.

T: (Write $\frac{1}{2} \div 2 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 2 = \frac{1}{4}$.

T: (Write $\frac{1}{2} \div 2 = \frac{1}{4}$. Beneath it, write $\frac{1}{2} \div 3 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 3 = \frac{1}{6}$.

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$. Beneath it, write $\frac{1}{2} \div 4 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 4 = \frac{1}{8}$.

T: (Write $\frac{1}{2} \div 7 =$ ____.) On your personal white board, complete the number sentence.

S: (Write $\frac{1}{2} \div 7 = \frac{1}{14}$.)

Continue with the following possible sequence: $\frac{1}{3} \div 2$, $\frac{1}{3} \div 3$, $\frac{1}{3} \div 4$, $\frac{1}{3} \div 9$, $\frac{1}{5} \div 3$, $\frac{1}{5} \div 5$, $\frac{1}{5} \div 7$, $\frac{1}{4} \div 4$, and $\frac{1}{8} \div 6$.

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Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The time normally allotted for the Application Problem has been reallocated to the Concept Development to provide adequate time for solving the word problems.

Suggested Delivery of Instruction for Solving Lesson 27 Word Problems

1. Model the problem.

Have two pairs of student work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

MP.3 As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish work on that question, sharing his or her work and thinking with a peer. All students should write their equations and statements of the answer.

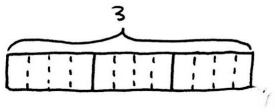
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solution.

Problem 1

Mrs. Silverstein bought 3 mini cakes for a birthday party. She cuts each cake into quarters and plans to serve each guest 1 quarter of a cake. How many guests can she serve with all her cakes? Draw a model to support your response.

Method 1

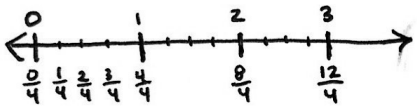


4 fourths in 1 whole.
12 fourths in 3 wholes.

$3 \div \frac{1}{4} = 12$

Mrs. Silverstein can serve 12 guests a quarter of a cake.

Method 2



Mrs. Silverstein can serve 12 guests if she cuts each cake into quarters.

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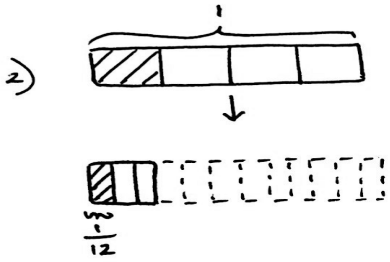
In this problem, students are asked to divide a whole number (3) by a unit fraction ($\frac{1}{4}$) and draw a model. A tape diagram or a number line would both be acceptable models to support their responses. The reference to the unit fraction as a *quarter* provides a bit of complexity. There are 4 fourths in 1 whole and 12 fourths in 3 wholes.

Problem 2

Mr. Pham has $\frac{1}{4}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.



$\frac{1}{4} \div 3$
= 1 fourth ÷ 3
= 3 twelfths ÷ 3
= 1 twelfth

✓ $\frac{1}{12} \times 3 = \frac{3}{12} = \frac{1}{4}$

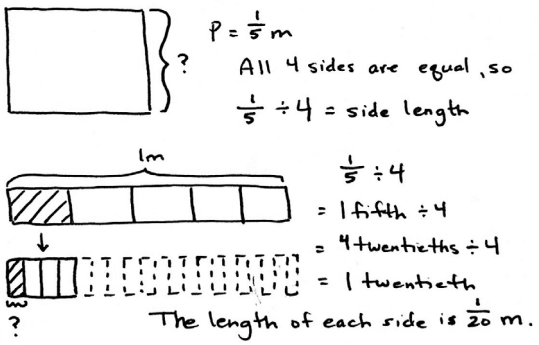
Mr. Pham will eat $\frac{1}{12}$ of the lasagna each night.

Problem 2 is intentionally similar to Problem 1. Although the numbers used in the problems are identical, careful reading reveals that 3 is now the divisor rather than the dividend. While drawing a supporting tape diagram, students should recognize that dividing a fourth into 3 equal parts creates a new unit—twelfths. The model shows that the fraction $\frac{1}{4}$ is equal to $\frac{3}{12}$; therefore, a division sentence using unit form (3 twelfths ÷ 3) is easy to solve. Facilitate a quick discussion about the similarities and differences of Problems 1 and 2. What do students notice about the division expressions and the solutions?

Problem 3

The perimeter of a square is $\frac{1}{5}$ meter.

a. Find the length of each side in meters. Draw a picture to support your response.
b. How long is each side in centimeters?



$P = \frac{1}{5}$ m
All 4 sides are equal, so
$\frac{1}{5} \div 4$ = side length

$\frac{1}{5} \div 4$
= 1 fifth ÷ 4
= 4 twentieths ÷ 4
= 1 twentieth

The length of each side is $\frac{1}{20}$ m.

b) $\frac{1}{20}$ m = ? cm

Each side is 5 cm.

✓ $\frac{1}{20} \times 4 = \frac{4}{20} = \frac{1}{5}$

$\frac{1}{20}$ m = $\frac{1}{20} \times 1$m
= $\frac{1}{20} \times 100$ cm
= $\frac{1 \times 100^{5}}{1 \quad 20}$ cm
= 5 cm

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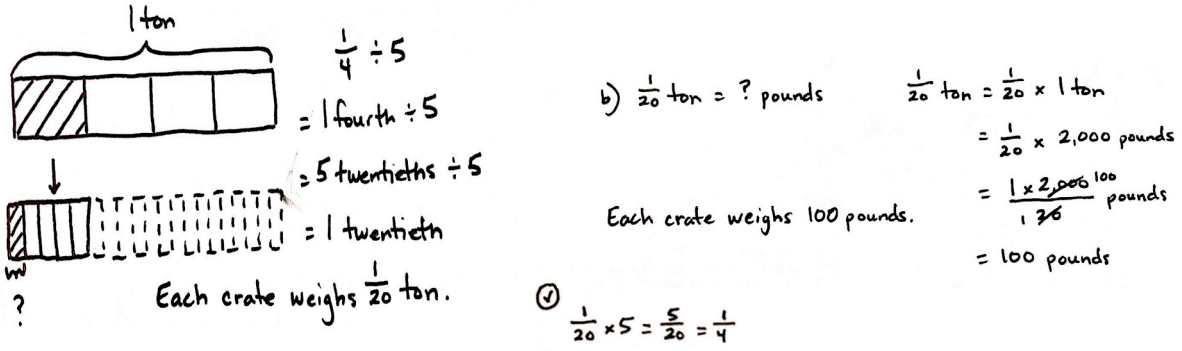
This problem requires that students recall their measurement work from Grades 3 and 4 involving perimeter. Students must know that all four side lengths of a square are equivalent; therefore, the unknown side length can be found by dividing the perimeter by 4 ($\frac{1}{5}$ m ÷ 4). The tape diagram shows clearly that dividing a fifth into 4 equal parts creates a new unit, twentieths, and that $\frac{1}{5}$ is equal to $\frac{4}{20}$. Students may use a division expression using unit form (4 twentieths ÷ 4) to solve this problem very easily. This problem also provides an opportunity to point out a partitive division interpretation to students. While the model was drawn to depict 1 fifth divided into 4 equal parts, the question mark clearly asks "What is $\frac{1}{4}$ of $\frac{1}{5}$?" That is $\frac{1}{4} \times \frac{1}{5}$.

Part (b) requires that students rename $\frac{1}{20}$ meter as centimeters. This conversion mirrors the work done in Lesson 20. Since 1 meter is equal to 100 centimeters, students can multiply to find that $\frac{1}{20}$ m is equivalent to $\frac{100}{20}$ cm, or 5 cm.

Problem 4

A pallet holding 5 identical crates weighs $\frac{1}{4}$ ton.

a. How many tons does each crate weigh? Draw a picture to support your response.
b. How many pounds does each crate weigh?



The numbers in this problem are similar to those used in Problem 3, and the resulting quotient is again $\frac{1}{20}$. Engage students in a discussion about why the answer is the same in Problems 3 and 4 but was not the same in Problems 1 and 2, despite both sets of problems using similar numbers. Is this just a coincidence? Additionally, Problem 4 presents another opportunity for students to interpret the division here as $\frac{1}{4} = 5 \times$ ___.

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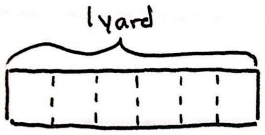
Problem 5

Faye has 5 pieces of ribbon, each 1 yard long. She cuts each ribbon into sixths.

a. How many sixths will she have after cutting all the ribbons?
b. How long will each of the sixths be in inches?



$1 \div \frac{1}{6} = 6$

6 sixths in 1 yard.

30 sixths in 5 yards.

Faye will have 30 sixths after cutting all the ribbons.

b) Each sixth is $\frac{1}{6}$ of a yard.

Each sixth is 6 inches long.

$\frac{1}{6}$ yd. = ? in.

$\frac{1}{6}$ yd = $\frac{1}{6}$ × 1 yd.

$= \frac{1}{6} \times 36$ in.

$= \frac{1 \times 36}{6}$ in.

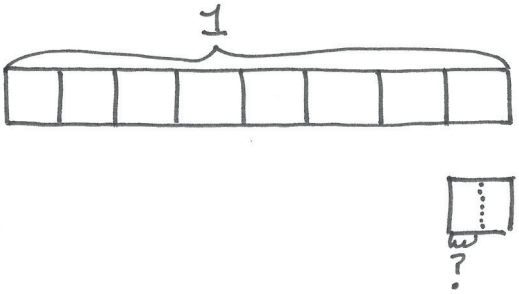
$= 6$ in

In Problem 5, since Faye has 5 pieces of ribbon of equal length, students have the choice of drawing a tape diagram showing how many sixths are in 1 yard (and then multiplying that number by 5) or drawing a tape diagram showing all 5 yards to find 30 sixths in total.

Problem 6

A glass pitcher is filled with water. $\frac{1}{8}$ of the water is poured equally into 2 glasses.

a. What fraction of the water is in each glass?
b. If each glass has 3 fluid ounces of water in it, how many fluid ounces of water were in the full pitcher?
c. If $\frac{1}{4}$ of the remaining water is poured out of the pitcher to water a plant, how many cups of water are left in the pitcher?



a) $\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{16}$

or

$\frac{1}{8} \div 2 = \frac{1}{16}$

$\frac{1}{16}$ of the water is in each glass.

b) $3 \times 2 = 6$

1 unit = 6

8 units = 48

There are 48 fluid ounces in the full pitcher.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Problem 6 in this lesson may be especially difficult for English language learners. The teacher may wish to have students act out this problem to keep track of the different questions asked about the water.

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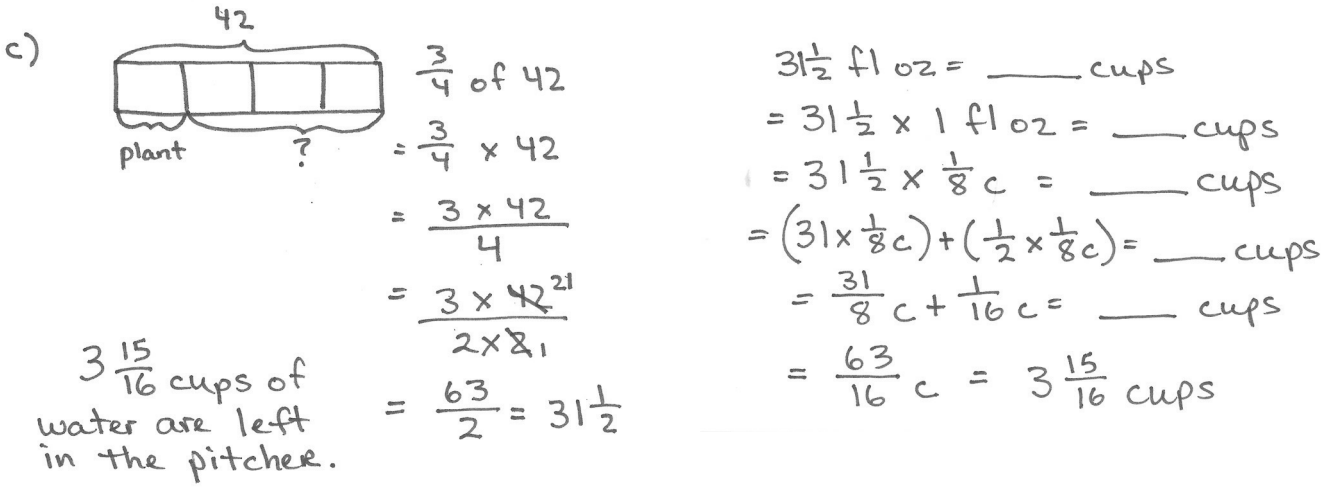


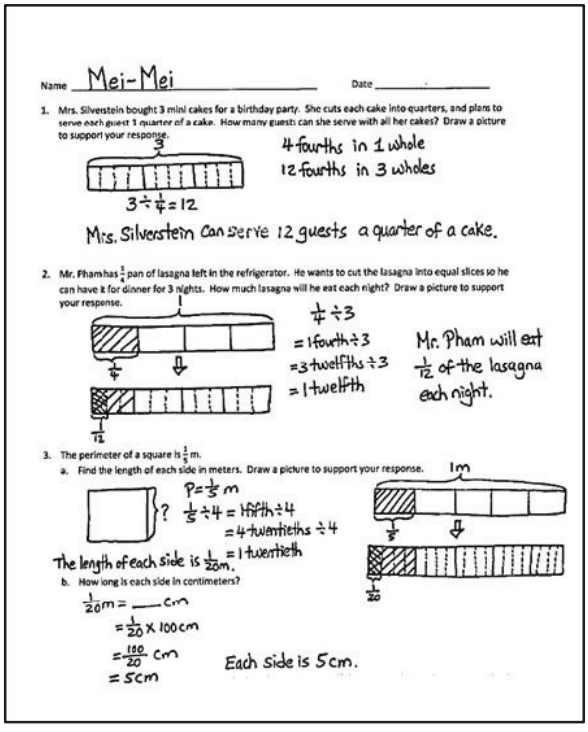
In Part (a), to find what fraction of the water is in each glass, students might divide the unit fraction ($\frac{1}{8}$) by 2 or multiply $\frac{1}{2} \times \frac{1}{8}$. Part (b) requires students to show that since both glasses hold 3 fluid ounces of water each, 1 unit (or $\frac{1}{8}$ of the total water) is equal to 6 fluid ounces. Multiplying 6 fluid ounces by 8 provides the total amount of water (48 fluid ounces) that was originally in the pitcher. Part (c) is a complex, multi-step problem that may require careful discussion. Since $\frac{1}{8}$ of the water (or 6 fluid ounces) has already been poured out, subtraction yields 42 fluid ounces of water left in the pitcher. After 1 fourth of the remaining water is used for the plant, $\frac{3}{4}$ of the water in the pitcher is $31\frac{1}{2}$ fluid ounces. Students must then convert $31\frac{1}{2}$ fluids ounces into cups.

Student Debrief (10 minutes)

Lesson Objective: Solve problems involving fraction division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


Name Mei-Mei Date

1. Mrs. Silverstein bought 3 mini cakes for a birthday party. She cuts each cake into quarters, and plans to serve each guest 1 quarter of a cake. How many guests can she serve with all her cakes? Draw a picture to support your response.

4 fourths in 1 whole
12 fourths in 3 wholes
$3 \div \frac{1}{4} = 12$
Mrs. Silverstein can serve 12 guests a quarter of a cake.

2. Mr. Pham has $\frac{1}{3}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.

$\frac{1}{3} \div 3$
$= 1$ fourth $\div 3$
$= 3$ twelfths $\div 3$
$= 1$ twelfth
Mr. Pham will eat $\frac{1}{12}$ of the lasagna each night.

3. The perimeter of a square is $\frac{1}{5}$ m.
a. Find the length of each side in meters. Draw a picture to support your response.

$P = \frac{1}{5}$ m
$\frac{1}{5} \div 4 = 1$ fifth $\div 4$
$= 4$ twentieths $\div 4$
$= 1$ twentieth
The length of each side is $\frac{1}{20}$ m.
b. How long is each side in centimeters?
$\frac{1}{20}$ m = ____ cm
$= \frac{1}{20} \times 100$ cm
$= \frac{100}{20}$ cm
$= 5$ cm
Each side is 5 cm.

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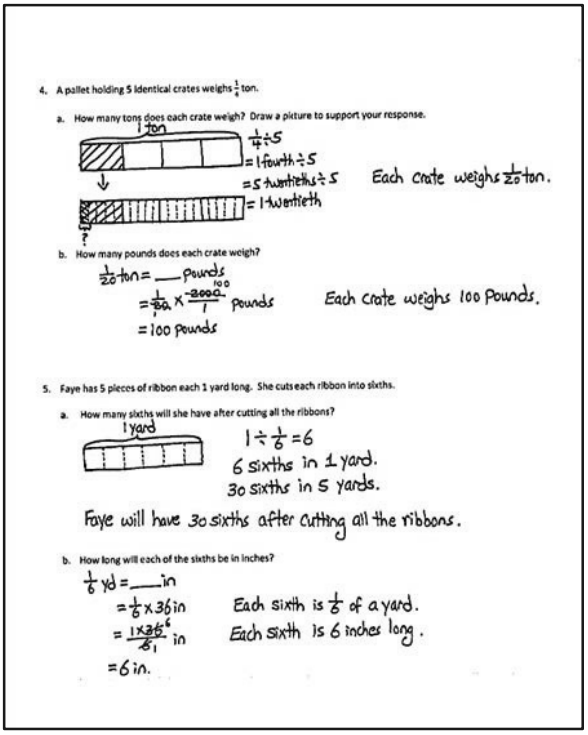
Any combination of the questions below may be used to lead the discussion.

- What did you notice about Problems 1 and 2? What are the similarities and differences? What did you notice about the division expressions and the solutions?
- What did you notice about the solutions in Problems 3(a) and 4(a)? Share your answer and explain it to a partner.
- Why is the answer the same in Problems 3 and 4, but not the same in Problems 1 and 2, despite using similar numbers in both sets of problems? Is this just a coincidence? Can you create similar pairs of problems and see if the resulting quotient is always equivalent (e.g., $\frac{1}{3} \div 2$ and $\frac{1}{2} \div 3$)?
- How did you solve for Problem 6? What strategy did you use? Explain it to a partner.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


4. A pallet holding 5 identical crates weighs $\frac{1}{4}$ ton.

a. How many tons does each crate weigh? Draw a picture to support your response.

1 ton

$\frac{1}{4} \div 5$

= 1 fourth ÷ 5

= 5 twentieths ÷ 5

= 1 twentieth

Each crate weighs $\frac{1}{20}$ ton.

b. How many pounds does each crate weigh?

$\frac{1}{20}$ ton = ___ pounds

$= \frac{1}{20} \times \frac{2000}{1}$ pounds

= 100 pounds

Each crate weighs 100 pounds.

5. Faye has 5 pieces of ribbon each 1 yard long. She cuts each ribbon into sixths.

a. How many sixths will she have after cutting all the ribbons?

1 yard

$1 \div \frac{1}{6} = 6$

6 sixths in 1 yard.

30 sixths in 5 yards.

Faye will have 30 sixths after cutting all the ribbons.

b. How long will each of the sixths be in inches?

$\frac{1}{6}$ yd = ___ in

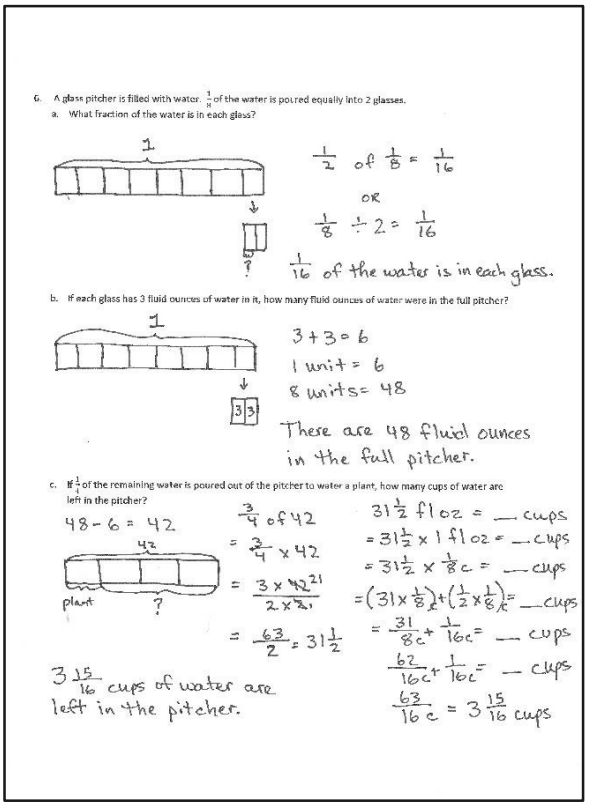
$= \frac{1}{6} \times 36$ in

$= \frac{1 \times 36^{6}}{6_{1}}$ in

= 6 in.

Each sixth is $\frac{1}{6}$ of a yard.

Each sixth is 6 inches long.


6. A glass pitcher is filled with water. $\frac{1}{8}$ of the water is poured equally into 2 glasses.

a. What fraction of the water is in each glass?

1

$\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{16}$

OR

$\frac{1}{8} \div 2 = \frac{1}{16}$

$\frac{1}{16}$ of the water is in each glass.

b. If each glass has 3 fluid ounces of water in it, how many fluid ounces of water were in the full pitcher?

1

3 3

3 + 3 = 6

1 unit = 6

8 units = 48

There are 48 fluid ounces in the full pitcher.

c. If $\frac{3}{4}$ of the remaining water is poured out of the pitcher to water a plant, how many cups of water are left in the pitcher?

48 − 6 = 42

42

plant ?

$\frac{3}{4}$ of 42

$= \frac{3}{4} \times 42$

$= \frac{3 \times 42^{21}}{2 \times 2_{1}}$

$= \frac{63}{2} = 31\frac{1}{2}$

$3\frac{15}{16}$ cups of water are left in the pitcher.

$31\frac{1}{2}$ fl oz = ___ cups

$= 31\frac{1}{2} \times 1$ fl oz = ___ cups

$= 31\frac{1}{2} \times \frac{1}{8}$ c = ___ cups

$= (31 \times \frac{1}{8}) + (\frac{1}{2} \times \frac{1}{8})$ = ___ cups

$= \frac{31}{8}$c + $\frac{1}{16}$c = ___ cups

$\frac{62}{16}$c + $\frac{1}{16}$c = ___ cups

$\frac{63}{16}$ c = $3\frac{15}{16}$ cups

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Name _____ Date _____

1. Mrs. Silverstein bought 3 mini cakes for a birthday party. She cuts each cake into quarters and plans to serve each guest 1 quarter of a cake. How many guests can she serve with all her cakes? Draw a picture to support your response.

2. Mr. Pham has $\frac{1}{4}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.

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3. The perimeter of a square is $\frac{1}{5}$ of a meter.

   a. Find the length of each side in meters. Draw a picture to support your response.

   b. How long is each side in centimeters?

4. A pallet holding 5 identical crates weighs $\frac{1}{4}$ of a ton.

   a. How many tons does each crate weigh? Draw a picture to support your response.

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b. How many pounds does each crate weigh?

5. Faye has 5 pieces of ribbon, each 1 yard long. She cuts each ribbon into sixths.

a. How many sixths will she have after cutting all the ribbons?

b. How long will each of the sixths be in inches?

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6. A glass pitcher is filled with water. $\frac{1}{8}$ of the water is poured equally into 2 glasses.
   a. What fraction of the water is in each glass?

   b. If each glass has 3 fluid ounces of water in it, how many fluid ounces of water were in the full pitcher?

   c. If $\frac{1}{4}$ of the remaining water is poured out of the pitcher to water a plant, how many cups of water are left in the pitcher?

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Name _____ Date _____

1. Kevin divides 3 pieces of paper into fourths. How many fourths does he have? Draw a picture to support your response.

2. Sybil has $\frac{1}{2}$ of a pizza left over. She wants to share the pizza with 3 of her friends. What fraction of the original pizza will Sybil and her 3 friends each receive? Draw a picture to support your response.

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Name _____ Date _____

1. Kelvin ordered four pizzas for a birthday party. The pizzas were cut in eighths. How many slices were there? Draw a picture to support your response.

2. Virgil has $\frac{1}{6}$ of a birthday cake left over. He wants to share the leftover cake with 3 friends. What fraction of the original cake will each of the 4 people receive? Draw a picture to support your response.

3. A pitcher of water contains $\frac{1}{4}$ liters of water. The water is poured equally into 5 glasses.

   a. How many liters of water are in each glass? Draw a picture to support your response.

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b. Write the amount of water in each glass in milliliters.

4. Drew has 4 pieces of rope 1 meter long each. He cuts each rope into fifths.

a. How many fifths will he have after cutting all the ropes?

b. How long will each of the fifths be in centimeters?

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5. A container is filled with blueberries. $\frac{1}{6}$ of the blueberries is poured equally into two bowls.

   a. What fraction of the blueberries is in each bowl?

   b. If each bowl has 6 ounces of blueberries in it, how many ounces of blueberries were in the full container?

   c. If $\frac{1}{5}$ of the remaining blueberries is used to make muffins, how many pounds of blueberries are left in the container?

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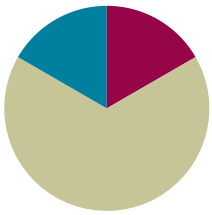
Lesson 28

Objective: Write equations and word problems corresponding to tape and number line diagrams.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Concept Development	(40 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Count by Fractions **5.NF.7** (5 minutes)
- Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers **5.NF.7** (5 minutes)

Count by Fractions (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 29.

T: Count by tenths to 20 tenths. (Write as students count.)
S: 1 tenth, 2 tenths, …, 20 tenths.
T: Let's count by tenths again. This time, when we arrive at a whole number, say the whole number. (Write as students count.)
S: 1 tenth, 2 tenths, …, 1, 11 tenths, 12 tenths, …, 2.
T: Let's count by tenths again. This time, say the tenths in decimal form. (Write as students count.)
S: Zero point 1, zero point 2, ….
T: How many tenths are in 1 whole?
S: 10 tenths.
T: (Write 1 = 10 tenths. Beneath it, write 2 = ____ tenths.) How many tenths are in 2 wholes?
S: 20 tenths.
T: 3 wholes?
S: 30 tenths.
T: (Write 9 = ____ tenths.) On your personal white board, fill in the unknown number.
S: (Write 9 = 90 tenths.)
T: (Write 10 = ____ tenths.) Fill in the unknown number.
S: (Write 10 = 100 tenths.)

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Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 25–26 and prepares students for today's lesson.

T: (Write $2 \div \frac{1}{3} =$ ____) Say the division sentence with the answer.
S: $2 \div \frac{1}{3} = 6$.
T: (Write $2 \div \frac{1}{3} = 6$. Beneath it, write $3 \div \frac{1}{3} =$ ____.) Say the division sentence with the answer.
S: $3 \div \frac{1}{3} = 9$.
T: (Write $3 \div \frac{1}{3} = 9$. Beneath it, write $8 \div \frac{1}{3} =$ ____.) On your personal white board, write the division sentence with the answer.
S: (Write $8 \div \frac{1}{3} = 24$.)

Continue with the following suggested problems: $2 \div \frac{1}{4}$, $5 \div \frac{1}{6}$, and $9 \div \frac{1}{6}$.

T: (Write $\frac{1}{2} \div 2 =$ ____.) Say the division sentence with the answer.
S: $\frac{1}{2} \div 2 = \frac{1}{4}$.
T: (Write $\frac{1}{2} \div 2 = \frac{1}{4}$. Beneath it, write $\frac{1}{4} \div 2 =$ ____.) Say the division sentence with the answer.
S: $\frac{1}{4} \div 2 = \frac{1}{8}$.
T: (Write $\frac{1}{4} \div 2 = \frac{1}{8}$. Erase the board and write $\frac{1}{3} \div 2 =$ ____.) On your personal white board, write the sentence with the answer.
S: (Write $\frac{1}{3} \div 2 = \frac{1}{6}$.)

Continue the process with the following possible suggested problems: $\frac{1}{6} \div 2$ and $\frac{1}{3} \div 3$.

Concept Development (40 minutes)

Materials: (S) Problem Set, personal white board

Note: Today's lesson involves creating word problems, which can be time intensive. The time for the Application Problem has been included in the Concept Development.

Note: Students create word problems from expressions and visual models in the form of tape diagrams. In Problem 1, guide students to identify what the whole and the divisor are in the expressions before they start writing the word problems. After about 10 minutes of working time, guide students to analyze the tape diagrams in Problems 2, 3, and 4. After the discussion, allow students to work for another 10 minutes. Finally, review the answers and have students share their answers with the class.

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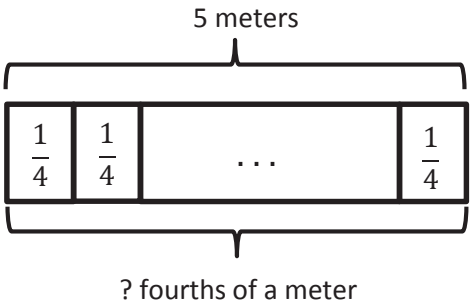
Problems 1–2

1. **Create and solve a division story problem about 5 meters of rope that is modeled by the tape diagram below.**



T: Let's take a look at Problem 1 on our Problem Set and read it out loud together. What's the whole in the tape diagram?

S: 5.

T: 5 what?

S: 5 meters of rope.

T: What else can you tell me about this tape diagram? Turn and share with a partner.

S: The 5 meters of rope is being cut into fourths. → The 5 meters of rope is being cut into pieces that are 1 fourth meter long. How many pieces can be cut? → This is a division drawing because a whole is being partitioned into equal parts. → We're trying to find out how many fourths are in 5.

T: Since we seem to agree that this is a picture of division, what would the division expression look like? Turn and talk.

S: Since 5 is the whole, it is the dividend. The one-fourths are the equal parts, so that is the divisor. → $5 \div \frac{1}{4}$.

T: Work with your partner to write a story about this diagram, then solve for the answer. (A possible response appears on the student work example of the Problem Set.)

T: (Allow students time to work.) How can we be sure that 20 fourths is correct? How do we check a division problem?

S: Multiply the quotient and the divisor.

T: What would our checking equation look like? Write it with your partner and solve.

S: $20 \times \frac{1}{4} = \frac{20}{4} = 5$.

T: Were we correct? How do you know?

S: Yes. Our product matches the dividend with which we started.

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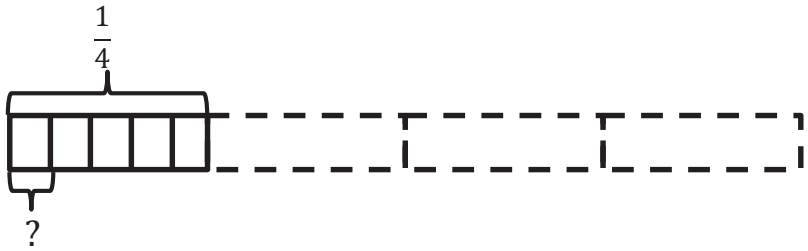
2. Create and solve a story problem about $\frac{1}{4}$ pound of almonds that is modeled by the tape diagram below.



T: Let's now look at Problem 2 on the Problem Set and read it together.

S: (Read aloud.)

T: Look at the tape diagram. What's the whole, or dividend, in this problem?

S: $\frac{1}{4}$ pound of almonds.

T: What else can you tell me about this tape diagram? Turn and share with a partner.

S: The 1 fourth is being cut into 5 parts. → I counted 5 boxes. It means the one-fourth is cut into 5 equal units, and we have to find how much 1 unit is. When you find the value of 1 equal part, that is division. → I see that we could find $\frac{1}{5}$ of $\frac{1}{4}$. That would be $\frac{1}{4} \times \frac{1}{5}$. That's the same as dividing by 5 and finding 1 part.

T: We must find how much of a whole pound of almonds is in each of the units. Say the division expression.

MP.4

S: $\frac{1}{4} \div 5$.

T: I noticed some of you were thinking about multiplication here. What multiplication expression would also give us the part that has the question mark?

S: $\frac{1}{4} \times \frac{1}{5}$.

T: Write the expression on your paper, and then work with a partner to write a division story and solve. (A possible response appears on the student work example of the Problem Set).

T: How can we check our division work?

S: Multiply the answer and the divisor.

T: Check it now.

S: (Write $\frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$.)

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Problem 3 a. $2 \div \frac{1}{3}$ b. $\frac{1}{3} \div 4$ c. $\frac{1}{4} \div 3$ d. $3 \div \frac{1}{5}$

T: (Write the four expressions on the board.) What do all of these expressions have in common?

S: They are division expressions. → They all have unit fractions and whole numbers. → Problems (b) and (c) have dividends that are unit fractions. → Problems (a) and (d) have divisors that are unit fractions.

T: What does each number in the expression represent? Turn and discuss with a partner.

S: The first number is the whole, and the second number is the divisor. → The first number tells how much there is in the beginning. It's the dividend. The second number tells how many in each group or how many equal groups we need to make. → In Problem (a), 2 is the whole, and $\frac{1}{3}$ is the divisor. → In Problems (b) and (c), both expressions have a fraction divided by a whole number.

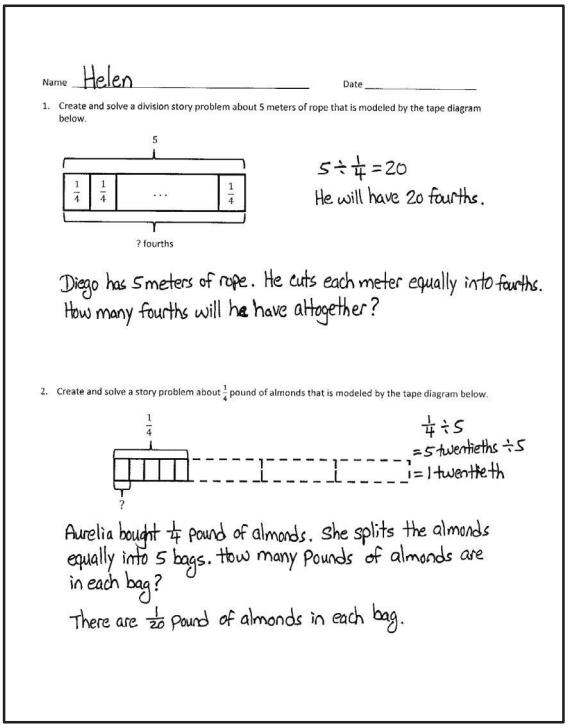
T: Compare these expressions to the word problems we just wrote. Turn and talk.

S: Problems (a) and (d) are like Problem 1, and the other two are like Problem 2. → Problems (a) and (d) have a whole number dividend just like Problem 1. The others have fraction dividends like Problem 2. → Our tape diagram for (a) should look like the one for Problem 1. → The first one is asking how many fractional units in the wholes like Problems (a) and (d). The others are asking what kind of unit you get when you split a fraction into equal parts. → Problems (b) and (c) will look like Problem 2.

T: Work with a partner to draw a tape diagram for each expression. Then, write a story to match your diagram and solve. Be sure to use multiplication to check your work. (Possible responses appear on the student work example of the Problem Set. Be sure to include in the class discussion all the interpretations of division because some students may write stories that take on a multiplication quality.)

Problem Set (10 minutes)

The Problem Set forms the basis for today's lesson. Please see the vignette in the Concept Development for modeling suggestions.



Name Helen Date

1. Create and solve a division story problem about 5 meters of rope that is modeled by the tape diagram below.

$5 \div \frac{1}{4} = 20$
He will have 20 fourths.

Diego has 5 meters of rope. He cuts each meter equally into fourths. How many fourths will he have altogether?

2. Create and solve a story problem about $\frac{1}{4}$ pound of almonds that is modeled by the tape diagram below.

$\frac{1}{4} \div 5$
= 5 twentieths ÷ 5
= 1 twentieth

Aurelia bought $\frac{1}{4}$ pound of almonds. She splits the almonds equally into 5 bags. How many pounds of almonds are in each bag?

There are $\frac{1}{20}$ pound of almonds in each bag.

Student Debrief (10 minutes)

Lesson Objective: Write equations and word problems corresponding to tape and number line diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

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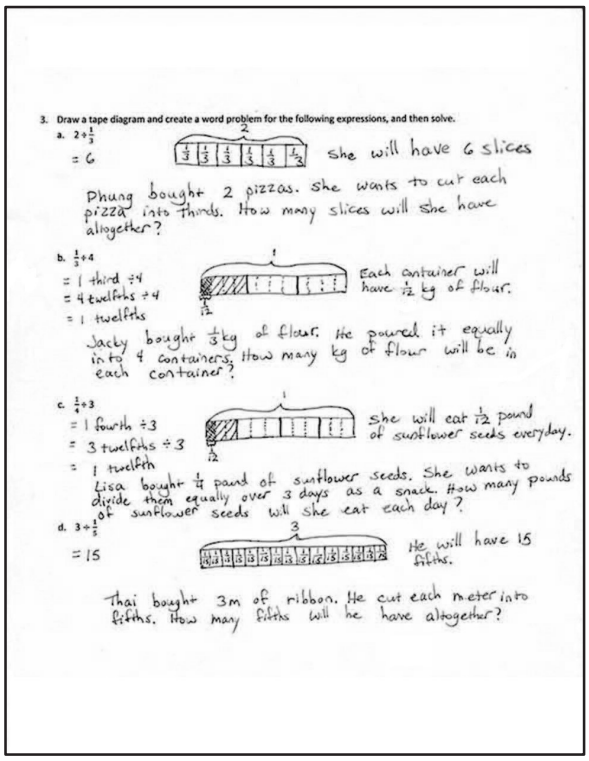
Any combination of the questions below may be used to lead the discussion.

- In Problem 3, what do you notice about (a) and (b), (a) and (d), and (b) and (c)?
- Compare your stories and solutions for Problem 3 with a partner.
- Compare and contrast Problems 1 and 2. What are the similarities or differences of these two problems?
- Share your solutions for Problems 1 and 2 and explain them to a partner.



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

NOTES ON MULTIPLE MEANS OF EXPRESSION AND ACTION:

Comparing and contrasting is often required in English language arts, science, and social studies classes. Use the same graphic organizers that are successfully used in these classes for math class. Although Venn diagrams are often used to help students organize their thinking when comparing and contrasting, this is not the only possible graphic organizer. To add variety, charts listing similarities in a center column and differences in two outer columns can also be used.

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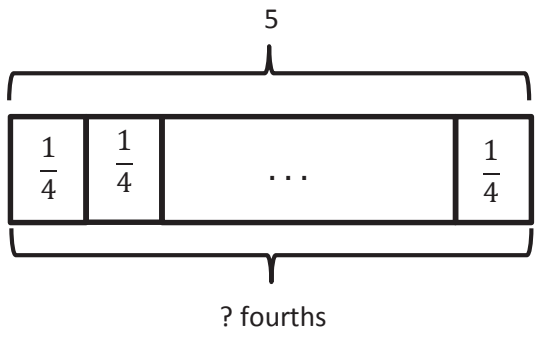
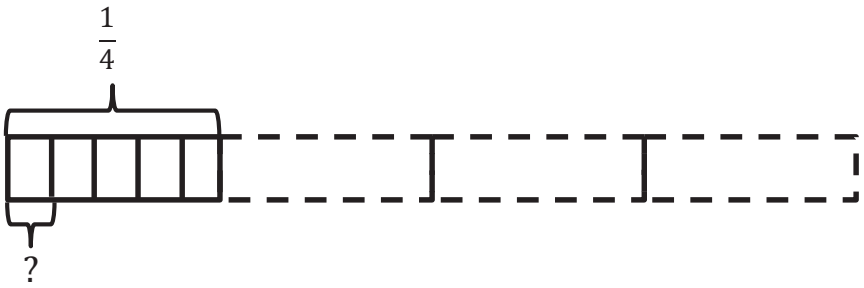
Name _____ Date _____

1. Create and solve a division story problem about 5 meters of rope that is modeled by the tape diagram below.



2. Create and solve a story problem about $\frac{1}{4}$ pound of almonds that is modeled by the tape diagram below.



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3. Draw a tape diagram and create a word problem for the following expressions, and then solve.

   a. $2 \div \frac{1}{3}$

   b. $\frac{1}{3} \div 4$

   c. $\frac{1}{4} \div 3$

   d. $3 \div \frac{1}{5}$

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Name _____ Date _____

Create a word problem for the following expressions, and then solve.

a. $4 \div \frac{1}{2}$

b. $\frac{1}{2} \div 4$

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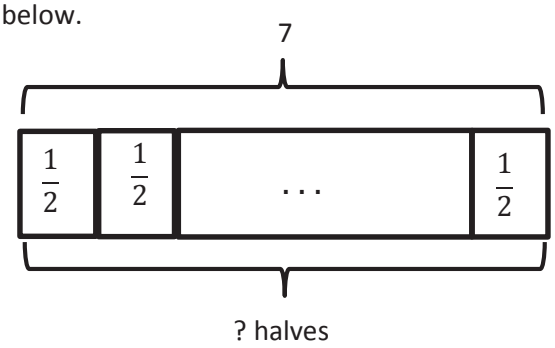
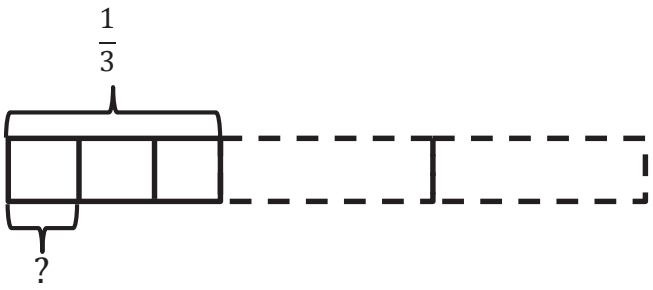
Name _____ Date _____

1. Create and solve a division story problem about 7 feet of rope that is modeled by the tape diagram below.



2. Create and solve a story problem about $\frac{1}{3}$ pound of flour that is modeled by the tape diagram below.



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3. Draw a tape diagram and create a word problem for the following expressions. Then, solve and check.

a. $2 \div \frac{1}{4}$

b. $\frac{1}{4} \div 2$

c. $\frac{1}{3} \div 5$

d. $3 \div \frac{1}{10}$

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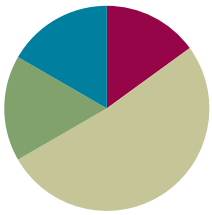
Lesson 29

Objective: Connect division by a unit fraction to division by 1 tenth and 1 hundredth.

Suggested Lesson Structure

Fluency Practice	(9 minutes)
Application Problem	(10 minutes)
Concept Development	(31 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (9 minutes)

- Count by Fractions **5.NF.7** (5 minutes)
- Divide Whole Numbers by Unit Fractions and Fractions by Whole Numbers **5.NF.7** (4 minutes)

Count by Fractions (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: Count by tenths to 20 tenths. When you reach a whole number, say the whole number. (Write as students count.)

S: 1 tenth, 2 tenths, 3 tenths, 4 tenths, 5 tenths, 6 tenths, 7 tenths, 8 tenths, 9 tenths, 1 whole, 11 tenths, 12 tenths, 13 tenths, 14 tenths, 15 tenths, 16 tenths, 17 tenths, 18 tenths, 19 tenths, 2 wholes.

T: How many tenths are in 1 whole?

S: 10 tenths.

T: 2 wholes?

S: 20 tenths.

T: 3 wholes?

S: 30 tenths.

T: 9 wholes.

S: 90 tenths.

T: 10 wholes?

S: 100 tenths.

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T: (Write 10 = 100 tenths. Beneath it, write 20 = ____ tenths.) On your personal white board, fill in the unknown.

S: (Write 20 = 200 tenths.)

Continue the process with 30, 50, 70, and 90.

T: (Write 90 = 900 tenths. Beneath it, write 91 = ____ tenths.) On your personal white board, fill in the unknown.

S: (Write 91 = 910 tenths.)

Continue the process with 92, 82, 42, 47, 64, 64.1, 64.2, and 83.5.

Divide Whole Numbers by Unit Fractions and Fractions by Whole Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 25–27 and prepares students for today's lesson.

T: (Write $2 \div \frac{1}{2} =$ ____.) Say the division sentence with the answer.

S: $2 \div \frac{1}{2} = 4$.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2} =$ ____.) Say the division sentence with the answer.

S: $3 \div \frac{1}{2} = 6$.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $8 \div \frac{1}{2} =$ ____.) On your personal white board, complete the division sentence.

S: (Write $8 \div \frac{1}{2} = 16$.)

Continue the process with $5 \div \frac{1}{4}$, $7 \div \frac{1}{3}$, $1 \div \frac{1}{10}$, $2 \div \frac{1}{10}$, $7 \div \frac{1}{10}$, and $10 \div \frac{1}{10}$.

T: (Write $\frac{1}{2} \div 3 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 3 = \frac{1}{6}$.

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$. Beneath it, write $\frac{1}{3} \div 4 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{3} \div 4 = \frac{1}{12}$.

T: (Write $\frac{1}{3} \div 4 = \frac{1}{12}$. Beneath it, write $\frac{1}{8} \div 5 =$ ____.) On your personal white board, complete the division sentence.

S: (Write $\frac{1}{8} \div 5 = \frac{1}{40}$.)

T: (Write $\frac{1}{4} \div 5 =$ ____.) Say the division sentence with the answer.

S: $\frac{1}{4} \div 5 = \frac{1}{20}$.

Continue the process with $7 \div \frac{1}{4}$, $\frac{1}{4} \div 7$, $5 \div \frac{1}{6}$, $\frac{1}{6} \div 5$, $\frac{1}{8} \div 7$, and $9 \div \frac{1}{8}$.

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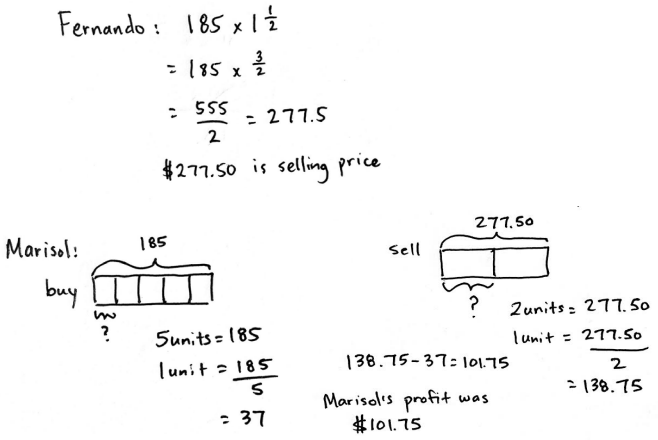
Application Problem (10 minutes)

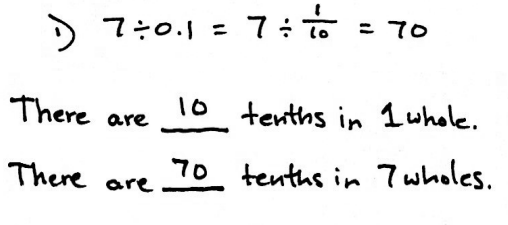
Fernando bought a jacket for \$185 and sold it for $1\frac{1}{2}$ times what he paid. Marisol spent $\frac{1}{5}$ as much as Fernando on the same jacket but sold it for $\frac{1}{2}$ as much as Fernando did. How much money did Marisol make? Explain your thinking using a diagram.

Note: This problem is a multi-step problem requiring a high level of organization. Scaling language and fraction multiplication from Topic G, coupled with fraction of a set and subtraction, warrant the extra time given to today's Application Problem.



Concept Development (31 minutes)

Materials: (S) Personal white board



Problem 1: 7 ÷ 0.1

T: (Post Problem 1 on the board.) Read the division expression using unit form.

S: 7 ones divided by 1 tenth.

T: Rewrite this expression using a fraction.

S: (Write $7 \div \frac{1}{10}$.)

T: (Write $7 \div \frac{1}{10}$.) What question does this division expression ask us?

S: How many tenths are in 7? → 7 is one tenth of what number?

T: (Write *There are ____ tenths in 1 whole*.) Let's start with just 1 whole. How many tenths are in 1 whole?

S: 10 tenths.

MP.2

T: (Write 10 in the blank, and then below it write, *There are _____ tenths in 7 wholes*.) So, if there are 10 tenths in 1 whole, how many are in 7 wholes?

S: 70 tenths.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

The same place value mats that were used in previous modules can be used in this lesson to support students who are struggling. Have students start Problem 1 by drawing or placing 7 disks in the ones column. Follow the same dialogue that is written in the lesson. Have students physically decompose the 7 wholes into 70 tenths, which can then be divided by one-tenth.

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T: (Write 70 in the blank.) Explain how you know. Turn and talk.

S: There are 10 tenths in 1, 20 tenths in 2, and 30 tenths in 3, so there are 70 tenths in 7. → 7 is 7 times greater than 1, and 70 tenths is 7 times more than 10 tenths. → Seven times 10 is 70 so there are 70 tenths in 7.

MP.2

T: Let's think about it another way. 7 is one-tenth of what number? Explain to your partner how you know.

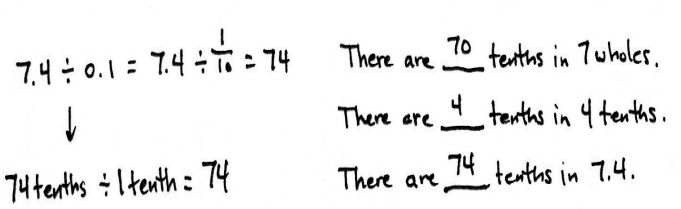
S: It's 70 because I think of a tape diagram with 10 parts, and 1 part is 7. 7 × 10 is 70. → I think of place value. Just move each digit one place to the left. It's ten times as much.

Problem 2: 7.4 ÷ 0.1

T: (Post Problem 2 on the board.) Rewrite this division expression using a fraction for the divisor.

S: (Write $7.4 \div \frac{1}{10}$.)

T: Compare this problem to the one we just solved. What do you notice? Turn and talk.



S: There still are 7 wholes, but now there are also 4 more tenths. → The whole in this problem is just 4 tenths more than in Problem 1. → There are 74 tenths instead of 70 tenths. → We can ask ourselves, 7.4 is 1 tenth of what number?

T: We already know part of this problem. (Write *There are _____ tenths in 7 wholes*.) How many tenths are in 7 wholes?

S: 70.

T: (Write 70 in the blank, and below it, write *There are _____ tenths in 4 tenths*.) How many tenths are in 4 tenths?

S: 4.

T: (Point to 7 ones.) So, if there are 70 tenths in 7 wholes, and (point to 4 tenths) 4 tenths in 4 tenths, how many tenths are in 7 and 4 tenths?

S: 74.

T: Work with your partner to rewrite this expression using only tenths to name the whole and divisor.

S: (Write 74 tenths ÷ 1 tenth.)

T: Look at our new expression. How many tenths are in 74 tenths?

S: 74 tenths.

T: (Write 6 ÷ 0.1.) Read this expression.

S: 6 divided by 1 tenth.

T: How many tenths are in 6? Show me on your personal white board.

S: (Write and show 60 tenths.)

T: 6 is 1 tenth of what number?

S: 60.

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T: (Erase 6 and replace with 6.2.) How many tenths in 6.2?

S: (Write 62 tenths.)

T: 6.2 is 1 tenth of what number?

S: 62.

Continue the process with 9 ÷ 0.1, 9.8 ÷ 0.1, 12 ÷ 0.1, and 12.6 ÷ 0.1.

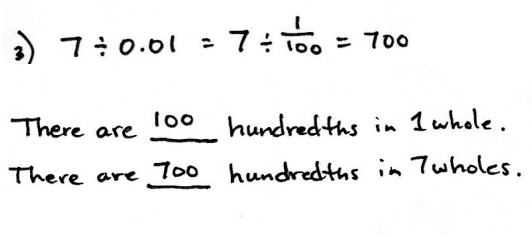
Problem 3: **a. 7 ÷ 0.01** **b. 7.4 ÷ 0.01** **c. 7.49 ÷ 0.01**

T: (Post Problem 3 (a) on the board.) Read this expression.

S: 7 divided by 1 hundredth.

T: Rewrite this division expression using a fraction for the divisor.

S: (Write $7 \div \frac{1}{100}$.)



T: We can think of this as finding how many hundredths are in 7. Will your thinking need to change to solve this? Turn and talk.

S: No, because the question is really the same. How many smaller units are in the whole? → The units we are counting are different, but that doesn't really change how we find the answer.

T: Will our quotient be greater or less than our last problem? Again, talk with your partner.

S: The quotient will be greater because we are counting units that are much smaller, so there'll be more of them in the wholes. → It's the same basic idea, but since our divisor has gotten smaller, the quotient should be larger than before.

T: Before we think about how many hundredths are in 7 wholes, let's find how many hundredths are in 1 whole. (Write on the board: *There are _____ hundredths in 1 whole*.) Fill in the blank.

S: 100.

T: (Write 100 in the blank. Write, *There are _____ hundredths in 7 wholes*.) Knowing this, how many hundredths are in 7 wholes?

S: 700.

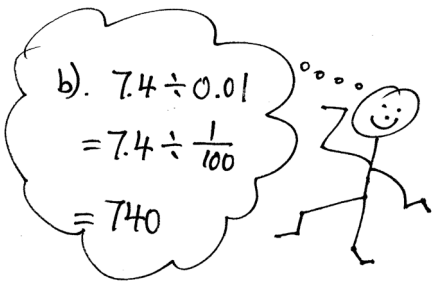
T: (Write 700 in the blank. Then, post Problem 3 (b) on the board.) What is the whole in this division expression?

S: 7 and 4 tenths.

T: How will you solve this problem? Turn and talk.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Generally speaking, it is better to use unit form to read decimal numbers. For example, *seven and four-tenths* is generally preferable to *seven point four*. *Seven point four* is appropriate when teachers or students are trying to express what they need to write. Similarly, it is preferable to read fractions in unit form, too. For example, it's better to say *two-thirds*, rather than *two over three* unless referring to how the fraction is written.



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S: It's only 4 more tenths than the one we just solved. We need to figure out how many hundredths are in 4 tenths.
→ We know there are 700 hundredths in 7 wholes, and this is 4 tenths more than that. There are 10 hundredths in
1 tenth, so there must be 40 hundredths in 4 tenths.

T: How many hundredths are in 7 wholes?

S: 700.

T: How many hundredths in 4 tenths?

S: 40.

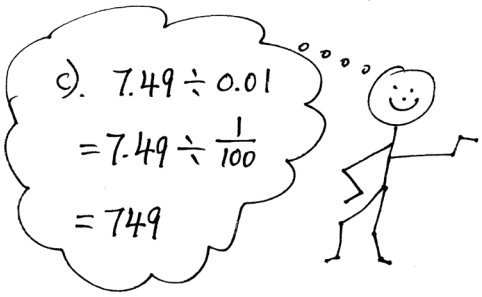
T: How many hundredths in 7.4?

S: 740.

T: Asked another way, if 7.4 is 1 hundredth, what is the whole?

S: 740.

T: (Post Problem 3 (c) on the board.) Work with a partner to solve this problem. Be prepared to explain your thinking.



S: (Work and show 7.49 ÷ 0.01 = 749.)

T: Explain your thinking as you solved.

S: 7.49 is just 9 hundredths more in the dividend than 7.4 ÷ 0.01, so the answer must be 749. → There are 700 hundredths in 7, and 49 hundredths in
49 hundredths. That's 749 hundredths all together.

T: Let's try some more. Think first... how many hundredths are in 6? Show me.

S: (Show 600.)

T: Show me how many hundredths are in 6.2?

S: (Show 620.)

T: 6.02?

S: (Show 602.)

T: 12.6?

S: (Show 1,260.)

T: 12.69?

S: (Show 1,269.)

T: What patterns are you noticing as we find the number of hundredths in each of these quantities?

S: The digits stay the same, but they are in a larger place value in the quotient. → I'm beginning to notice that, when we divide by a hundredth, each digit shifts two places to the left. It's like multiplying by 100.

MP.2

T: That helps us think of our division expression differently. When we divide by a hundredth, we can think, "This number is 1 hundredth of what whole?" or "What number is this 1 hundredth of?"

T: (Write $7 \div \frac{1}{100}$ on the board.) What number is 7 one hundredths of?

S: 700.

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T: Explain to your partner how you know.

MP.2

S: It's like thinking 7 times 100 because 7 is one of a hundred parts. → It's place value again, but this time, the digits shifted two places to the right.

T: You can use that way of thinking about these expressions, too.

Problem Set (10 minutes)

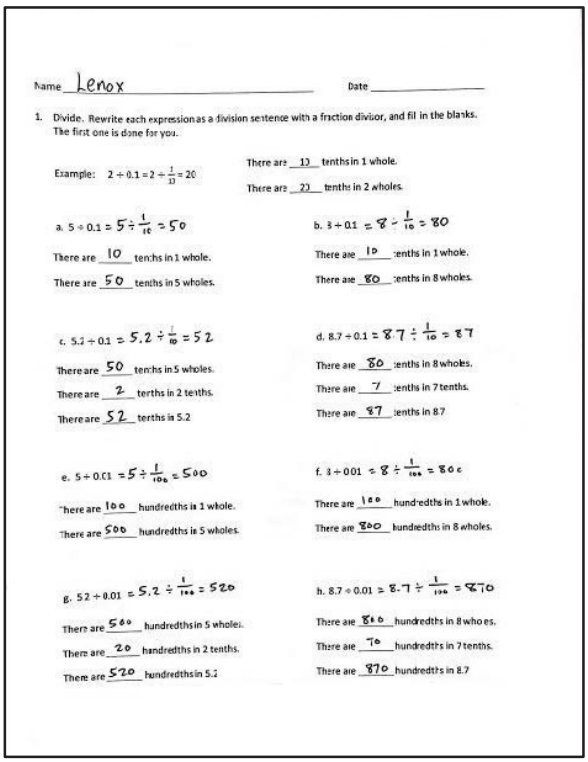
Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Connect division by a unit fraction to division by 1 tenth and 1 hundredth.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.



Name Lenox Date

1. Divide. Rewrite each expression as a division sentence with a fraction divisor, and fil in the blanks. The first one is done for you.

Example: $2 \div 0.1 = 2 \div \frac{1}{10} = 20$

There are 10 tenths in 1 whole.

There are 20 tenths in 2 wholes.

a. $5 \div 0.1 = 5 \div \frac{1}{10} = 50$

There are 10 tenths in 1 whole.

There are 50 tenths in 5 wholes.

b. $8 \div 0.1 = 8 \div \frac{1}{10} = 80$

There are 10 tenths in 1 whole.

There are 80 tenths in 8 wholes.

c. $5.2 \div 0.1 = 5.2 \div \frac{1}{10} = 52$

There are 50 tenths in 5 wholes.

There are 2 tenths in 2 tenths.

There are 52 tenths in 5.2

d. $8.7 \div 0.1 = 8.7 \div \frac{1}{10} = 87$

There are 80 tenths in 8 wholes.

There are 7 tenths in 7 tenths.

There are 87 tenths in 8.7

e. $5 \div 0.01 = 5 \div \frac{1}{100} = 500$

There are 100 hundredths in 1 whole.

There are 500 hundredths in 5 wholes.

f. $8 \div 0.01 = 8 \div \frac{1}{100} = 800$

There are 100 hundredths in 1 whole.

There are 800 hundredths in 8 wholes.

g. $5.2 \div 0.01 = 5.2 \div \frac{1}{100} = 520$

There are 500 hundredths in 5 wholes.

There are 20 hundredths in 2 tenths.

There are 520 hundredths in 5.2

h. $8.7 \div 0.01 = 8.7 \div \frac{1}{100} = 870$

There are 800 hundredths in 8 wholes.

There are 70 hundredths in 7 tenths.

There are 870 hundredths in 8.7

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, did you notice the relationship between (a) and (c), (b) and (d), (e) and (g), (f) and (h)?
- What is the relationship between Problems 2(a) and 2(b)? (The quotient of (b) is triple that of (a).)
- What strategy did you use to solve Problem 3? Share your strategy and explain to a partner.
- How did you answer Problem 4? Share your thinking with a partner.
- Compare your answer for Problem 5 to your partner's.
- Connect the work of Module 1, the movement on the place value chart, to the division work of this lesson. (Back then, the focus was on conversion between units. However, it's important to note that place value work asks the same questions, "How many tenths are in 1 whole?" "How many hundredths are in a tenth?" Further, the partitive division interpretation leads naturally to a discussion of multiplication by powers of 10. That is, if 6 is 1 hundredth, what is the whole? ($6 \times 100 = 600$.) This echoes the work students have done on the place value chart.)

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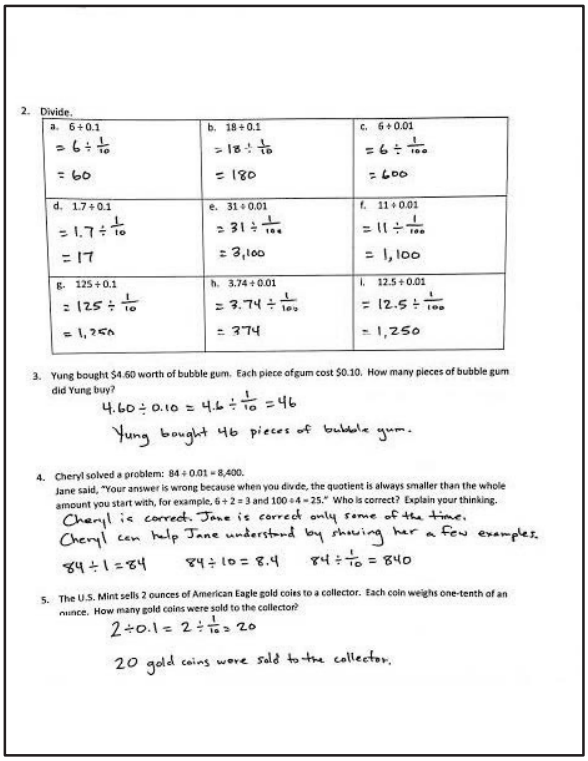
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



2. Divide.

a. $6 \div 0.1$ $= 6 \div \frac{1}{10}$ $= 60$	b. $18 \div 0.1$ $= 18 \div \frac{1}{10}$ $= 180$	c. $6 \div 0.01$ $= 6 \div \frac{1}{100}$ $= 600$
d. $1.7 \div 0.1$ $= 1.7 \div \frac{1}{10}$ $= 17$	e. $31 \div 0.01$ $= 31 \div \frac{1}{100}$ $= 3{,}100$	f. $11 \div 0.01$ $= 11 \div \frac{1}{100}$ $= 1{,}100$
g. $125 \div 0.1$ $= 125 \div \frac{1}{10}$ $= 1{,}250$	h. $3.74 \div 0.01$ $= 3.74 \div \frac{1}{100}$ $= 374$	i. $12.5 \div 0.01$ $= 12.5 \div \frac{1}{100}$ $= 1{,}250$

3. Yung bought $4.60 worth of bubble gum. Each piece ofgum cost $0.10. How many pieces of bubble gum did Yung buy?

$4.60 \div 0.10 = 4.6 \div \frac{1}{10} = 46$

Yung bought 46 pieces of bubble gum.

4. Cheryl solved a problem: $84 \div 0.01 = 8{,}400$.
Jane said, "Your answer is wrong because when you divde, the quotient is always smaller than the whole amount you start with, for example, $6 \div 2 = 3$ and $100 \div 4 = 25$." Who is correct? Explain your thinking.

Cheryl is correct. Jane is correct only some of the time. Cheryl can help Jane understand by showing her a few examples.

$84 \div 1 = 84$ $84 \div 10 = 8.4$ $84 \div \frac{1}{10} = 840$

5. The U.S. Mint sells 2 ounces of American Eagle gold coins to a collector. Each coin weighs one-tenth of an ounce. How many gold coins were sold to the collector?

$2 \div 0.1 = 2 \div \frac{1}{10} = 20$

20 gold coins were sold to the collector.

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Name _____ Date _____

1. Divide. Rewrite each expression as a division sentence with a fraction divisor, and fill in the blanks. The first one is done for you.

Example: $2 \div 0.1 = 2 \div \frac{1}{10} = 20$

There are 10 tenths in 1 whole.

There are 20 tenths in 2 wholes.

a. $5 \div 0.1$

There are _____ tenths in 1 whole.

There are _____ tenths in 5 wholes.

b. $8 \div 0.1$

There are _____ tenths in 1 whole.

There are _____ tenths in 8 wholes.

c. $5.2 \div 0.1$

There are _____ tenths in 5 wholes.

There are _____ tenths in 2 tenths.

There are _____ tenths in 5.2.

d. $8.7 \div 0.1$

There are _____ tenths in 8 wholes.

There are _____ tenths in 7 tenths.

There are _____ tenths in 8.7.

e. $5 \div 0.01$

There are _____ hundredths in 1 whole.

There are _____ hundredths in 5 wholes.

f. $8 \div 0.01$

There are _____ hundredths in 1 whole.

There are _____ hundredths in 8 wholes.

g. $5.2 \div 0.01$

There are _____ hundredths in 5 wholes.

There are _____ hundredths in 2 tenths.

There are _____ hundredths in 5.2.

h. $8.7 \div 0.01$

There are _____ hundredths in 8 wholes.

There are _____ hundredths in 7 tenths.

There are _____ hundredths in 8.7.

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2. Divide.

a. $6 \div 0.1$	b. $18 \div 0.1$	c. $6 \div 0.01$
d. $1.7 \div 0.1$	e. $31 \div 0.01$	f. $11 \div 0.01$
g. $125 \div 0.1$	h. $3.74 \div 0.01$	i. $12.5 \div 0.01$

3. Yung bought $4.60 worth of bubble gum. Each piece of gum cost $0.10. How many pieces of bubble gum did Yung buy?

4. Cheryl solved a problem: $84 \div 0.01 = 8{,}400$.

   Jane said, "Your answer is wrong because when you divide, the quotient is always smaller than the whole amount you start with, for example, $6 \div 2 = 3$ and $100 \div 4 = 25$." Who is correct? Explain your thinking.

5. The U.S. Mint sells 2 ounces of American Eagle gold coins to a collector. Each coin weighs one-tenth of an ounce. How many gold coins were sold to the collector?

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Name _____ Date _____

1. 8.3 is equal to

_____ tenths

_____ hundredths

2. 28 is equal to

_____ hundredths

_____ tenths

3. $15.09 \div 0.01 =$ _____

4. $267.4 \div \frac{1}{10} =$ _____

5. $632.98 \div \frac{1}{100} =$ _____

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Name _____ Date _____

1. Divide. Rewrite each expression as a division sentence with a fraction divisor, and fill in the blanks. The first one is done for you.

Example: $4 \div 0.1 = 4 \div \frac{1}{10} = 40$

There are __10__ tenths in 1 whole.

There are __40__ tenths in 4 wholes.

a. $9 \div 0.1$

There are _____ tenths in 1 whole.

There are _____ tenths in 9 wholes.

b. $6 \div 0.1$

There are _____ tenths in 1 whole.

There are _____ tenths in 6 wholes.

c. $3.6 \div 0.1$

There are _____ tenths in 3 wholes.

There are _____ tenths in 6 tenths.

There are _____ tenths in 3.6.

d. $12.8 \div 0.1$

There are _____ tenths in 12 wholes.

There are _____ tenths in 8 tenths.

There are _____ tenths in 12.8.

e. $3 \div 0.01$

There are _____ hundredths in 1 whole.

There are _____ hundredths in 3 wholes.

f. $7 \div 0.01$

There are _____ hundredths in 1 whole.

There are _____ hundredths in 7 wholes.

g. $4.7 \div 0.01$

There are _____ hundredths in 4 wholes.

There are _____ hundredths in 7 tenths.

There are _____ hundredths in 4.7.

h. $11.3 \div 0.01$

There are _____ hundredths in 11 wholes.

There are _____ hundredths in 3 tenths.

There are _____ hundredths in 11.3.

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2. Divide.

a. $2 \div 0.1$	b. $23 \div 0.1$	c. $5 \div 0.01$
d. $7.2 \div 0.1$	e. $51 \div 0.01$	f. $31 \div 0.1$
g. $231 \div 0.1$	h. $4.37 \div 0.01$	i. $24.5 \div 0.01$

3. Giovanna is charged $0.01 for each text message she sends. Last month, her cell phone bill included a $12.60 charge for text messages. How many text messages did Giovanna send?

4. Geraldine solved a problem: $68.5 \div 0.01 = 6{,}850$.

   Ralph said, "This is wrong because a quotient can't be greater than the whole you start with. For example, $8 \div 2 = 4$ and $250 \div 5 = 50$." Who is correct? Explain your thinking.

5. The price for an ounce of gold on September 23, 2013, was $1,326.40. A group of 10 friends decide to equally share the cost of 1 ounce of gold. How much money will each friend pay?

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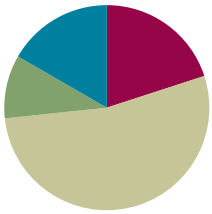
Lesson 30

Objective: Divide decimal dividends by non-unit decimal divisors.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers **5.NBT.7** (9 minutes)
- Divide Decimals **5.NBT.7** (3 minutes)

Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers (9 minutes)

Materials: (S) Divide Whole Numbers by Fractions and Fractions by Whole Numbers Sprint

Note: This fluency activity reviews Lessons 26–28.

Divide Decimals (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 29.

T: (Write 1 ÷ 0.1 = ____.) How many tenths are in 1?
S: 10 tenths.
T: 2?
S: 20 tenths.
T: 3?
S: 30 tenths.
T: 9?
S: 90 tenths.
T: (Write 10 ÷ 0.1 = ____.) On your personal white board, complete the equation, answering how many tenths are in 10.
S: (Write 10 ÷ 0.1 = 100.)

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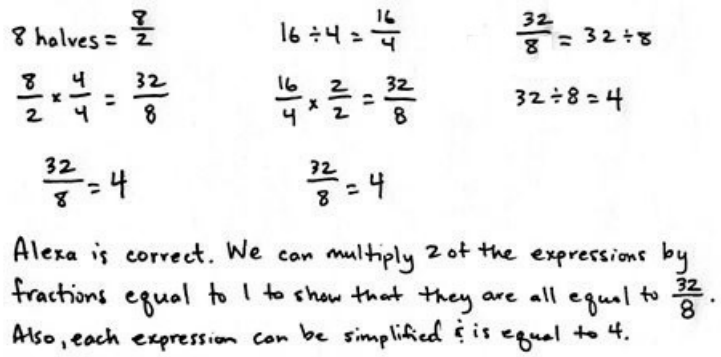
T: (Write 20 ÷ 0.1 = ____.) If there are 100 tenths in 10, how many tenths are in 20?
S: 200 tenths.
T: 30?
S: 300 tenths.
T: 70?
S: 700 tenths.
T: (Write 75 ÷ 0.1 = ____.) On your personal white board, complete the equation.
S: (Write 75 ÷ 0.1 = 750.)
T: (Write 75.3 ÷ 0.1 = ____.) Complete the equation.
S: (Write 75.3 ÷ 0.1 = 753.)

Continue this process with the following possible sequence: 0.63 ÷ 0.1, 6.3 ÷ 0.01, 63 ÷ 0.1, and 630 ÷ 0.01.

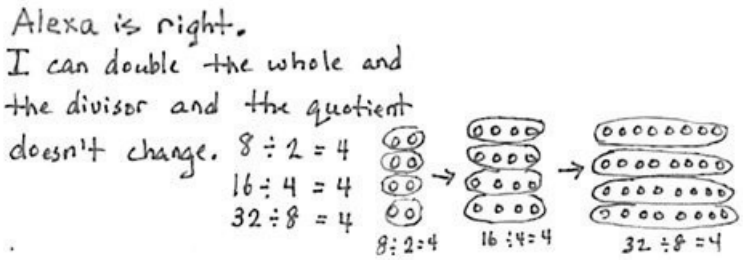
Application Problem (6 minutes)

Alexa claims that 16 ÷ 4, $\frac{32}{8}$, and 8 halves are all equivalent expressions. Is Alexa correct? Explain how you know.

Method 1



Method 2



Note: This Application Problem reminds students that, when you multiply (or divide) both the divisor and the dividend by the same factor, the quotient stays the same or, alternatively, that when multiplying (or dividing), it can be thought of as the fraction having the same value. This concept is critical to the Concept Development in this lesson.

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Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: **a. 2 ÷ 0.1** **b. 2 ÷ 0.2** **c. 2.4 ÷ 0.2** **d. 2.4 ÷ 0.4**

T: (Post Problem 1(a), 2 ÷ 0.1, on the board.) We did this yesterday. How many tenths are in 2?

S: 20.

T: (Write = 20.) Tell a partner how you know.

S: I can count by tenths. 1 tenth, 2 tenths, 3 tenths,… all the way up to 20 tenths, which is 2 wholes. → There are 10 tenths in 1, so there are 20 tenths in 2. → Dividing by 1 tenth is the same as multiplying by 10, and 2 times 10 is 20.

a. $2 \div 0.1 = \frac{2}{0.1}$

$= \frac{2}{0.1} \times \frac{10}{10}$

$= \frac{20}{1}$

$= 20$

T: We also know that any division expression can be rewritten as a fraction. Rewrite this expression as a fraction.

S: (Show $\frac{2}{0.1}$.)

T: That fraction looks different from most we've seen before. What's different about it?

S: The denominator has a decimal point; that's weird.

T: It is different, but it's a perfectly acceptable fraction. We can rename this fraction so that the denominator is a whole number. What have we learned that allows us to rename fractions without changing their value?

S: We can multiply by a fraction equal to 1.

T: What fraction equal to 1 will rename the denominator as a whole number? Turn and talk.

S: Multiplying by $\frac{2}{2}$ is easy, but that would just make the denominator 0.2. That's not a whole number. → I think it is fun to multiply by $\frac{13}{13}$, but then we'll still have 1.3 as the denominator. → I'll multiply by $\frac{10}{10}$. That way, I'll be able to keep the digits the same. → If we just want a whole number, $\frac{20}{20}$ would work. Any fraction with a numerator and denominator that are multiples of 10 would work, really.

T: I overheard a lot of suggestions for ways to rename this denominator as a whole number. I'd like you to try some of your suggestions. Be prepared to share your results about what worked and what didn't. (Allow students time to work and experiment.)

S: (Work and experiment.)

T: Let's share some of the equivalent fractions we've created.

S: (Share while teacher records on the board. Possible examples include $\frac{20}{1}$, $\frac{40}{2}$, $\frac{100}{5}$, and $\frac{200}{10}$.)

T: Show me these fractions written as division sentences with the quotient.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The presence of decimals in the denominators in this lesson may pique the interest of students working above grade level. These students can be encouraged to investigate and operate with complex fractions (fractions whose numerator, denominator, or both contain a fraction).

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S: (Work and show 20 ÷ 1 = 20, 40 ÷ 2 = 20, 100 ÷ 5 = 20, etc.)

T: What do you notice about all of these division sentences?

S: The quotients are all 20.

T: Since all of the quotients are equal to each other, can we say then that these number sentences are equivalent as well? (Write 2 ÷ 0.1 = 20 ÷ 1 = 40 ÷ 2, etc.)

$\frac{2}{0.1} \times \frac{10}{10} = \frac{20}{1}$ $\frac{2}{0.1} \times \frac{50}{50} = \frac{100}{5}$

$\frac{2}{0.1} \times \frac{20}{20} = \frac{40}{2}$ $\frac{2}{0.1} \times \frac{100}{100} = \frac{200}{10}$

S: The answers for all of the expressions are the same, so they are equivalent expressions. → Since they are equal but don't look alike, they remind me of equal fractions.

T: These are all equivalent expressions. When we multiply by a fraction equal to 1, we create equal fractions and an equivalent division expression.

T: (Post Problem 1(b), 2 ÷ 0.2, on the board.) Let's use this thinking as we find the value of this expression. Turn and talk about what you think the quotient will be.

b). $2 \div 0.2 = \frac{2}{0.2}$

$= \frac{2}{0.2} \times \frac{10}{10}$

$= \frac{20}{2}$

$= 10$

S: I can count by 2 tenths. 2 tenths, 4 tenths, 6 tenths,…, 20 tenths. That was 10. The quotient must be 10. → Two is like 2.0 or 20 tenths. 20 tenths divided by 2 tenths is going to be 10. → The divisor in this problem is twice as large as the one we just did, so the quotient will be half as big. Half of 20 is 10.

T: Let's see if our thinking is correct. Rewrite this division expression as a fraction.

S: (Work and show $\frac{2}{0.2}$.)

T: What do you notice about the denominator?

S: It's not a whole number. → It's a decimal.

T: How will you find an equal fraction with a whole number divisor? Share your ideas.

S: We have to multiply it by a fraction equal to 1. → I think multiplying by $\frac{5}{5}$ would work. That will make the divisor exactly 1. → $\frac{10}{10}$ would work again. That would make $\frac{20}{2}$. → This time, any numerator and denominator that is a multiple of 5 would work.

T: I heard the fraction 10 tenths being mentioned during both discussions. What if our divisor were 0.3? If we multiplied by $\frac{10}{10}$, what would the new denominator be?

S: 3.

T: What if the divisor were 0.8?

S: 8.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Place value mats can be used here to support struggling learners. The same concepts that students studied in Module 1 apply here. By writing the divisor and dividend on a place value mat, students can see that 2 ones divided by 2 tenths is equal to 10 since the digit 2 in the ones place is 10 times greater than a 2 in the tenths place.

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T: What about 1.2?

S: 12.

T: What do you notice about the decimal point and digits when we use tenths to rename?

S: The digits stay the same, but they shift to the left. → The digits shift so that the numerator and the denominator are 10 times as much.

T: Multiply the fraction by 10 tenths.

S: (Show $\frac{2}{0.2} \times \frac{10}{10} = \frac{20}{2}$.)

T: What division expression does our renamed fraction represent?

S: 20 divided by 2.

T: What's the quotient?

S: 10.

T: Let's be sure. To check our answer (write $\frac{2}{0.2} = 10$), we multiply the quotient by the…?

S: Divisor.

T: Show me.

S: (Show 10 × 0.2 = 2 or 10 × 2 tenths = 20 tenths.)

T: (Post Problem 1(c), 2.4 ÷ 0.2, on the board.) Share your thoughts about what the quotient might be for this expression.

S: I think it is 12. I counted by 2 tenths again and got 12. → 2.4 is only 4 tenths more than the last problem, and there are two groups of 2 tenths in 4 tenths, so that makes 12 altogether.
→ I'm thinking 24 tenths divided by 2 tenths is going to be 12.
→ I'm starting to think of it like whole number division. It almost looks like 24 divided by 2, which is 12.

c). $2.4 \div 0.2 = \frac{2.4}{0.2}$

$= \frac{2.4}{0.2} \times \frac{10}{10}$

$= \frac{24}{2}$

$= 12$

T: Rewrite this division expression as a fraction.

S: (Write and show $\frac{2.4}{0.2}$.)

T: This time, we have a decimal in both the divisor and whole. Remind me. What will you do to rename the divisor as a whole number?

S: Multiply by $\frac{10}{10}$.

T: What will happen to the numerator when you multiply by $\frac{10}{10}$?

S: It will be renamed as a whole number, too.

T: Show me.

S: (Work and show $\frac{2.4}{0.2} \times \frac{10}{10} = \frac{24}{2}$.)

T: Say the fraction as a division sentence with the quotient.

S: 24 divided by 2 equals 12.

T: Check your work.

S: (Check work.)

d). $2.4 \div 0.4 = \frac{2.4}{0.4}$

$= \frac{2.4}{0.4} \times \frac{10}{10}$

$= \frac{24}{4}$

$= 6$

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T: (Post Problem 1(d), 2.4 ÷ 0.4, on the board.) Work this one independently.

S: (Work and share.)

Problem 2: **a. 1.6 ÷ 0.04** **b. 1.68 ÷ 0.04** **c. 1.68 ÷ 0.12**

T: (Post Problem 2(a), 1.6 ÷ 0.04, on the board.) Rewrite this expression as a fraction.

S: (Write $\frac{1.6}{0.04}$.)

T: How is this expression different from the ones we just evaluated?

S: This one is dividing by hundredths. → Our divisor is 4 hundredths rather than 4 tenths.

T: Our divisor is still not a whole number, and now it's a hundredth. Will multiplying by 10 tenths create a whole number divisor?

S: No, 4 hundredths times 10 is just 4 tenths. That's still not a whole number.

T: Since our divisor is now a hundredth, the most efficient way to rename it as a whole number is to multiply by 100 hundredths. Multiply and show me the equivalent fraction.

S: (Show $\frac{1.6}{0.04} \times \frac{100}{100} = \frac{160}{4}$.)

T: Say the division expression.

S: 160 divided by 4.

T: This expression is equivalent to 1.6 divided by 0.04. What is the quotient?

S: 40.

T: So, 1.6 divided by 0.04 is also equal to what number?

S: 40.

T: Show me the multiplication sentence you can use to check.

S: (Show 40 × 0.04 = 1.6 or 40 × 4 hundredths = 160 hundredths.)

a). $1.6 \div 0.04 = \frac{1.6}{0.04}$

$= \frac{1.6}{0.04} \times \frac{100}{100}$

$= \frac{160}{4}$

$= 40$

b). $1.68 \div 0.04 = \frac{1.68}{0.04}$

$= \frac{1.68}{0.04} \times \frac{100}{100}$

$= \frac{168}{4}$

$= 42$

c). $1.68 \div 0.12 = \frac{1.68}{0.12}$

$= \frac{1.68}{0.12} \times \frac{100}{100}$

$= \frac{168}{12}$

$= 14$

T: (Post Problem 2(b), 1.68 ÷ 0.04, on the board.) Work with your partner to solve and check.

S: (Work.)

T: (Post Problem 2(c), 1.68 ÷ 0.12, on the board.) Work independently to find the quotient. Check your work with a partner after each step.

S: (Work and share.)

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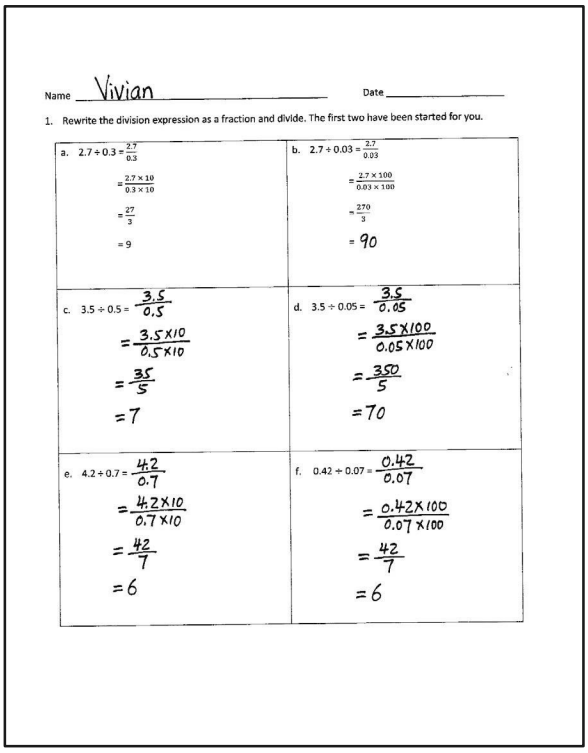
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Vivian Date

1. Rewrite the division expression as a fraction and divide. The first two have been started for you.

a. $2.7 \div 0.3 = \frac{2.7}{0.3}$
$= \frac{2.7 \times 10}{0.3 \times 10}$
$= \frac{27}{3}$
$= 9$

b. $2.7 \div 0.03 = \frac{2.7}{0.03}$
$= \frac{2.7 \times 100}{0.03 \times 100}$
$= \frac{270}{3}$
$= 90$

c. $3.5 \div 0.5 = \frac{3.5}{0.5}$
$= \frac{3.5 \times 10}{0.5 \times 10}$
$= \frac{35}{5}$
$= 7$

d. $3.5 \div 0.05 = \frac{3.5}{0.05}$
$= \frac{3.5 \times 100}{0.05 \times 100}$
$= \frac{350}{5}$
$= 70$

e. $4.2 \div 0.7 = \frac{4.2}{0.7}$
$= \frac{4.2 \times 10}{0.7 \times 10}$
$= \frac{42}{7}$
$= 6$

f. $0.42 \div 0.07 = \frac{0.42}{0.07}$
$= \frac{0.42 \times 100}{0.07 \times 100}$
$= \frac{42}{7}$
$= 6$

Student Debrief (10 minutes)

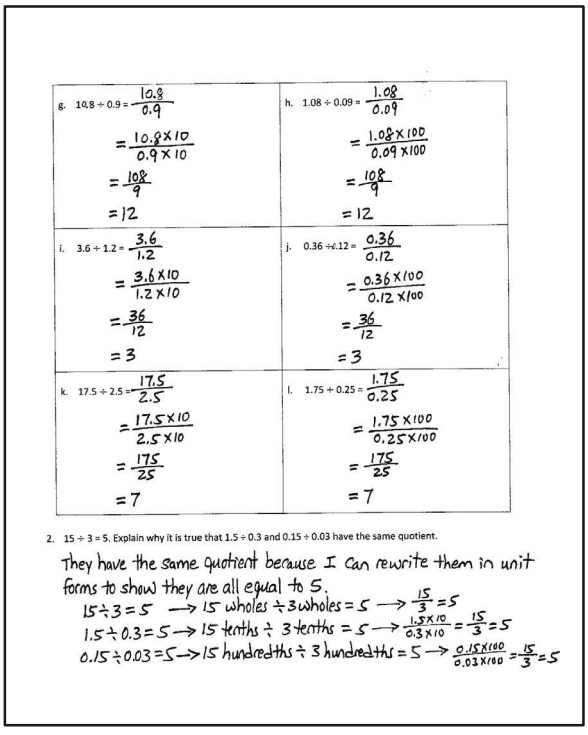
Lesson Objective: Divide decimal dividends by non-unit decimal divisors.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what did you notice about the relationship between (a) and (b), (c) and (d), (e) and (f), (g) and (h), (i) and (j), and (k) and (l)?
- Share your explanation for Problem 2 with a partner.
- In Problem 3, what is the connection between (a) and (b)? How did you solve (b)? Did you solve it mentally or by recalculating everything?
- Share and compare your solution for Problem 4 with a partner.
- How did you solve Problem 5? Did you use drawings to help you solve the problem? Share and compare your strategy with a partner.
- Use the work you completed today to help you find the quotient of 0.08 ÷ 0.4.



g. $10.8 \div 0.9 = \frac{10.8}{0.9}$
$= \frac{10.8 \times 10}{0.9 \times 10}$
$= \frac{108}{9}$
$= 12$

h. $1.08 \div 0.09 = \frac{1.08}{0.09}$
$= \frac{1.08 \times 100}{0.09 \times 100}$
$= \frac{108}{9}$
$= 12$

i. $3.6 \div 1.2 = \frac{3.6}{1.2}$
$= \frac{3.6 \times 10}{1.2 \times 10}$
$= \frac{36}{12}$
$= 3$

j. $0.36 \div 0.12 = \frac{0.36}{0.12}$
$= \frac{0.36 \times 100}{0.12 \times 100}$
$= \frac{36}{12}$
$= 3$

k. $17.5 \div 2.5 = \frac{17.5}{2.5}$
$= \frac{17.5 \times 10}{2.5 \times 10}$
$= \frac{175}{25}$
$= 7$

l. $1.75 \div 0.25 = \frac{1.75}{0.25}$
$= \frac{1.75 \times 100}{0.25 \times 100}$
$= \frac{175}{25}$
$= 7$

2. $15 \div 3 = 5$. Explain why it is true that $1.5 \div 0.3$ and $0.15 \div 0.03$ have the same quotient.

They have the same quotient because I can rewrite them in unit forms to show they are all equal to 5.
$15 \div 3 = 5 \rightarrow$ 15 wholes ÷ 3 wholes = 5 $\rightarrow \frac{15}{3} = 5$
$1.5 \div 0.3 = 5 \rightarrow$ 15 tenths ÷ 3 tenths = 5 $\rightarrow \frac{1.5 \times 10}{0.3 \times 10} = \frac{15}{3} = 5$
$0.15 \div 0.03 = 5 \rightarrow$ 15 hundredths ÷ 3 hundredths = 5 $\rightarrow \frac{0.15 \times 100}{0.03 \times 100} = \frac{15}{3} = 5$

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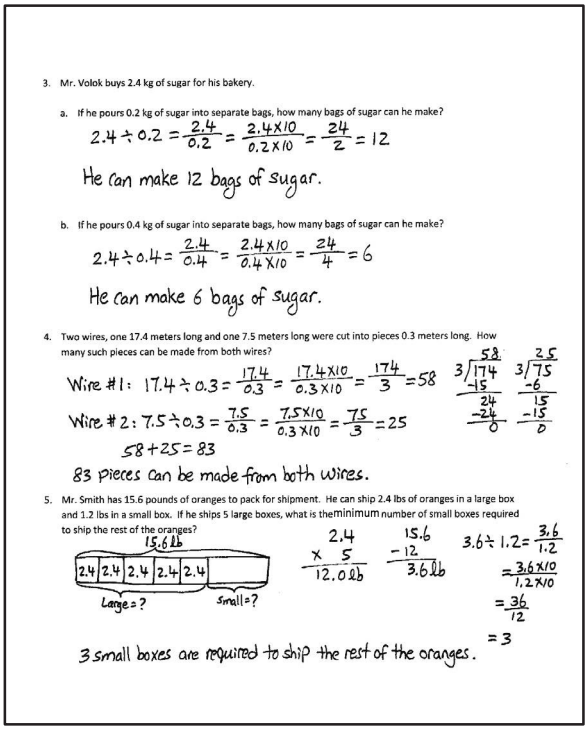
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



3. Mr. Volok buys 2.4 kg of sugar for his bakery.

a. If he pours 0.2 kg of sugar into separate bags, how many bags of sugar can he make?

$2.4 \div 0.2 = \frac{2.4}{0.2} = \frac{2.4 \times 10}{0.2 \times 10} = \frac{24}{2} = 12$

He can make 12 bags of sugar.

b. If he pours 0.4 kg of sugar into separate bags, how many bags of sugar can he make?

$2.4 \div 0.4 = \frac{2.4}{0.4} = \frac{2.4 \times 10}{0.4 \times 10} = \frac{24}{4} = 6$

He can make 6 bags of sugar.

4. Two wires, one 17.4 meters long and one 7.5 meters long were cut into pieces 0.3 meters long. How many such pieces can be made from both wires?

Wire #1: $17.4 \div 0.3 = \frac{17.4}{0.3} = \frac{17.4 \times 10}{0.3 \times 10} = \frac{174}{3} = 58$

Wire #2: $7.5 \div 0.3 = \frac{7.5}{0.3} = \frac{7.5 \times 10}{0.3 \times 10} = \frac{75}{3} = 25$

$58 + 25 = 83$

83 pieces can be made from both wires.

5. Mr. Smith has 15.6 pounds of oranges to pack for shipment. He can ship 2.4 lbs of oranges in a large box and 1.2 lbs in a small box. If he ships 5 large boxes, what is the minimum number of small boxes required to ship the rest of the oranges?

$2.4 \times 5 = 12.0$ lb

$15.6 - 12 = 3.6$ lb

$3.6 \div 1.2 = \frac{3.6}{1.2} = \frac{3.6 \times 10}{1.2 \times 10} = \frac{36}{12} = 3$

3 small boxes are required to ship the rest of the oranges.

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A

Number Correct: _____

Divide Whole Numbers by Fractions and Fractions by Whole Numbers

1.	$\frac{1}{2} \div 2 =$	
2.	$\frac{1}{2} \div 3 =$	
3.	$\frac{1}{2} \div 4 =$	
4.	$\frac{1}{2} \div 7 =$	
5.	$7 \div \frac{1}{2} =$	
6.	$6 \div \frac{1}{2} =$	
7.	$5 \div \frac{1}{2} =$	
8.	$3 \div \frac{1}{2} =$	
9.	$2 \div \frac{1}{5} =$	
10.	$3 \div \frac{1}{5} =$	
11.	$4 \div \frac{1}{5} =$	
12.	$7 \div \frac{1}{5} =$	
13.	$\frac{1}{5} \div 7 =$	
14.	$\frac{1}{3} \div 2 =$	
15.	$2 \div \frac{1}{3} =$	
16.	$\frac{1}{4} \div 2 =$	
17.	$2 \div \frac{1}{4} =$	
18.	$\frac{1}{5} \div 2 =$	
19.	$2 \div \frac{1}{5} =$	
20.	$3 \div \frac{1}{4} =$	
21.	$\frac{1}{4} \div 3 =$	
22.	$\frac{1}{4} \div 4 =$	

23.	$4 \div \frac{1}{4} =$	
24.	$\frac{1}{3} \div 3 =$	
25.	$\frac{2}{3} \div 3 =$	
26.	$\frac{1}{4} \div 2 =$	
27.	$\frac{3}{4} \div 2 =$	
28.	$\frac{1}{5} \div 2 =$	
29.	$\frac{3}{5} \div 2 =$	
30.	$\frac{1}{6} \div 2 =$	
31.	$\frac{5}{6} \div 2 =$	
32.	$\frac{5}{6} \div 3 =$	
33.	$\frac{1}{6} \div 3 =$	
34.	$3 \div \frac{1}{6} =$	
35.	$6 \div \frac{1}{6} =$	
36.	$7 \div \frac{1}{7} =$	
37.	$8 \div \frac{1}{8} =$	
38.	$9 \div \frac{1}{9} =$	
39.	$\frac{1}{8} \div 7 =$	
40.	$9 \div \frac{1}{8} =$	
41.	$\frac{1}{8} \div 7 =$	
42.	$7 \div \frac{1}{6} =$	
43.	$9 \div \frac{1}{7} =$	
44.	$\frac{1}{8} \div 9 =$	

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B

Number Correct: _____

Improvement: _____

Divide Whole Numbers by Fractions and Fractions by Whole Numbers

1.	$\frac{1}{2} \div 2 =$	
2.	$\frac{1}{5} \div 3 =$	
3.	$\frac{1}{5} \div 4 =$	
4.	$\frac{1}{5} \div 7 =$	
5.	$7 \div \frac{1}{5} =$	
6.	$6 \div \frac{1}{5} =$	
7.	$5 \div \frac{1}{5} =$	
8.	$3 \div \frac{1}{5} =$	
9.	$2 \div \frac{1}{2} =$	
10.	$3 \div \frac{1}{2} =$	
11.	$4 \div \frac{1}{2} =$	
12.	$7 \div \frac{1}{2} =$	
13.	$\frac{1}{2} \div 7 =$	
14.	$\frac{1}{4} \div 2 =$	
15.	$2 \div \frac{1}{4} =$	
16.	$\frac{1}{3} \div 2 =$	
17.	$2 \div \frac{1}{3} =$	
18.	$\frac{1}{2} \div 2 =$	
19.	$2 \div \frac{1}{2} =$	
20.	$4 \div \frac{1}{3} =$	
21.	$\frac{1}{3} \div 4 =$	
22.	$\frac{1}{3} \div 3 =$	

23.	$3 \div \frac{1}{3} =$	
24.	$\frac{1}{4} \div 4 =$	
25.	$\frac{3}{4} \div 4 =$	
26.	$\frac{1}{3} \div 3 =$	
27.	$\frac{2}{3} \div 3 =$	
28.	$\frac{1}{6} \div 2 =$	
29.	$\frac{5}{6} \div 2 =$	
30.	$\frac{1}{5} \div 5 =$	
31.	$\frac{3}{5} \div 5 =$	
32.	$\frac{3}{5} \div 4 =$	
33.	$\frac{1}{5} \div 6 =$	
34.	$6 \div \frac{1}{5} =$	
35.	$6 \div \frac{1}{4} =$	
36.	$7 \div \frac{1}{6} =$	
37.	$8 \div \frac{1}{7} =$	
38.	$9 \div \frac{1}{8} =$	
39.	$\frac{1}{8} \div 8 =$	
40.	$9 \div \frac{1}{9} =$	
41.	$\frac{1}{9} \div 8 =$	
42.	$7 \div \frac{1}{7} =$	
43.	$9 \div \frac{1}{6} =$	
44.	$\frac{1}{8} \div 6 =$	

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Name _____ Date _____

1. Rewrite the division expression as a fraction and divide. The first two have been started for you.

a. $2.7 \div 0.3 = \frac{2.7}{0.3}$ $= \frac{2.7 \times 10}{0.3 \times 10}$ $= \frac{27}{3}$ $= 9$	b. $2.7 \div 0.03 = \frac{2.7}{0.03}$ $= \frac{2.7 \times 100}{0.03 \times 100}$ $= \frac{270}{3}$ $=$
c. $3.5 \div 0.5$	d. $3.5 \div 0.05$
e. $4.2 \div 0.7$	f. $0.42 \div 0.07$

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g. $10.8 \div 0.9$	h. $1.08 \div 0.09$
i. $3.6 \div 1.2$	j. $0.36 \div 0.12$
k. $17.5 \div 2.5$	l. $1.75 \div 0.25$

2. $15 \div 3 = 5$. Explain why it is true that $1.5 \div 0.3$ and $0.15 \div 0.03$ have the same quotient.

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3. Mr. Volok buys 2.4 kg of sugar for his bakery.

   a. If he pours 0.2 kg of sugar into separate bags, how many bags of sugar can he make?

   b. If he pours 0.4 kg of sugar into separate bags, how many bags of sugar can he make?

4. Two wires, one 17.4 meters long and one 7.5 meters long, were cut into pieces 0.3 meters long. How many such pieces can be made from both wires?

5. Mr. Smith has 15.6 pounds of oranges to pack for shipment. He can ship 2.4 pounds of oranges in a large box and 1.2 pounds in a small box. If he ships 5 large boxes, what is the minimum number of small boxes required to ship the rest of the oranges?

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Name _____ Date _____

Rewrite the division expression as a fraction and divide.

a. $3.2 \div 0.8$	b. $3.2 \div 0.08$
c. $7.2 \div 0.9$	d. $0.72 \div 0.09$

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Name _____ Date _____

1. Rewrite the division expression as a fraction and divide. The first two have been started for you.

a. $2.4 \div 0.8 = \frac{2.4}{0.8}$ $= \frac{2.4 \times 10}{0.8 \times 10}$ $= \frac{24}{8}$ =	b. $2.4 \div 0.08 = \frac{2.4}{0.08}$ $= \frac{2.4 \times 100}{0.08 \times 100}$ $= \frac{240}{8}$ =
c. $4.8 \div 0.6$	d. $0.48 \div 0.06$
e. $8.4 \div 0.7$	f. $0.84 \div 0.07$

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g. $4.5 \div 1.5$	h. $0.45 \div 0.15$
i. $14.4 \div 1.2$	j. $1.44 \div 0.12$

2. Leann says $18 \div 6 = 3$, so $1.8 \div 0.6 = 0.3$ and $0.18 \div 0.06 = 0.03$. Is Leann correct? Explain how to solve these division problems.

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3. Denise is making bean bags. She has 6.4 pounds of beans.

   a. If she makes each bean bag 0.8 pounds, how many bean bags will she be able to make?

   b. If she decides instead to make mini bean bags that are half as heavy, how many can she make?

4. A restaurant's small salt shakers contain 0.6 ounces of salt. Its large shakers hold twice as much. The shakers are filled from a container that has 18.6 ounces of salt. If 8 large shakers are filled, how many small shakers can be filled with the remaining salt?

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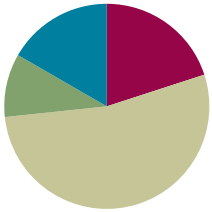
Lesson 31

Objective: Divide decimal dividends by non-unit decimal divisors.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Multiply Decimals by 10 and 100 **5.NBT.1** (4 minutes)
- Divide by 1 Tenth and 1 Hundredth **5.NBT.7** (3 minutes)
- Divide Decimals **5.NBT.7** (5 minutes)

Multiply Decimals by 10 and 100 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 31.

T: (Write 3 × 10 = ____.) Say the multiplication sentence with the answer.
S: 3 × 10 = 30.
T: (Write 3 × 10 = 30. Beneath it, write 20 × 10 = ____.) Say the multiplication sentence with the answer.
S: 20 × 10 = 200.
T: (Write 20 × 10 = 200. Beneath it, write 23 × 10 = ____.) Say the multiplication sentence with the answer.
S: 23 × 10 = 230.
T: (Write 2.3 × 10 = ____. Point to 2.3.) How many tenths is 2 and 3 tenths?
S: 23 tenths.
T: On your personal white boards, write the multiplication sentence with the answer.
S: (Write 2.3 × 10 = 23.)
T: (Write 2.34 × 100 = ____. Point to 2.34.) How many hundredths is 2 and 34 hundredths?
S: 234 hundredths.
T: Write the multiplication sentence with the answer.

3 X 10 = 30
20 X 10 = 200
23 X 10 = 230
2.3 X 10 = 23
2.34 X 100 = 234
23.4 X 10 = 234

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S: (Write 2.34 × 100 = 234.)
T: (Write 23.4 × 10 = ____. Point to 23.4.) How many tenths is 23 and 4 tenths?
S: 234 tenths.
T: Write the multiplication sentence with the answer.
S: (Write 23.4 × 10 = 234.)

Continue with the following possible sequence: 47.3 × 10, 4.73 × 100, 8.2 × 10, 38.2 × 10, and 6.17 × 100.

Divide by 1 Tenth and 1 Hundredth (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 29.

$1 \div 0.1 = 10$
$10 \div 0.1 = 100$
$20 \div 0.1 = 200$
$65 \div 0.1 = 650$
$65.2 \div 0.1 = 652$
$0.08 \div 0.1 = \frac{8}{100} \div \frac{1}{10}$

T: (Write 1 ÷ 0.1 = ____.) How many tenths are in 1?
S: 10 tenths.
T: 2?
S: 20 tenths.
T: 3?
S: 30 tenths.
T: 7?
S: 70 tenths.
T: (Write 10 ÷ 0.1 = ____.) On your personal white board, write the complete number sentence, answering how many tenths are in 10.
S: (Write 10 ÷ 0.1 = 100.)
T: (Write 20 ÷ 0.1 = ____.) If there are 100 tenths in 10, how many tenths are in 20?
S: 200 tenths.
T: 30?
S: 300 tenths.
T: 90?
S: 900 tenths.
T: (Write 65 ÷ 0.1 = ____.) On your personal white board, write the complete number sentence.
S: (Write 65 ÷ 0.1 = 650.)
T: (Write 65.2 ÷ 0.1 = ____.) Write the complete number sentence.
S: (Write 65.2 ÷ 0.1 = 652.)
T: (Write 0.08 ÷ 0.1 = $\frac{\quad}{100} \div \frac{\quad}{10}$.) Complete the division sentence.
S: (Write 0.08 ÷ 0.1 = $\frac{8}{100} \div \frac{1}{10}$.)

Continue with the following possible sequence: 0.36 ÷ 0.1, 3.6 ÷ 0.01, 36 ÷ 0.1, and 360 ÷ 0.01.

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Divide Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 29–30.

$15 \div 5 = \underline{3}$

$1.5 \div 0.5 = \frac{1.5}{0.5}$

$= \frac{1.5 \times 10}{0.5 \times 10}$

$= \frac{15}{5}$

$= 3$

T: (Write 15 ÷ 5 = ____.) Say the division sentence with the answer.

S: 15 ÷ 5 = 3.

T: (Write 15 ÷ 5 = 3. Beneath it, write 1.5 ÷ 0.5 = ____.) Say the division sentence in tenths.

S: 15 tenths ÷ 5 tenths.

T: Write 15 tenths ÷ 5 tenths as a fraction.

S: (Write $\frac{1.5}{0.5}$.)

T: (Beneath 1.5 ÷ 0.5, write $\frac{1.5 \times 10}{0.5 \times 10}$.) On your personal white board, rewrite the fraction using whole numbers.

S: (Write $\frac{1.5 \times 10}{0.5 \times 10}$. Beneath it, write $\frac{15}{5}$.)

T: (Beneath $\frac{1.5 \times 10}{0.5 \times 10}$, write $\frac{15}{5}$. Beneath it, write = ____.) Fill in your answer.

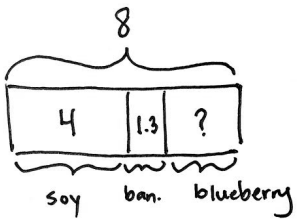
S: (Write = 3.)

Continue with the following possible sequence: 1.5 ÷ 0.05, 0.12 ÷ 0.3, 1.04 ÷ 4, 4.8 ÷ 1.2, and 0.48 ÷ 1.2.

Application Problem (6 minutes)

A café makes ten 8-ounce fruit smoothies. Each smoothie is made with 4 ounces of soy milk and 1.3 ounces of banana flavoring. The rest is blueberry juice. How much of each ingredient will be necessary to make the smoothies?

Note: This two-step problem requires decimal subtraction and multiplication, reviewing concepts from Module 1. Some students will be comfortable performing these calculations mentally, while others may need to sketch a quick visual model. Developing versatility with decimals by reviewing strategies for multiplying decimals serves as a quick warm-up for today's lesson.



$8 - 4 - 1.3 = 2.7$

2.7 oz. blueberry

Soy: $4 \times 10 = 40$

40 oz.

banana: $1.3 \times 10 = 13$

13 oz.

blueberry: $2.7 \times 10 = 27$

27 oz.

The café will need 40 oz. of soy milk, 13 oz. of banana flavoring, & 27 oz. of blueberry juice.

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Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: **a. 34.8 ÷ 0.6** **b. 7.36 ÷ 0.08**

T: (Post Problem 1(a), 34.8 ÷ 0.6, on the board.) Rewrite this division expression as a fraction.

S: (Work and show $\frac{34.8}{0.6}$.)

T: (Write = $\frac{34.8}{0.6}$.) How can we express the divisor as a whole number?

S: Multiply by a fraction equal to 1.

T: Tell a neighbor which fraction equal to 1 you'll use.

$34.8 \div 0.6 = \frac{348}{0.6}$

$= \frac{34.8}{0.6} \times \frac{10}{10}$

$= \frac{348}{6}$ $\approx 360 \div 6 = 60$

$= 58$

$$\begin{array}{r} 58 \\ 6\overline{)348} \\ -30 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}$$

S: I could multiply by 5 fifths, which would make the divisor 3, but I'm not sure I want to multiply 34.8 by 5. That's not as easy. → If we multiply by 10 tenths, that would make both the numerator and the denominator whole numbers. → There are a lot of choices. If I use 10 tenths, the digits will all stay the same—they will just move to a larger place value.

T: As always, we have many fractions equal to 1 that would create a whole number divisor. Which fraction would be most efficient?

S: 10 tenths.

T: (Write × $\frac{10}{10}$.) Multiply, and then show me the equivalent fraction.

S: (Work and show $\frac{348}{6}$.)

T: (Write = $\frac{348}{6}$.) This isn't mental math like the basic facts we saw yesterday, so before we divide, let's estimate to give us an idea of a reasonable quotient. Think of a multiple of 6 that is close to 348 and divide. (Write ≈ _____ ÷ 6.) Turn and share your ideas with a partner.

S: I can round 348 to 360. → I can use mental math to divide 360 by 6 = 60.

T: (Fill in the blank to obtain ≈ 360 ÷ 6 = 60.) Now, use the division algorithm to find the actual quotient.

S: (Work.)

T: What is 34.8 ÷ 0.6? How many 6 tenths are in 34.8?

S: 58.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Some students may require a refresher on the process of long division. This example dialogue might help:

T: Can we divide 3 hundreds by 6, or must we decompose?

S: We need to decompose.

T: Let's work with 34 tens then. What is 34 tens divided by 6?

S: 5 tens.

T: What is 5 tens times 6?

S: 30 tens.

T: How many tens remain?

S: 4 tens.

T: Can we divide 4 tens by 6?

S: Not without decomposing.

T: 4 tens is equal to 40 ones, plus the 8 ones in our whole makes 48 ones. What is 48 ones divided by 6?

S: 8 ones.

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T: Is our quotient reasonable?

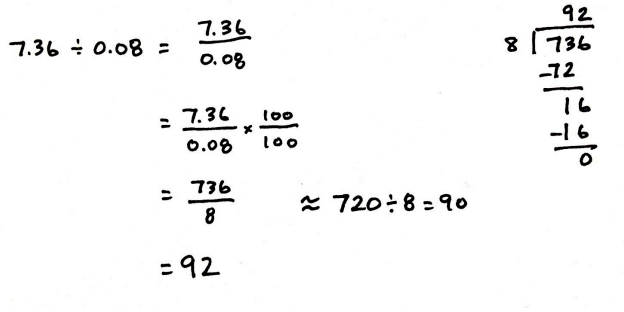
S: Yes, our estimate was 60.

T: (Post Problem 1(b), 7.36 ÷ 0.08, on the board.) Work with a partner to find the quotient. Remember to rename your fraction so that the denominator is a whole number.

S: (Work and share.)

T: What is 7.36 ÷ 0.08? How many 8 hundredths are in 7.36?

S: 92.



T: Is the quotient reasonable considering your estimate?

S: Yes, our estimate was 100. → We got an estimate of 90, so 92 is reasonable.

Problem 2: **a. 21.56 ÷ 0.98** **b. 45.5 ÷ 0.7** **c. 4.55 ÷ 0.7**

T: (Post Problem 2(a) on the board.) Rewrite this division expression as a fraction.

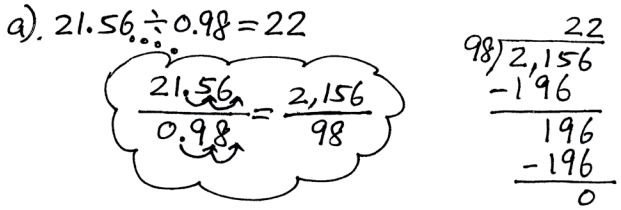
S: (Work and show $\frac{21.56}{0.98}$.)

T: We know that, before we divide, we'll want to rename the divisor as a whole number. Remind me how we'll do that.

S: Multiply the fraction by $\frac{100}{100}$.

T: Then, what would the fraction be after multiplying?

S: $\frac{2,156}{98}$.



T: In this case, both the divisor and whole become 100 times greater. When we write the number that is 100 times as much, we must shift the digits two places to the …?

S: Left.

T: Rather than writing the multiplication sentence to show this, I'm going to record that thinking using arrows. (Draw a thought bubble around the fraction and use arrows to show the change in value of the divisor and whole.)

MP.7 T: Is this fraction equivalent to the one we started with? Turn and talk.

S: It looks a little different, but it shows the fraction we got when we multiplied by 100 hundredths. It's equal. → Both the divisor and whole were multiplied by the same amount, so the two fractions are still equal.

T: Because it is an equal fraction, the division will give us the same quotient as dividing 21.56 by 0.98. Estimate 98.

S: 100.

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T: (Write ≈ _____ ÷ 100.) Now, estimate the whole, 2,156, as a number that we can easily divide by 100. Turn and talk.

S: 100 times 22 is 2,200. → 2,156 is between 21 hundreds and 22 hundreds. It's closer to 22 hundreds. I'll round to 2,200.

T: Record your estimated quotient, and then work with a partner to divide.

S: (Work and share.)

T: Say the quotient.

S: 22.

T: Is that reasonable?

S: Yes.

T: (Post Problem 2(b), 45.5 ÷ 0.7, on the board.) Rewrite this expression as a fraction and show a thought bubble as you rename the divisor as a whole number.

S: (Work and show $\frac{455}{7}$.)

T: Work independently to estimate, and then find the quotient. Check your work with a neighbor as you go.

S: (Work and share.)

T: (Check student work and discuss reasonableness of the quotient. Post Problem 2(c), 4.55 ÷ 0.7, on the board.) Use a thought bubble to show this expression as a fraction with a whole number divisor.

S: (Work and show $\frac{45.5}{7}$.)

T: How is this problem similar to and different from the previous one? Turn and talk.

S: The digits are all the same, but the whole is smaller this time. → The whole still has a decimal point in it. → The whole is 1 tenth the size of the previous whole.

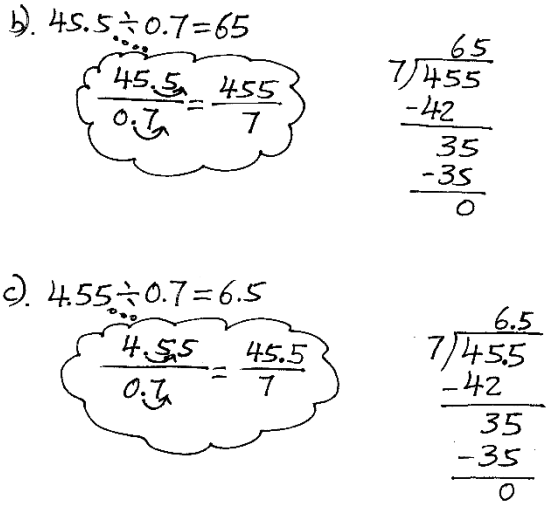
T: We still have a divisor of 7, but this time, our whole is 45 and 5 tenths. Is the whole more than or less than it was in the previous problem?

S: Less than.

T: So, will the quotient be more than 65 or less than 65? Turn and talk.

S: Our whole is smaller, so we can make fewer groups of 7 from it. The quotient will be less than 65. → The whole is 1 tenth as large, so the quotient will be, too.

T: Divide.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Unit form is a powerful means of representing these dividends so that students can more easily see the multiples of the rounded divisor. Expressing 2,156 as 21 hundreds + 56 ones may allow students to estimate more accurately.

Similarly, students should be using easily identifiable multiples to find an estimated quotient. Remind students about the relationship between multiplication and division so they can think of the following division sentences as multiplication equations:

- 2,200 ÷ 100 = ___
  Think: 100 × ___ = 2,200
- 490 ÷ 7 = ___
  Think: 7 × ___ = 490

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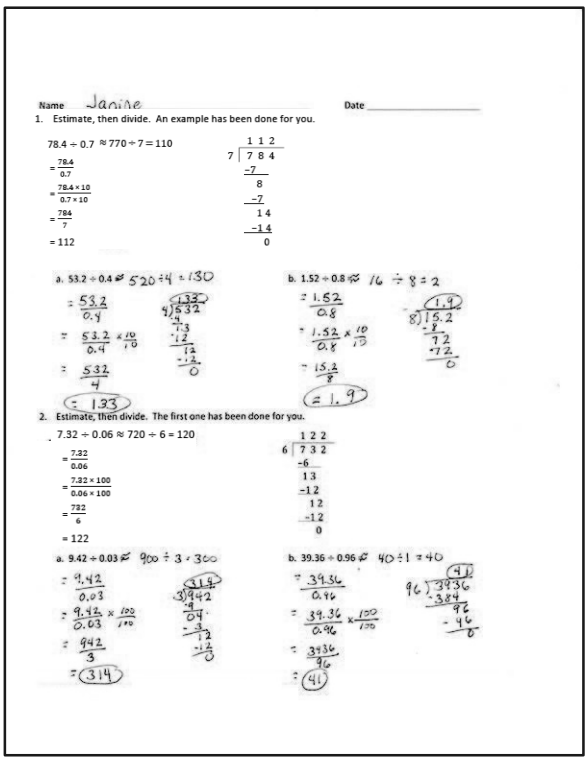
S: (Work.)
T: What is the quotient?
S: 6 and 5 tenths.
T: Does that make sense?
S: Yes.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Janine Date
1. Estimate, then divide. An example has been done for you.
78.4 ÷ 0.7 ≈ 770 ÷ 7 = 110
= 78.4/0.7
= (78.4 × 10)/(0.7 × 10)
= 784/7
= 112
112 / 7|784, -7, 8, -7, 14, -14, 0
a. 53.2 ÷ 0.4 ≈ 520 ÷ 4 = 130
= 53.2/0.4
= (53.2 × 10)/(0.4 × 10)
= 532/4
= 133
133 / 4|532, -4, 13, -12, 12, -12, 0
b. 1.52 ÷ 0.8 ≈ 16 ÷ 8 = 2
= 1.52/0.8
= (1.52 × 10)/(0.8 × 10)
= 15.2/8
= 1.9
1.9 / 8|15.2, -8, 72, -72, 0
2. Estimate, then divide. The first one has been done for you.
7.32 ÷ 0.06 ≈ 720 ÷ 6 = 120
= 7.32/0.06
= (7.32 × 100)/(0.06 × 100)
= 732/6
= 122
122 / 6|732, -6, 13, -12, 12, -12, 0
a. 9.42 ÷ 0.03 ≈ 900 ÷ 3 = 300
= 9.42/0.03
= (9.42 × 100)/(0.03 × 100)
= 942/3
= 314
314 / 3|942, -9, 04, -3, 12, -12, 0
b. 39.36 ÷ 0.96 ≈ 40 ÷ 1 = 40
= 39.36/0.96
= (39.36 × 100)/(0.96 × 100)
= 3936/96
= 41
41 / 96|3936, -384, 96, -96, 0

Student Debrief (10 minutes)

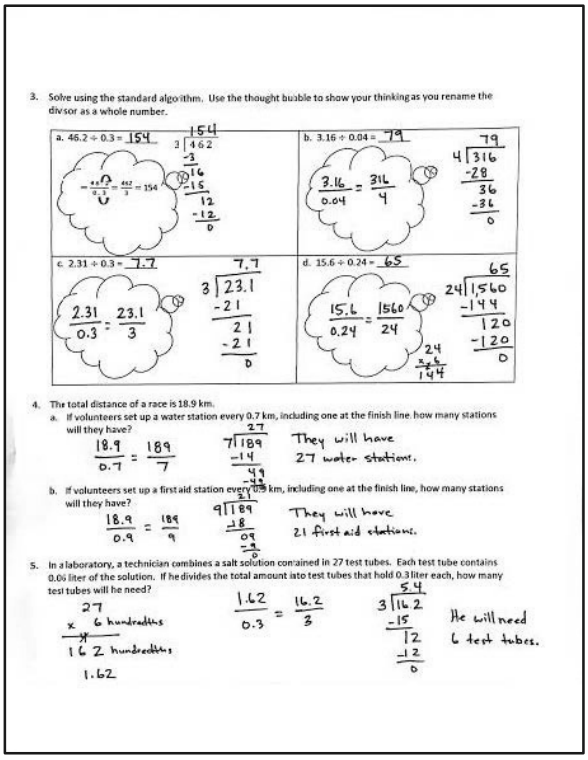
Lesson Objective: Divide decimal dividends by non-unit decimal divisors.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Look at the example in Problem 1. What is another way to estimate the quotient? (Students could say 78 divided by 1 is equal to 78.) Compare the two estimated sentences, 770 ÷ 7 = 110 and 78 ÷ 1 = 78. Why is the actual quotient equal to 112? Does it make sense?
- In Problems 1(a) and 1(b), is your actual quotient close to your estimated quotients?
- In Problems 2(a) and 2(b), is your actual quotient close to your estimated quotients?


3. Solve using the standard algorithm. Use the thought bubble to show your thinking as you rename the divisor as a whole number.
a. 46.2 ÷ 0.3 = 154
462 ÷ 3 = 154
154 / 3|462, -3, 16, -15, 12, -12, 0
b. 3.16 ÷ 0.04 = 79
3.16/0.04 = 316/4
79 / 4|316, -28, 36, -36, 0
c. 2.31 ÷ 0.3 = 7.7
2.31/0.3 = 23.1/3
7.7 / 3|23.1, -21, 21, -21, 0
d. 15.6 ÷ 0.24 = 65
15.6/0.24 = 1560/24
65 / 24|1,560, -144, 120, -120, 0
24 × 6 = 144
4. The total distance of a race is 18.9 km.
a. If volunteers set up a water station every 0.7 km, including one at the finish line, how many stations will they have?
18.9/0.7 = 189/7
27 / 7|189, -14, 49, -49, 0
They will have 27 water stations.
b. If volunteers set up a first aid station every 0.9 km, including one at the finish line, how many stations will they have?
18.9/0.9 = 189/9
21 / 9|189, -18, 09, -9, 0
They will have 21 first aid stations.
5. In a laboratory, a technician combines a salt solution contained in 27 test tubes. Each test tube contains 0.06 liter of the solution. If he divides the total amount into test tubes that hold 0.3 liter each, how many test tubes will he need?
27 × 6 hundredths = 162 hundredths
162 hundredths = 1.62
1.62/0.3 = 16.2/3
5.4 / 3|16.2, -15, 12, -12, 0
He will need 6 test tubes.

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- How did you solve Problem 4? Share and explain your strategy to a partner.
- How did you solve Problem 5? Did you draw a tape diagram to help you solve? Share and compare your strategy with a partner.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Name _____ Date _____

1. Estimate and then divide. An example has been done for you.

$78.4 \div 0.7 \approx 770 \div 7 = 110$

$= \frac{78.4}{0.7}$

$= \frac{78.4 \times 10}{0.7 \times 10}$

$= \frac{784}{7}$

$= 112$

$$\begin{array}{r} 1\ 1\ 2 \\ 7 \overline{)\ 7\ 8\ 4} \\ -7 \\ \hline 8 \\ -7 \\ \hline 1\ 4 \\ -1\ 4 \\ \hline 0 \end{array}$$

a. $53.2 \div 0.4 \approx$

b. $1.52 \div 0.8 \approx$

2. Estimate and then divide. The first one has been done for you.

$7.32 \div 0.06 \approx 720 \div 6 = 120$

$= \frac{7.32}{0.06}$

$= \frac{7.32 \times 100}{0.06 \times 100}$

$= \frac{732}{6}$

$= 122$

$$\begin{array}{r} 1\ 2\ 2 \\ 6 \overline{)\ 7\ 3\ 2} \\ -6 \\ \hline 1\ 3 \\ -1\ 2 \\ \hline 1\ 2 \\ -1\ 2 \\ \hline 0 \end{array}$$

a. $9.42 \div 0.03 \approx$

b. $39.36 \div 0.96 \approx$

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3. Solve using the standard algorithm. Use the thought bubble to show your thinking as you rename the divisor as a whole number.

a. 46.2 ÷ 0.3 = _____

$3 \overline{)\,462}$

$= \frac{46.2}{0.3} = \frac{462}{3} = 154$

b. 3.16 ÷ 0.04 = _____

c. 2.31 ÷ 0.3 = _____

d. 15.6 ÷ 0.24 = _____

4. The total distance of a race is 18.9 km.

   a. If volunteers set up a water station every 0.7 km, including one at the finish line, how many stations will they have?

   b. If volunteers set up a first aid station every 0.9 km, including one at the finish line, how many stations will they have?

5. In a laboratory, a technician combines a salt solution contained in 27 test tubes. Each test tube contains 0.06 liter of the solution. If he divides the total amount into test tubes that hold 0.3 liter each, how many test tubes will he need?

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Name _____ Date _____

Estimate first, and then solve using the standard algorithm. Show how you rename the divisor as a whole number.

1. $6.39 \div 0.09$

2. $82.14 \div 0.6$

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Name _____ Date _____

1. Estimate and then divide. An example has been done for you.

$78.4 \div 0.7 \approx 770 \div 7 = 110$

$= \frac{78.4}{0.7}$

$= \frac{78.4 \times 10}{0.7 \times 10}$

$= \frac{784}{7}$

$= 112$

$$\begin{array}{r} 1\ 1\ 2 \\ 7 \overline{)\ 7\ 8\ 4} \\ -7 \\ \hline 8 \\ -7 \\ \hline 1\ 4 \\ -1\ 4 \\ \hline 0 \end{array}$$

a. $61.6 \div 0.8 \approx$

b. $5.74 \div 0.7 \approx$

2. Estimate and then divide. An example has been done for you.

$7.32 \div 0.06 \approx 720 \div 6 = 120$

$= \frac{7.32}{0.06}$

$= \frac{7.32 \times 100}{0.06 \times 100}$

$= \frac{732}{6}$

$= 122$

$$\begin{array}{r} 1\ 2\ 2 \\ 6 \overline{)\ 7\ 3\ 2} \\ -6 \\ \hline 1\ 3 \\ -1\ 2 \\ \hline 1\ 2 \\ -1\ 2 \\ \hline 0 \end{array}$$

a. $4.74 \div 0.06 \approx$

b. $19.44 \div 0.54 \approx$

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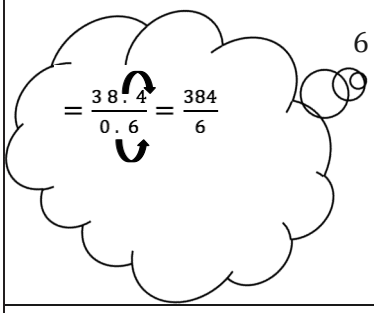
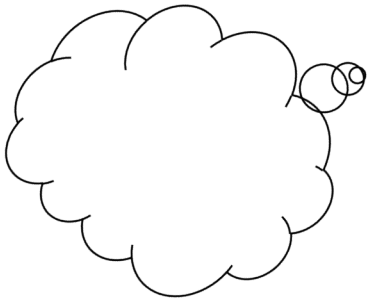
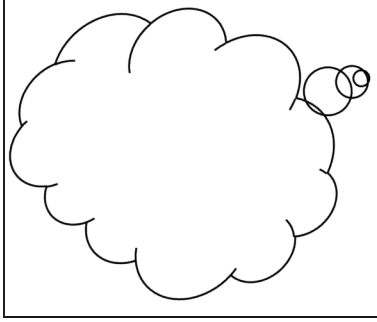
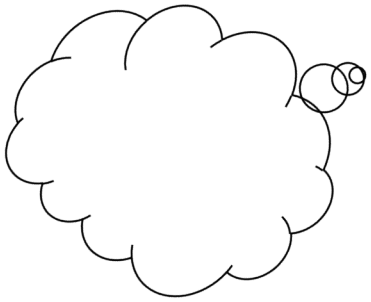
3. Solve using the standard algorithm. Use the thought bubble to show your thinking as you rename the divisor as a whole number.

a. 38.4 ÷ 0.6 = _____ 	b. 7.52 ÷ 0.08 = _____ 
c. 12.45 ÷ 0.5 = _____ 	d. 5.6 ÷ 0.16 = _____ 

4. Lucia is making a 21.6 centimeter beaded string to hang in the window. She decides to put a green bead every 0.4 centimeters and a purple bead every 0.6 centimeters. How many green beads and how many purple beads will she need?

5. A group of 14 friends collects 0.7 pound of blueberries and decides to make blueberry muffins. They put 0.05 pound of berries in each muffin. How many muffins can they make if they use all the blueberries they collected?

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Topic H

Interpretation of Numerical Expressions

5.OA.1, 5.OA.2

Focus Standards:	5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
	5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 +7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Instructional Days:	2	
Coherence -Links from:	G4–M5	Fraction Equivalence, Ordering, and Operations
	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions
	G6–M4	Expressions and Equations

The module concludes with Topic H, in which numerical expressions involving fraction-by-fraction multiplication are interpreted and evaluated (**5.OA.1**, **5.OA.2**). Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

A Teaching Sequence Toward Mastery of Interpretation of Numerical Expressions

Objective 1: **Interpret and evaluate numerical expressions including the language of scaling and fraction division.**
(Lesson 32)

Objective 2: **Create story contexts for numerical expressions and tape diagrams, and solve word problems.**
(Lesson 33)

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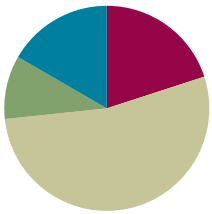
Lesson 32

Objective: Interpret and evaluate numerical expressions including the language of scaling and fraction division.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Order of Operations **5.OA.1** (3 minutes)
- Divide by 1 Tenth and 1 Hundredth **5.NBT.7** (3 minutes)
- Divide Decimals **5.NBT.7** (6 minutes)

Order of Operations (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: (Write 12 ÷ 3 + 1 = ____.) On your personal white board, write the complete number sentence.

S: (Write 12 ÷ 3 + 1 = 5.)

T: (Write 12 ÷ (3 + 1) = ____.) Copy the expression on your personal white board.

S: (Write 12 ÷ (3 + 1) = ____.)

T: Write the complete number sentence, performing the operation inside the parentheses.

S: (Beneath 12 ÷ (3 + 1) = ____, write 12 ÷ 4 = 3 = ____.)

$12 \div 3 + 1 = 5$

$12 \div (3+1) = 3$

Continue with the following possible sequence: 20 – 4 ÷ 2, (20 – 4) ÷ 2, 24 ÷ 6 – 2, and 24 ÷ (6 – 2).

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Divide by 1 Tenth and 1 Hundredth (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 29.

T: (Write 1 ÷ 0.1 = ____.) How many tenths are in 1?
S: 10 tenths.
T: 2?
S: 20 tenths.
T: 3?
S: 30 tenths.
T: 6?
S: 60 tenths.
T: (Write 10 ÷ 0.1 = ____.) On your board, write the complete number sentence answering how many tenths are in 10.
S: (Write 10 ÷ 0.1 = 100.)
T: (Write 20 ÷ 0.1 = ____.) If there are 100 tenths in 10, how many tenths are in 20?
S: 200 tenths.
T: 30?
S: 300 tenths.
T: 80?
S: 800 tenths.
T: (Write 43 ÷ 0.1 = ____.) On your board, write the complete number sentence.
S: (Write 43 ÷ 0.1 = 430.)
T: (Write 43.5 ÷ 0.1 = ____.) Write the complete number sentence.
S: (Write 43.5 ÷ 0.1 = 435.)
T: (Write 0.04 ÷ 0.1 = $\frac{\quad}{100} \div \frac{\quad}{10}$.) Complete the division sentence.
S: (Write 0.04 ÷ 0.1 = $\frac{4}{100} \div \frac{1}{10}$.)

$1 \div 0.1 = 10$
$10 \div 0.1 = 100$
$20 \div 0.1 = 200$
$43 \div 0.1 = 430$
$43.5 \div 0.1 = 435$
$0.04 \div 0.1 = \frac{4}{100} \div \frac{1}{10}$

Continue with the following possible sequence: 0.97 ÷ 0.1, 9.7 ÷ 0.01, 97 ÷ 0.1, and 970 ÷ 0.01.

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Divide Decimals (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 29–30.

T: (Write $\frac{8}{2}$ = ____.) Say the division sentence with the answer.

S: 8 ÷ 2 = 4.

T: (Write $\frac{8}{2}$ = 4. Beneath it, write $\frac{0.8}{0.2}$ = ____.) What is 8 tenths ÷ 2 tenths?

S: 4.

T: (Write $\frac{0.8}{0.02}$ = ____.) On your board, complete the division sentence.

S: (Write 0.8 ÷ 0.02 = 40.)

$\frac{8}{2} = 4$

$\frac{0.8}{0.2} = 4$

$\frac{0.8}{0.02} = 40$

Continue this process with the following possible sequence: 12 ÷ 3, 1.2 ÷ 0.3, 1.2 ÷ 0.03, and 12 ÷ 0.3.

T: (Write 23.84 ÷ 0.2 = ____.) On your board, write the division sentence as a fraction.

S: (Write $\frac{23.84}{0.2}$.)

T: (Write = $\frac{23.84}{0.2}$.) What do we need to multiply the divisor by to make it a whole number?

S: 10.

T: Multiply your numerator and denominator by 10.

S: (Write $\frac{23.84 \times 10}{0.2 \times 10}$.)

T: (Beneath 23.84 ÷ 0.2 = ____, write $\frac{23.84 \times 10}{0.2 \times 10}$.) Use the standard algorithm to solve, and then write your answer.

S: (Write 23.84 ÷ 0.2 = 119.2 after solving.)

$23.84 \div 0.2 = \frac{23.84}{0.2}$

$= \frac{23.84 \times 10}{0.2 \times 10}$

$= \frac{238.4}{2}$

$= 119.2$

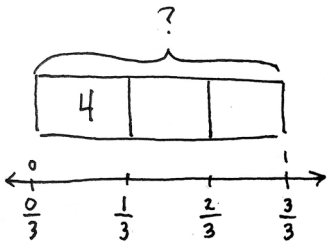
$2\overline{)238.4}$ → 119.2; −2; 03; −2; 18; −18; 04; −4; 0

Continue this process with these possible problems: 5.76 ÷ 0.4, 9.54 ÷ 0.03, and 98.4 ÷ 0.12.

Application Problem (6 minutes)

Four baby socks can be made from $\frac{1}{3}$ skein of yarn. How many baby socks can be made from a whole skein? Draw a model to show your thinking.

Note: This Application Problem is a partitive fraction division problem intended to give students more experience with this interpretation of division (See Module 4 Lesson 25 Problem 2 and accompanying UDL box for further clarification.)



$4 \div \frac{1}{3} = 4 \times 3 = 12$

12 socks can be made from 1 whole skein.

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Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: Write word form expressions numerically.

a. Twice the sum of $\frac{3}{5}$ and $1\frac{1}{2}$

b. Half the sum of $\frac{3}{5}$ and $1\frac{1}{2}$

c. The difference between $\frac{3}{4}$ and $\frac{1}{2}$ divided by 3

a). $2 \times \left(\frac{3}{5} + 1\frac{1}{2}\right)$

$\left(\frac{3}{5} + 1\frac{1}{2}\right) \times 2$

T: (Post Problem 1(a) on the board.) Read the expression aloud with me.

S: (Read.)

T: What do you notice about this expression that would help us translate these words into symbols? Turn and talk.

S: It says *twice*; that means we have to make two copies of something or multiply by 2. → It says *the sum of,* so something is being added. → The part that says *sum of* needs to be added before multiplying, so we'll need parentheses.

T: Let's take a look at the first part of the expression. On your personal white board, show me how can we write *twice* numerically.

S: (Write 2 ×.)

T: (Write 2 × _____ on the board.) Great, what is being multiplied by 2?

S: The sum of 3 fifths and 1 and one-half.

T: Show me the sum of 3 fifths and 1 and one-half numerically.

S: (Write $\frac{3}{5} + 1\frac{1}{2}$.)

T: (Write $\frac{3}{5} + 1\frac{1}{2}$ in the blank.) Are we finished? Turn and talk.

S: (Share.)

T: What else is needed?

S: Parentheses.

T: Why? Turn and talk.

S: You must have parentheses around the addition expression so that the addition is done before the multiplication. If you don't, the answer will be $\frac{6}{5} + 1\frac{1}{2}$. → Because otherwise, you just go left to right and multiply first.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The complexity of language in the word form expressions may pose challenges for English language learners. Consider drawing tape diagrams for each expression. Then, have students match each expression to its tape diagram before writing the numerical expression.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Because of the commutativity of addition and multiplication, there are multiple ways of writing expressions involving these operations. Have students be creative and experiment with their expressions. Be cautious, though, that students do not overgeneralize this property and try to apply it to subtraction and division.

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T: (Draw parentheses around the addition expression.) Work with a partner to find another way to write this expression numerically.

S: (Work and show $(\frac{3}{5} + 1\frac{1}{2}) \times 2$.)

T: (Post Problem 1(b), *half the sum of* $\frac{3}{5}$ *and* $1\frac{1}{2}$, on the board.) Read this expression out loud with me.

S: (Read.)

T: Compare this expression to the previous one. Turn and talk.

S: (Share.)

T: Without evaluating this expression, will the value of this expression be more than or less than the previous one? Turn and talk.

S: It's less. The other one multiplied the sum by 2, and this one is one-half of the same sum. → Taking half of a number is less than doubling it.

T: This expression again includes the sum of 3 fifths and 1 and one-half. (Write $\frac{3}{5} + 1\frac{1}{2}$ on the board.) We need to show one-half of it. Tell a neighbor how we can show one-half of this expression.

S: We can multiply it by $\frac{1}{2}$. → We can show $\frac{1}{2}$ times the addition expression, or we can show the addition expression times $\frac{1}{2}$. → Taking a half of something is the same as dividing it by 2, so we could divide the addition expression by 2. → We could divide the addition expression by 2 but write it like a fraction with 2 as the denominator.

T: Work with a partner to write this expression numerically in at least two different ways.

S: (Work and share.)

T: (Select students to share their work and explain their thought process.)

T: (Post Problem 1(c), *the difference between* $\frac{3}{4}$ *and* $\frac{1}{2}$ *divided by 3,* on the board.) Read this expression aloud with me.

S: (Read.)

T: Talk with a neighbor about what is happening in this expression.

S: *Difference* means subtract, so one half is being subtracted from three fourths. → The last part says *divided by 3,* so we can put parentheses around the subtraction expression and then write ÷ 3. → Since it says *divided by 3*, we can write that like a fraction with 3 as the denominator. → Dividing by 3 is the same as taking a third of something, so we can multiply the subtraction expression by $\frac{1}{3}$.

T: Work independently to write this expression numerically. Share your work with a neighbor when you're finished.

S: (Work and share.)

b). $\frac{1}{2} \times (\frac{3}{5} + 1\frac{1}{2})$

$(\frac{3}{5} + 1\frac{1}{2}) \times \frac{1}{2}$

$\frac{\frac{3}{5} + 1\frac{1}{2}}{2}$

$(\frac{3}{5} + 1\frac{1}{2}) \div 2$

c). $(\frac{3}{4} - \frac{1}{2}) \div 3$

$\frac{\frac{3}{4} - \frac{1}{2}}{3}$

$(\frac{3}{4} - \frac{1}{2}) \times \frac{1}{3}$

$(\frac{3}{4} - \frac{1}{2}) \div 3$

$= (\frac{3}{4} - \frac{2}{4}) \div 3$

$= \frac{1}{4} \div 3$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

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T: Look at your numerical expression. Let's evaluate it. Let's put this expression in its simplest form. What is the first step?

S: Subtract one-half from 3 fourths.

T: Why must we do that first?

S: Because it is in parentheses. → Because we need to evaluate the numerator before dividing by 3.

T: Work with a partner, and then show me the difference between 3 fourths and one-half.

S: (Work and show $\frac{1}{4}$.)

T: Since we wrote our numerical expressions in different ways, tell your partner what your next step will be in evaluating your expression.

S: I need to multiply one-fourth times one-third. → I need to divide one-fourth by 3.

T: Complete the next step and then share your work with a partner.

S: (Work and share.)

T: What is this expression equal to in simplest form?

S: 1 twelfth.

T: Everyone got 1 twelfth?

S: Yes!

T: What does that show us about the different ways to write these expressions? Turn and talk.

S: (Share.)

Problem 2: Write numerical expressions in word form.

a. $\frac{2}{5} \times (\frac{1}{2} - 0.4)$ b. $(\frac{3}{4} + 1.25) \div \frac{1}{3}$ c. $(2 \div \frac{1}{5}) \times 0.3$

T: (Post Problem 2(a) on the board.) Now, we'll rewrite a numerical expression in word form. What is happening in this expression? Turn and talk.

S: In parentheses, there's the difference between one-half and 4 tenths. → The subtraction expression is being multiplied by 2 fifths.

a). $\frac{2}{5}$ times the difference between $\frac{1}{2}$ and 0.4.

b). The sum of $\frac{3}{4}$ and 1.25 divided by $\frac{1}{3}$.

c). The quotient of 2 and $\frac{1}{5}$ times 0.3.

T: Show me a word form expression for the operation outside the parentheses.

S: (Show $\frac{2}{5}$ times.)

T: (Write $\frac{2}{5}$ *times*_____ on the board.) We have 2 fifths times what?

S: The difference between one-half and 4 tenths.

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T: Exactly! (Write *the difference between* $\frac{1}{2}$ *and 0.4* in the blank, and then post Problem 2(b) on the board.) Work with a partner to write this expression using words.

S: (Work and show *the sum of* $\frac{3}{4}$ *and 1.25 divided by* $\frac{1}{3}$.)

T: Let's evaluate the numerical expression. What must we do first?

S: Add $\frac{3}{4}$ and 1.25.

T: Work with a partner to find the sum in its simplest form.

S: (Work and show 2.)

T: What's the next step?

S: Divide 2 by one-third.

T: How many thirds are in 1 whole?

S: 3.

T: How many are in 2 wholes?

S: 6.

T: (Post Problem 2(c) on the board.) Work independently to rewrite this expression using words. If you finish early, evaluate the expression. Check your work with a partner when you're both ready.

S: (Work and share.)

b). $(\frac{3}{4}+1.25)\div\frac{1}{3}$

$=(0.75+1.25)\div\frac{1}{3}$

$=2\div\frac{1}{3}$

$=2\times\frac{3}{1}$

$=6$

c). $(2\div\frac{1}{5})\times 0.3$

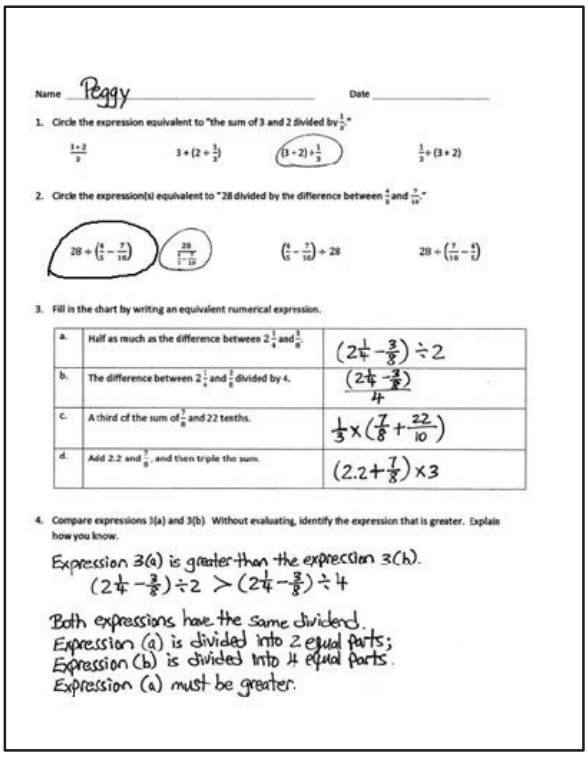
$=(2\times\frac{5}{1})\times 0.3$

$=10\times 0.3$

$=3$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



Name Peggy Date

1. Circle the expression equivalent to "the sum of 3 and 2 divided by $\frac{1}{2}$."

$\frac{3+2}{\frac{1}{2}}$ $3+(2\div\frac{1}{2})$ $(3+2)\div\frac{1}{2}$ $\frac{1}{2}+(3+2)$

2. Circle the expression(s) equivalent to "28 divided by the difference between $\frac{4}{5}$ and $\frac{7}{10}$."

$28\div(\frac{4}{5}-\frac{7}{10})$ $\frac{28}{\frac{4}{5}-\frac{7}{10}}$ $(\frac{4}{5}-\frac{7}{10})\div 28$ $28\div(\frac{7}{10}-\frac{4}{5})$

3. Fill in the chart by writing an equivalent numerical expression.

a.	Half as much as the difference between $2\frac{1}{4}$ and $\frac{3}{8}$.	$(2\frac{1}{4}-\frac{3}{8})\div 2$
b.	The difference between $2\frac{1}{4}$ and $\frac{3}{8}$ divided by 4.	$\frac{(2\frac{1}{4}-\frac{3}{8})}{4}$
c.	A third of the sum of $\frac{7}{8}$ and 22 tenths.	$\frac{1}{3}\times(\frac{7}{8}+\frac{22}{10})$
d.	Add 2.2 and $\frac{7}{8}$, and then triple the sum.	$(2.2+\frac{7}{8})\times 3$

4. Compare expressions 3(a) and 3(b). Without evaluating, identify the expression that is greater. Explain how you know.

Expression 3(a) is greater than the expression 3(b).
$(2\frac{1}{4}-\frac{3}{8})\div 2 > (2\frac{1}{4}-\frac{3}{8})\div 4$

Both expressions have the same dividend.
Expression (a) is divided into 2 equal parts;
Expression (b) is divided into 4 equal parts.
Expression (a) must be greater.

Student Debrief (10 minutes)

Lesson Objective: Interpret and evaluate numerical expressions including the language of scaling and fraction division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

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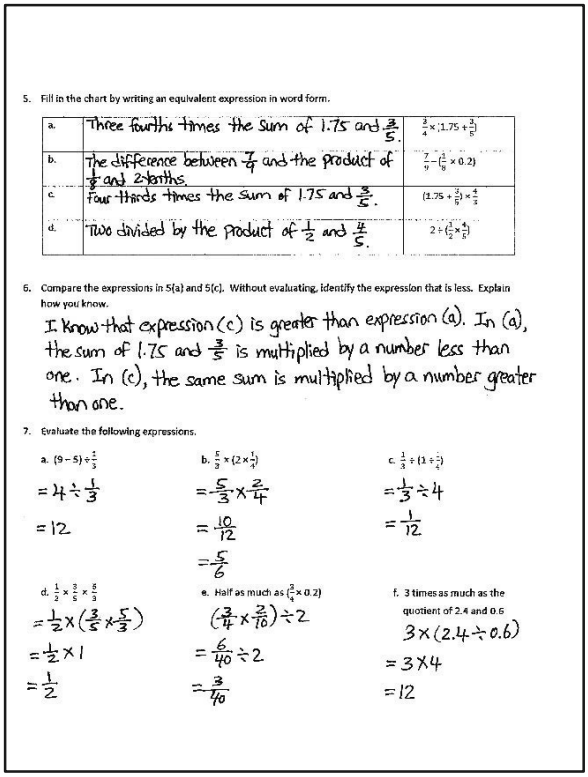
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- For Problems 1 and 2, explain to a partner how you chose the correct equivalent expression(s).
- Compare your answer for Problem 3 with a partner. Is there more than one correct answer?
- What's the relationship between Problems 5(a) and 5(c)?
- Share and compare your solutions for Problem 7 with a partner. Be careful with the order of operations; calculate the parenthesis first.
- Share and compare your answers for Problem 8 with a partner. For the two expressions that did not match the story problems, can you think of a story problem for them? Share your ideas with a partner.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



5. Fill in the chart by writing an equivalent expression in word form.

a.	Three fourths times the sum of 1.75 and $\frac{3}{5}$.	$\frac{3}{4} \times (1.75 + \frac{3}{5})$
b.	The difference between $\frac{7}{9}$ and the product of $\frac{1}{3}$ and 2-tenths.	$\frac{7}{9} - (\frac{1}{3} \times 0.2)$
c.	Four thirds times the sum of 1.75 and $\frac{3}{5}$.	$(1.75 + \frac{3}{5}) \times \frac{4}{3}$
d.	Two divided by the product of $\frac{1}{2}$ and $\frac{4}{5}$.	$2 \div (\frac{1}{2} \times \frac{4}{5})$

6. Compare the expressions in 5(a) and 5(c). Without evaluating, identify the expression that is less. Explain how you know.

I know that expression (c) is greater than expression (a). In (a), the sum of 1.75 and $\frac{3}{5}$ is multiplied by a number less than one. In (c), the same sum is multiplied by a number greater than one.

7. Evaluate the following expressions.

a. $(9-5) \div \frac{1}{3}$
$= 4 \div \frac{1}{3}$
$= 12$

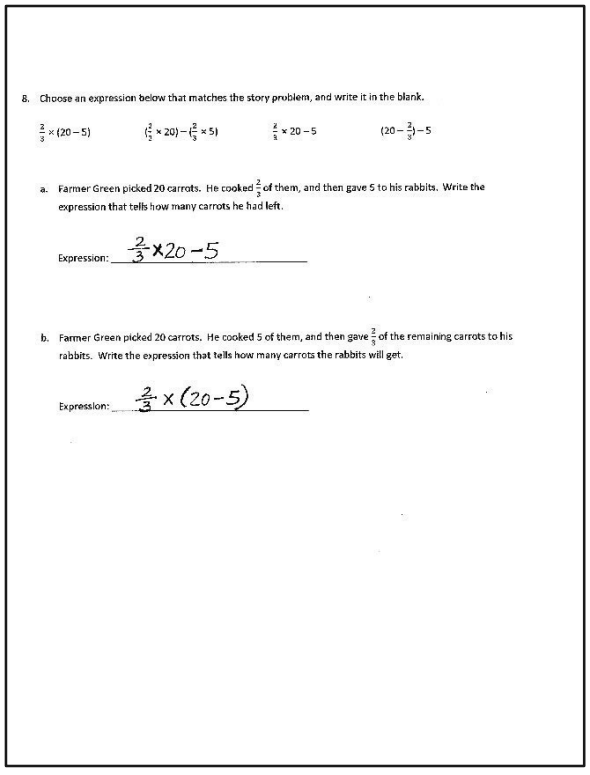
b. $\frac{5}{3} \times (2 \times \frac{1}{4})$
$= \frac{5}{3} \times \frac{2}{4}$
$= \frac{10}{12}$
$= \frac{5}{6}$

c. $\frac{1}{3} \div (1 \div \frac{1}{4})$
$= \frac{1}{3} \div 4$
$= \frac{1}{12}$

d. $\frac{1}{2} \times \frac{3}{5} \times \frac{5}{3}$
$= \frac{1}{2} \times (\frac{3}{5} \times \frac{5}{3})$
$= \frac{1}{2} \times 1$
$= \frac{1}{2}$

e. Half as much as $(\frac{3}{4} \times 0.2)$
$(\frac{3}{4} \times \frac{2}{10}) \div 2$
$= \frac{6}{40} \div 2$
$= \frac{3}{40}$

f. 3 times as much as the quotient of 2.4 and 0.6
$3 \times (2.4 \div 0.6)$
$= 3 \times 4$
$= 12$



8. Choose an expression below that matches the story problem, and write it in the blank.

$\frac{2}{3} \times (20-5)$ $(\frac{2}{3} \times 20) - (\frac{2}{3} \times 5)$ $\frac{2}{3} \times 20 - 5$ $(20 - \frac{2}{3}) - 5$

a. Farmer Green picked 20 carrots. He cooked $\frac{2}{3}$ of them, and then gave 5 to his rabbits. Write the expression that tells how many carrots he had left.

Expression: $\frac{2}{3} \times 20 - 5$

b. Farmer Green picked 20 carrots. He cooked 5 of them, and then gave $\frac{2}{3}$ of the remaining carrots to his rabbits. Write the expression that tells how many carrots the rabbits will get.

Expression: $\frac{2}{3} \times (20-5)$

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1. Circle the expression equivalent to *the sum of 3 and 2 divided by* $\frac{1}{3}$.

$\frac{3+2}{3}$ $3 + (2 \div \frac{1}{3})$ $(3 + 2) \div \frac{1}{3}$ $\frac{1}{3} \div (3 + 2)$

2. Circle the expression(s) equivalent to *28 divided by the difference between* $\frac{4}{5}$ *and* $\frac{7}{10}$.

$28 \div \left(\frac{4}{5} - \frac{7}{10}\right)$ $\frac{28}{\frac{4}{5} - \frac{7}{10}}$ $\left(\frac{4}{5} - \frac{7}{10}\right) \div 28$ $28 \div \left(\frac{7}{10} - \frac{4}{5}\right)$

3. Fill in the chart by writing an equivalent numerical expression.

a.	Half as much as the difference between $2\frac{1}{4}$ and $\frac{3}{8}$.	
b.	The difference between $2\frac{1}{4}$ and $\frac{3}{8}$ divided by 4.	
c.	A third of the sum of $\frac{7}{8}$ and 22 tenths.	
d.	Add 2.2 and $\frac{7}{8}$, and then triple the sum.	

4. Compare expressions 3(a) and 3(b). Without evaluating, identify the expression that is greater. Explain how you know.

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5. Fill in the chart by writing an equivalent expression in word form.

a.		$\frac{3}{4} \times (1.75 + \frac{3}{5})$
b.		$\frac{7}{9} - (\frac{1}{8} \times 0.2)$
c.		$(1.75 + \frac{3}{5}) \times \frac{4}{3}$
d.		$2 \div (\frac{1}{2} \times \frac{4}{5})$

6. Compare the expressions in 5(a) and 5(c). Without evaluating, identify the expression that is less. Explain how you know.

7. Evaluate the following expressions.

a. $(9 - 5) \div \frac{1}{3}$ b. $\frac{5}{3} \times (2 \times \frac{1}{4})$ c. $\frac{1}{3} \div (1 \div \frac{1}{4})$

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d. $\frac{1}{2} \times \frac{3}{5} \times \frac{5}{3}$

e. Half as much as $(\frac{3}{4} \times 0.2)$

f. 3 times as much as the quotient of 2.4 and 0.6

8. Choose an expression below that matches the story problem, and write it in the blank.

$\frac{2}{3} \times (20 - 5)$ $(\frac{2}{3} \times 20) - (\frac{2}{3} \times 5)$ $\frac{2}{3} \times 20 - 5$ $(20 - \frac{2}{3}) - 5$

a. Farmer Green picked 20 carrots. He cooked $\frac{2}{3}$ of them, and then gave 5 to his rabbits. Write the expression that tells how many carrots he had left.

Expression: _____

b. Farmer Green picked 20 carrots. He cooked 5 of them, and then gave $\frac{2}{3}$ of the remaining carrots to his rabbits. Write the expression that tells how many carrots the rabbits will get.

Expression: _____

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1. Write an equivalent expression in numerical form.

   A fourth as much as the product of two-thirds and 0.8

2. Write an equivalent expression in word form.

   a. $\frac{3}{8} \times (1 - \frac{1}{3})$

   b. $(1 - \frac{1}{3}) \div 2$

3. Compare the expressions in 2(a) and 2(b). Without evaluating, determine which expression is greater, and explain how you know.

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1. Circle the expression equivalent to *the difference between 7 and 4, divided by a fifth*.

$7 + (4 \div \frac{1}{5})$ $\frac{7-4}{5}$ $(7-4) \div \frac{1}{5}$ $\frac{1}{5} \div (7-4)$

2. Circle the expression(s) equivalent to *42 divided by the sum of* $\frac{2}{3}$ *and* $\frac{3}{4}$.

$(\frac{2}{3}+\frac{3}{4}) \div 42$ $(42 \div \frac{2}{3}) + \frac{3}{4}$ $42 \div (\frac{2}{3}+\frac{3}{4})$ $\frac{42}{\frac{2}{3}+\frac{3}{4}}$

3. Fill in the chart by writing the equivalent numerical expression or expression in word form.

	Expression in word form	Numerical expression
a.	A fourth as much as the sum of $3\frac{1}{8}$ and 4.5	
b.		$(3\frac{1}{8} + 4.5) \div 5$
c.	Multiply $\frac{3}{5}$ by 5.8; then halve the product	
d.		$\frac{1}{6} \times (4.8 - \frac{1}{2})$
e.		$8 - (\frac{1}{2} \div 9)$

4. Compare the expressions in 3(a) and 3(b). Without evaluating, identify the expression that is greater. Explain how you know.

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5. Evaluate the following expressions.

a. $(11 - 6) \div \frac{1}{6}$

b. $\frac{9}{5} \times (4 \times \frac{1}{6})$

c. $\frac{1}{10} \div (5 \div \frac{1}{2})$

d. $\frac{3}{4} \times \frac{2}{5} \times \frac{4}{3}$

e. 50 divided by the difference between $\frac{3}{4}$ and $\frac{5}{8}$

6. Lee is sending out 32 birthday party invitations. She gives 5 invitations to her mom to give to family members. Lee mails a third of the rest, and then she takes a break to walk her dog.

a. Write a numerical expression to describe how many invitations Lee has already mailed.

b. Which expression matches how many invitations still need to be sent out?

$32 - 5 - \frac{1}{3}(32 - 5)$ $\frac{2}{3} \times 32 - 5$ $(32 - 5) \div \frac{1}{3}$ $\frac{1}{3} \times (32 - 5)$

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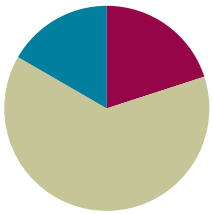
Lesson 33

Objective: Create story contexts for numerical expressions and tape diagrams, and solve word problems.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Concept Development (38 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)



Fluency Practice (12 minutes)

- Sprint: Divide Decimals **5.NBT.7** (8 minutes)
- Write Equivalent Expressions **5.OA.1** (4 minutes)

Sprint: Divide Decimals (8 minutes)

Materials: (S) Divide Decimals Sprint

Note: This fluency activity reviews Lessons 29–32.

Write Equivalent Expressions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 32.

T: (Write $2 \div \frac{1}{3} =$ ____.) What is $2 \div \frac{1}{3}$?

S: 6.

T: (Write $2 \div \frac{1}{3} + 4 =$ ____.) On your personal white board, write the complete number sentence.

S: (Write $2 \div \frac{1}{3} + 4$. Beneath it, write = 6 + 4. Beneath it, write = 10.)

$2 \div \frac{1}{3} = 6$

$2 \div \frac{1}{3} + 4$
$= 6 + 4$
$= 10$

Continue with the following possible sequence: $\frac{5+3}{5}$, $\frac{1}{3} \div (2 + 2)$, $(4 + 3) \div \frac{1}{4}$, and $(\frac{2}{5} - \frac{3}{10}) \div 5$.

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Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The time normally allotted for the Application Problem has been included in the Concept Development portion of today's lesson to give students the time necessary to write story problems.

Suggested Delivery of Instruction for Solving Lesson 33 Word Problems

1. Model the problem.

Have two pairs of student work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Consider selecting students who may have difficulty grasping the Problem Set to work at the board. Support them as they work, and praise their effort and perseverance. This approach can help improve classroom climate, where top students are usually selected to work at the board, and reinforce the notion that determination and persistence are key to learning math skills.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish work on that question, sharing their work and thinking with a peer. All should write their equations and statements of the answer.

3. Assess the solution for reasonableness.

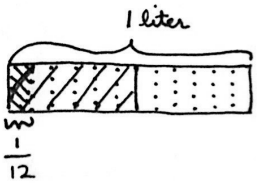
Give students one to two minutes to assess and explain the reasonableness of their solution.

Problem 1

Ms. Hayes has $\frac{1}{2}$ liter of juice. She distributes it equally to 6 students in her tutoring group.

a. How many liters of juice does each student get?

1a) $\frac{1}{2}$ liter $\div 6$
= 1 half $\div 6$
= 6 twelfths $\div 6$
= 1 twelfth



Each student gets $\frac{1}{12}$ liter of juice.

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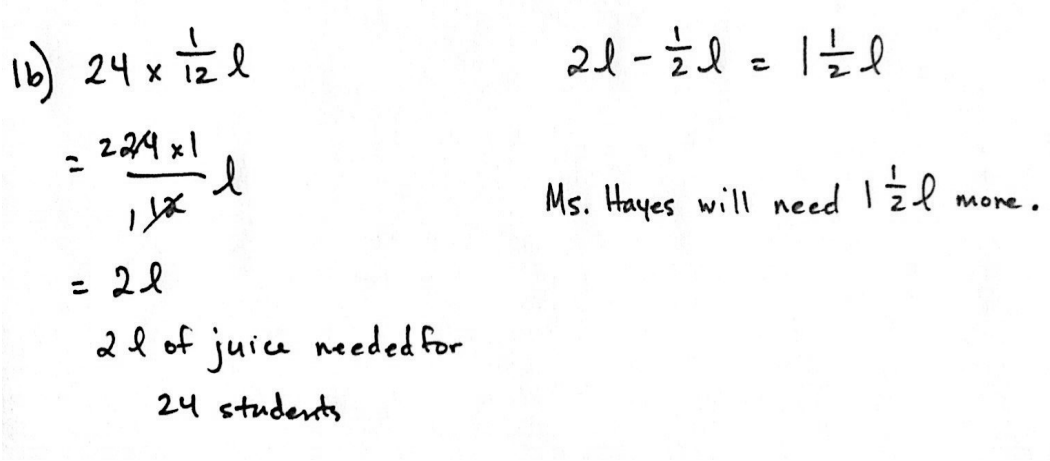
b. How many more liters of juice will Ms. Hayes need if she wants to give each of the 24 students in her class the same amount of juice found in Part (a)?



In this problem, Ms. Hayes is sharing equally, or dividing, one-half liter of juice among 6 students. Students should recognize this problem as $\frac{1}{2} \div 6$. A tape diagram shows that when halves are partitioned into 6 equal parts, twelfths are created. Likewise, the diagram shows that 1 half is equal to 6 twelfths, and when written in unit form, 6 twelfths divided by 6 is a simple problem. Each student receives one-twelfth liter of juice. In Part (b), students must find how much more juice is necessary to give a total of 24 students $\frac{1}{12}$ liter of juice. Some students may choose to solve by multiplying 24 by one-twelfth to find that a total of 2 liters of juice is necessary. Encourage interpretation as a scaling problem. Help students see that, since 24 students is 4 times more students than 6 students, Ms. Hayes will need 4 times more juice as well. 4 times one-half is, again, equal to 2 liters of juice. Either way, Ms. Hayes will need $1\frac{1}{2}$ more liters of juice.

Problem 2

Lucia has 3.5 hours left in her workday as a car mechanic. Lucia needs $\frac{1}{2}$ of an hour to complete one oil change.

a. How many oil changes can Lucia complete during the rest of her workday?
b. Lucia can complete two car inspections in the same amount of time it takes her to complete one oil change. How long does it take her to complete one car inspection?
c. How many inspections can she complete in the rest of her workday?

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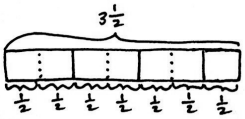
2a) How many half hours in 3.5 hours?

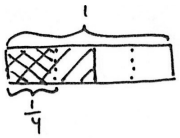


$3.5 \div \frac{1}{2}$

$= 3\frac{1}{2} \div \frac{1}{2}$

There are 2 halves in 1 whole.
There are 6 halves in 3 wholes
There is 1 half in 1 half.
There are 7 halves in $3\frac{1}{2}$.

2b) $\frac{1}{2} \div 2 = \frac{1}{4}$



Lucia can complete one car inspection in 1/4 hour.

2c) How many $\frac{1}{4}$ hours are in 3.5 hours?

$3.5 \div \frac{1}{4}$

$= 3\frac{2}{4} \div \frac{1}{4}$

There are 12 fourths in 3.
There are 2 fourths in 2/4.
There are 14 fourths in $3\frac{2}{4}$.

Lucia can complete 14 inspections in 3.5 hours.

2c) $7 \times 2 = 14$

Lucia can complete twice as many inspections as oil changes, so she can complete 14 inspections in 3.5 hours.

In Part (a), students are asked to find how many half-hours are in a 3.5-hour period. The presence of both decimal and fraction notation in these problems adds a layer of complexity. Students should be comfortable choosing which form of fractional number is most efficient for solving. This will vary by problem and, in many cases, by student. In this problem, many students may prefer to deal with 3.5 as a mixed number ($3\frac{1}{2}$). Then, a tape diagram clearly shows that $3\frac{1}{2}$ can be partitioned into 7 units of $\frac{1}{2}$. Others may prefer to express the half hour as 0.5. Still, others may begin their thinking with, "How many halves are in 1 whole?" and continue with similar prompts to find how many halves are in 3 wholes and 1 half.

In Part (b), students reason that since Lucia can complete 2 inspections in the time it takes her to complete just one oil change, $\frac{1}{2}$ may be divided by 2 to find the fraction of an hour that an inspection requires. Students may also reason that there are two 15-minute units in one half-hour period, and therefore, Lucia can complete an inspection in $\frac{1}{4}$ hour.

MP.7

In Part (c), a variety of approaches are also possible. Some may argue that, since Lucia can work twice as fast completing inspections, they need only to double the number of oil changes she could complete in 3.5 hours to find the number of inspections done. This type of thinking is evidence of a deeper understanding of a scaling principle. Other students may solve Part (c), just as they did Part (a), but using a divisor of $\frac{1}{4}$. In either case, Lucia can complete 14 inspections in 3.5 hours.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge high-achieving students (who may also be early finishers) to solve the problems more than one way. After looking at their work, challenge them by specifying the operation they must use to begin, or change the path of their approach by requiring a certain operation within their solution.

Challenge them further by asking them to use the same general context but to write a different question that results in the same answer as that obtained for the original problem.

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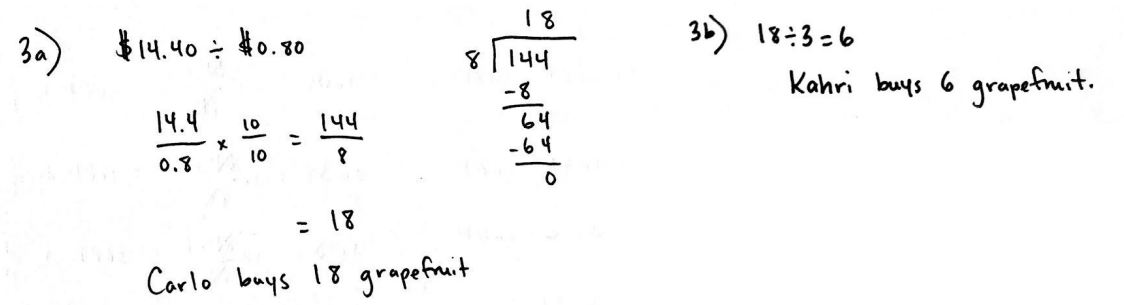
Problem 3

Carlo buys $14.40 worth of grapefruit. Each grapefruit costs $0.80.

a. How many grapefruits does Carlo buy?

b. At the same store, Kahri spends one-third as much money on grapefruit as Carlo. How many grapefruits does she buy?



Students divide a decimal dividend by a decimal divisor to solve Problem 3. This problem is made simpler by showing the division expression as a fraction. Then, multiplication by a fraction equal to 1 ($\frac{10}{10}$ or $\frac{100}{100}$, depending on whether 80 cents is expressed as 0.8 or 0.80) results in both a whole number divisor and dividend. From here, students must divide 144 by 8 to find a quotient of 18. Carlo buys 18 grapefruits with his money.

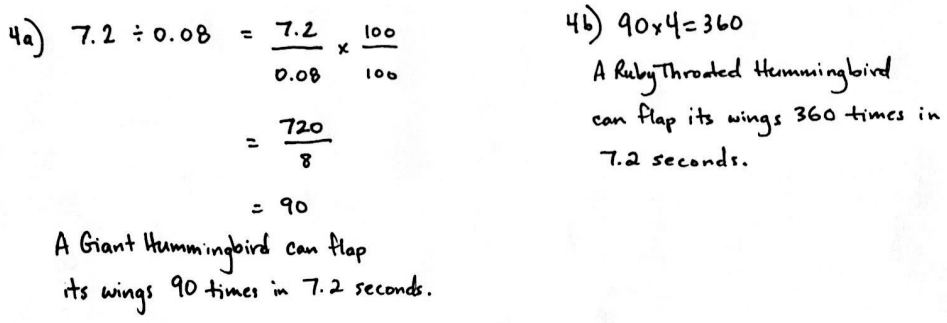
In Part (b), since Kahri spends one-third of her money on equally priced grapefruit, students should reason that she would be buying one-third the number of fruit. Therefore, 18 ÷ 3 shows that Kahri buys 6 grapefruit. Students may also choose the far less-direct method of solving a third of $14.40 and dividing that number ($4.80) by $0.80 to find the number of grapefruits purchased by Kahri.

Problem 4

Studies show that a typical giant hummingbird can flap its wings once in 0.08 of a second.

a. While flying for 7.2 seconds, how many times will a typical giant hummingbird flap its wings?

b. A ruby-throated hummingbird can flap its wings 4 times faster than a giant hummingbird. How many times will a ruby-throated hummingbird flap its wings in the same amount of time?



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Problem 4 is another decimal divisor/dividend problem. Similarly, students should express this division as a fraction and then multiply to rename the divisor as a whole number. Ultimately, students should find that the giant hummingbird can flap its wings 90 times in 7.2 seconds. Part (b) is another example of the usefulness of the scaling principle. Since a ruby-throated hummingbird can flap its wings 4 times faster than the giant hummingbird, students need only multiply 90 by 4 to find that a ruby-throated hummingbird can flap its wings a remarkable 360 times in 7.2 seconds. Though not very efficient, students could also divide 0.08 by 4 to find that it takes a ruby-throated hummingbird just 0.02 seconds to flap its wings once. Then, division of 7.2 by 0.02 (or 720 by 2, after renaming the divisor as a whole number) yields a quotient of 360.

Problem 5

Create a story context for the following expression.

$\frac{1}{3} \times (\$20 - \$3.20)$

5) Jamis had a $20 bill. He spent $3.20 of it on breakfast & one-third of the remaining money on a book. How much did Jamis spend on the book?

5) Wilma promises to share her change equally among her 3 children. She pays her $3.20 bill with a $20. How much does each child receive?

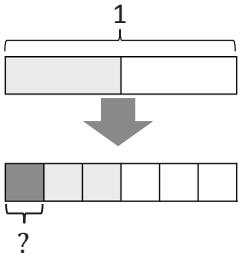
Working backward from expression to story may be challenging for some students. Since the expression given contains parentheses, the story created must first involve the subtraction of $3.20 from $20. For students in need of assistance, drawing a tape diagram first may be of help. Note that the story of Jamis interprets the multiplication of $\frac{1}{3}$ directly, whereas the story of Wilma interprets the expression as division by 3.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge early finishers in this lesson by encouraging them to go back to each problem and provide an alternate means for solution or an additional model to represent the problem. Students could discuss how their interpretation of each problem led them to solve it the way they did, and how and why alternate interpretations could lead to a different solution strategy.

Problem 6

Create a story context about painting a wall for the following tape diagram.



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Again, students are asked to create a story problem, this time using a given tape diagram and the context of painting a wall. The challenge here is that this tape diagram implies a two-step word problem. The whole, 1, is first partitioned into half, and then one of those halves is divided into thirds. The story students create should reflect this two-part drawing. Students should be encouraged to share aloud and discuss their stories and thought process for solving.

a) Ariel is painting her room and she finishes half of it before lunch. While painting she spends equal time kneeling, standing and climbing a ladder. What portion of the job was painted while kneeling?

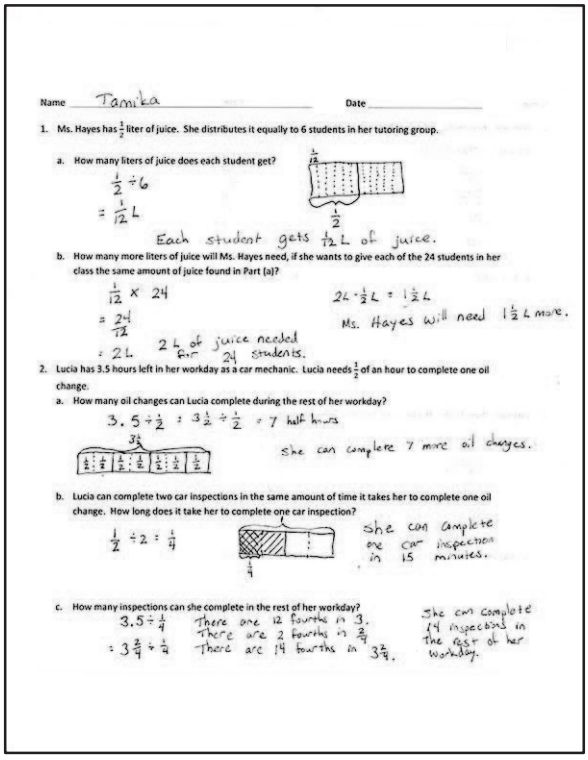
b) One half of a wall is painted white. The other half is painted to look like a Romanian flag, having equal parts blue, yellow and red. What fraction of the wall is painted yellow?

Student Debrief (10 minutes)

Lesson Objective: Create story contexts for numerical expressions and tape diagrams, and solve word problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


Name Tamika Date

1. Ms. Hayes has $\frac{1}{2}$ liter of juice. She distributes it equally to 6 students in her tutoring group.

a. How many liters of juice does each student get?

$\frac{1}{2} \div 6$

$= \frac{1}{12}$ L

Each student gets $\frac{1}{12}$ L of juice.

b. How many more liters of juice will Ms. Hayes need, if she wants to give each of the 24 students in her class the same amount of juice found in Part (a)?

$\frac{1}{12} \times 24$

$= \frac{24}{12}$

$= 2$ L

2 L of juice needed for 24 students.

$2 \text{ L} - \frac{1}{2} \text{ L} = 1\frac{1}{2}$ L

Ms. Hayes will need $1\frac{1}{2}$ L more.

2. Lucia has 3.5 hours left in her workday as a car mechanic. Lucia needs $\frac{1}{2}$ of an hour to complete one oil change.

a. How many oil changes can Lucia complete during the rest of her workday?

$3.5 \div \frac{1}{2} = 3\frac{1}{2} \div \frac{1}{2} = 7$ half hours

She can complete 7 more oil changes.

b. Lucia can complete two car inspections in the same amount of time it takes her to complete one oil change. How long does it take her to complete one car inspection?

$\frac{1}{2} \div 2 = \frac{1}{4}$

She can complete one car inspection in 15 minutes.

c. How many inspections can she complete in the rest of her workday?

$3.5 \div \frac{1}{4}$

$= 3\frac{2}{4} \div \frac{1}{4}$

There are 12 fourths in 3.
There are 2 fourths in $\frac{2}{4}$.
There are 14 fourths in $3\frac{2}{4}$.

She can complete 14 inspections in the rest of her workday.

Any combination of the questions below may be used to lead the discussion.

- For Problems 1–5, did you draw a tape diagram to help solve the problems? If so, share your drawings and explain them to a partner.
- For Problems 1–4, there are different ways to solve the problems. Share and compare your strategy with a partner.
- For Problems 5 and 6, share your story problem with a partner. Explain how you interpreted the expression in Problem 5 and the tape diagram in Problem 6.

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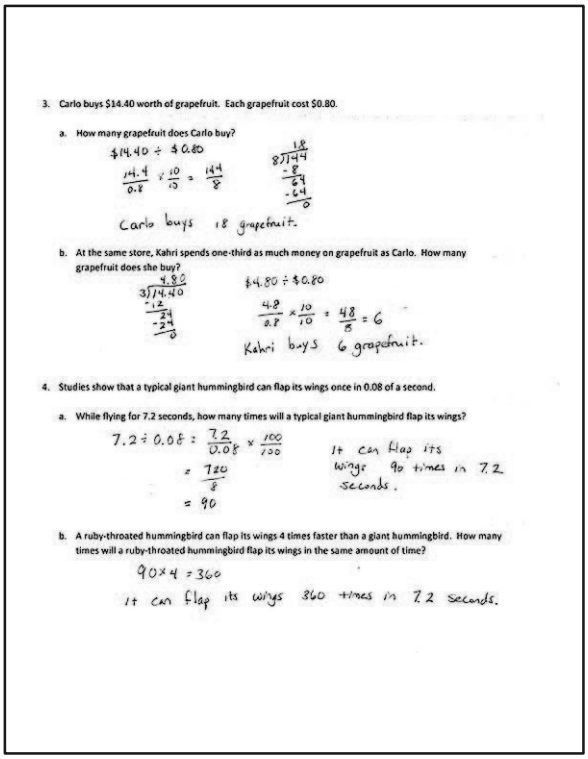
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



3. Carlo buys $14.40 worth of grapefruit. Each grapefruit cost $0.80.

a. How many grapefruit does Carlo buy?

$14.40 ÷ $0.80

$\frac{14.4}{0.8} \times \frac{10}{10} = \frac{144}{8}$

18
8)144
-8
64
-64
0

Carlo buys 18 grapefruit.

b. At the same store, Kahri spends one-third as much money on grapefruit as Carlo. How many grapefruit does she buy?

4.80
3)14.40
-12
24
-24
0

$4.80 ÷ $0.80

$\frac{4.8}{0.8} \times \frac{10}{10} = \frac{48}{8} = 6$

Kahri buys 6 grapefruit.

4. Studies show that a typical giant hummingbird can flap its wings once in 0.08 of a second.

a. While flying for 7.2 seconds, how many times will a typical giant hummingbird flap its wings?

$7.2 ÷ 0.08 = \frac{7.2}{0.08} \times \frac{100}{100}$

$= \frac{720}{8}$

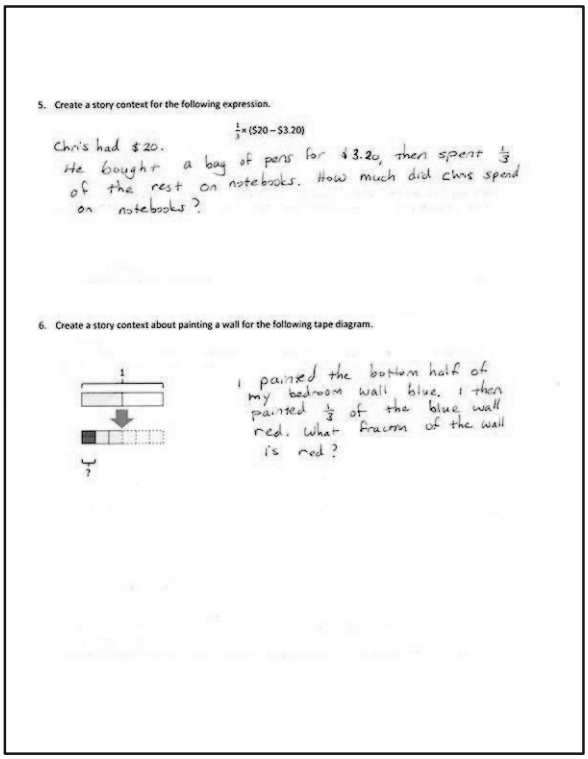
$= 90$

It can flap its wings 90 times in 7.2 seconds.

b. A ruby-throated hummingbird can flap its wings 4 times faster than a giant hummingbird. How many times will a ruby-throated hummingbird flap its wings in the same amount of time?

$90 \times 4 = 360$

It can flap its wings 360 times in 7.2 seconds.



5. Create a story context for the following expression.

$\frac{1}{3} \times (\$20 - \$3.20)$

Chris had $20. He bought a bag of pens for $3.20, then spent $\frac{1}{3}$ of the rest on notebooks. How much did Chris spend on notebooks?

6. Create a story context about painting a wall for the following tape diagram.

I painted the bottom half of my bedroom wall blue. I then painted $\frac{1}{3}$ of the blue wall red. What fraction of the wall is red?

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Number Correct: _____

A

Divide Decimals

1.	1 ÷ 1 =	
2.	1 ÷ 0.1 =	
3.	2 ÷ 0.1 =	
4.	7 ÷ 0.1 =	
5.	1 ÷ 0.1 =	
6.	10 ÷ 0.1 =	
7.	20 ÷ 0.1 =	
8.	60 ÷ 0.1 =	
9.	1 ÷ 1 =	
10.	1 ÷ 0.1 =	
11.	10 ÷ 0.1 =	
12.	100 ÷ 0.1 =	
13.	200 ÷ 0.1 =	
14.	800 ÷ 0.1 =	
15.	1 ÷ 0.1 =	
16.	1 ÷ 0.01 =	
17.	2 ÷ 0.01 =	
18.	9 ÷ 0.01 =	
19.	5 ÷ 0.01 =	
20.	50 ÷ 0.01 =	
21.	60 ÷ 0.01 =	
22.	20 ÷ 0.01 =	
23.	5 ÷ 0.1 =	
24.	0.5 ÷ 0.1 =	
25.	0.05 ÷ 0.1 =	
26.	0.08 ÷ 0.1 =	
27.	4 ÷ 0.01 =	
28.	40 ÷ 0.01 =	
29.	47 ÷ 0.01 =	
30.	59 ÷ 0.01 =	
31.	3 ÷ 0.1 =	
32.	30 ÷ 0.1 =	
33.	32 ÷ 0.1 =	
34.	32.5 ÷ 0.1 =	
35.	25 ÷ 5 =	
36.	2.5 ÷ 0.5 =	
37.	2.5 ÷ 0.05 =	
38.	3.6 ÷ 0.04 =	
39.	32 ÷ 0.08 =	
40.	56 ÷ 0.7 =	
41.	77 ÷ 1.1 =	
42.	4.8 ÷ 0.12 =	
43.	4.84 ÷ 0.4 =	
44.	9.63 ÷ 0.03 =	

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B

Number Correct: _____

Improvement: _____

Divide Decimals

1.	10 ÷ 1 =	
2.	1 ÷ 0.1 =	
3.	2 ÷ 0.1 =	
4.	8 ÷ 0.1 =	
5.	1 ÷ 0.1 =	
6.	10 ÷ 0.1 =	
7.	20 ÷ 0.1 =	
8.	70 ÷ 0.1 =	
9.	1 ÷ 1 =	
10.	1 ÷ 0.1 =	
11.	10 ÷ 0.1 =	
12.	100 ÷ 0.1 =	
13.	200 ÷ 0.1 =	
14.	900 ÷ 0.1 =	
15.	1 ÷ 0.1 =	
16.	1 ÷ 0.01 =	
17.	2 ÷ 0.01 =	
18.	7 ÷ 0.01 =	
19.	4 ÷ 0.01 =	
20.	40 ÷ 0.01 =	
21.	50 ÷ 0.01 =	
22.	80 ÷ 0.01 =	

23.	4 ÷ 0.1 =	
24.	0.4 ÷ 0.1 =	
25.	0.04 ÷ 0.1 =	
26.	0.07 ÷ 0.1 =	
27.	5 ÷ 0.01 =	
28.	50 ÷ 0.01 =	
29.	53 ÷ 0.01 =	
30.	68 ÷ 0.01 =	
31.	2 ÷ 0.1 =	
32.	20 ÷ 0.1 =	
33.	23 ÷ 0.1 =	
34.	23.6 ÷ 0.1 =	
35.	15 ÷ 5 =	
36.	1.5 ÷ 0.5 =	
37.	1.5 ÷ 0.05 =	
38.	3.2 ÷ 0.04 =	
39.	28 ÷ 0.07 =	
40.	42 ÷ 0.6 =	
41.	88 ÷ 1.1 =	
42.	3.6 ÷ 0.12 =	
43.	3.63 ÷ 0.3 =	
44.	8.44 ÷ 0.04 =	

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Name _____ Date _____

1. Ms. Hayes has $\frac{1}{2}$ liter of juice. She distributes it equally to 6 students in her tutoring group.

   a. How many liters of juice does each student get?

   b. How many more liters of juice will Ms. Hayes need if she wants to give each of the 24 students in her class the same amount of juice found in Part (a)?

2. Lucia has 3.5 hours left in her workday as a car mechanic. Lucia needs $\frac{1}{2}$ of an hour to complete one oil change.

   a. How many oil changes can Lucia complete during the rest of her workday?

   b. Lucia can complete two car inspections in the same amount of time it takes her to complete one oil change. How long does it take her to complete one car inspection?

   c. How many inspections can she complete in the rest of her workday?

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3. Carlo buys $14.40 worth of grapefruit. Each grapefruit costs $0.80.

   a. How many grapefruits does Carlo buy?

   b. At the same store, Kahri spends one-third as much money on grapefruits as Carlo. How many grapefruits does she buy?

4. Studies show that a typical giant hummingbird can flap its wings once in 0.08 of a second.

   a. While flying for 7.2 seconds, how many times will a typical giant hummingbird flap its wings?

   b. A ruby-throated hummingbird can flap its wings 4 times faster than a giant hummingbird. How many times will a ruby-throated hummingbird flap its wings in the same amount of time?

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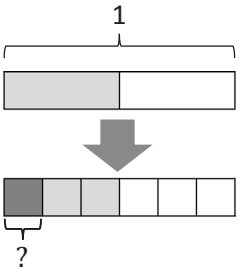
5. Create a story context for the following expression.

$$\frac{1}{3} \times (\$20 - \$3.20)$$

6. Create a story context about painting a wall for the following tape diagram.



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An entire commercial break is 3.6 minutes.

a. If each commercial takes 0.6 minutes, how many commercials will be played?

b. A different commercial break of the same length plays commercials half as long. How many commercials will play during this break?

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Name _____ Date _____

1. Chase volunteers at an animal shelter after school, feeding and playing with the cats.
   a. If he can make 5 servings of cat food from a third of a kilogram of food, how much does one serving weigh?

   b. If Chase wants to give this same serving size to each of 20 cats, how many kilograms of food will he need?

2. Anouk has 4.75 pounds of meat. She uses a quarter pound of meat to make one hamburger.
   a. How many hamburgers can Anouk make with the meat she has?

   b. Sometimes Anouk makes sliders. Each slider is half as much meat as is used for a regular hamburger. How many sliders could Anouk make with the 4.75 pounds?

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3. Ms. Geronimo has a $10 gift certificate to her local bakery.

   a. If she buys a slice of pie for $2.20 and uses the rest of the gift certificate to buy chocolate macaroons that cost $0.60 each, how many macaroons can Ms. Geronimo buy?

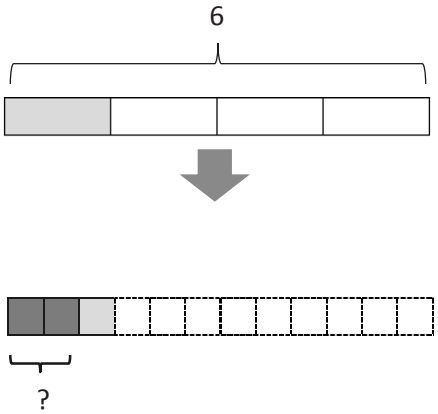
   b. If she changes her mind and instead buys a loaf of bread for $4.60 and uses the rest to buy cookies that cost $1\frac{1}{2}$ times as much as the macaroons, how many cookies can she buy?

4. Create a story context for the following expressions.

   a. $(5\frac{1}{4} - 2\frac{1}{8}) \div 4$

   b. $4 \times (\frac{4.8}{0.8})$

5. Create a story context for the following tape diagram.



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Name _____ Date _____

1. Multiply or divide. Draw a model to explain your thinking.

a. $\frac{1}{3} \times \frac{1}{4}$

b. $\frac{3}{4}$ of $\frac{1}{3}$

c. $\frac{3}{4} \times \frac{3}{5}$

d. $4 \div \frac{1}{3}$

e. $5 \div \frac{1}{4}$

f. $\frac{1}{4} \div 5$

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2. Multiply or divide using any method.

a. 1.5×32

b. 1.5×0.32

c. $12 \div 0.03$

d. $1.2 \div 0.3$

e. $12.8 \times \frac{3}{4}$

f. $102.4 \div 3.2$

3. Fill in the chart by writing an equivalent expression.

a.	One-fifth the sum of one-half and one-third	
b.	Two and one-half times the sum of nine and twelve	
c.	Twenty-four divided by the difference between $1\frac{1}{2}$ and $\frac{3}{4}$	

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4. A castle has to be guarded 24 hours a day. Five knights are ordered to split each day's guard duty equally. How long will each knight spend on guard duty in one day?

   a. Record your answer in hours.

   b. Record your answer in hours and minutes.

   c. Record your answer in minutes.

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5. On the blank, write a division expression that matches the situation.

   a. _____ Mark and Jada share 5 yards of ribbon equally. How much ribbon will each get?

   b. _____ It takes half of a yard of ribbon to make a bow. How many bows can be made with 5 yards of ribbon?

   c. Draw a diagram for each problem and solve.

   d. Could either of the problems also be solved by using $\frac{1}{2} \times 5$? If so, which one(s)? Explain your thinking.

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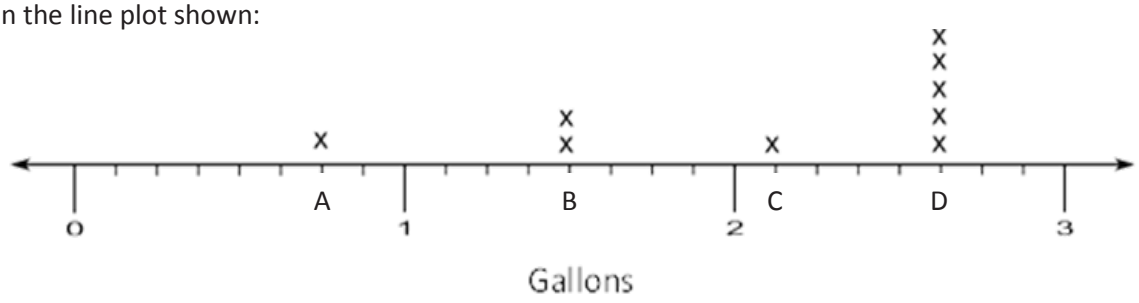
6. Jackson claims that multiplication always makes a number bigger. He gave the following examples:

   - If I take 6, and I multiply it by 4, I get 24, which is bigger than 6.
   - If I take $\frac{1}{4}$, and I multiply it by 2 (whole number), I get $\frac{2}{4}$, or $\frac{1}{2}$, which is bigger than $\frac{1}{4}$.

   Jackson's reasoning is incorrect. Give an example that proves he is wrong, and explain his mistake using pictures, words, or numbers.

7. Jill collected honey from 9 different beehives and recorded the amount collected, in gallons, from each hive in the line plot shown:



   a. She wants to write the value of each point marked on the number line above (Points A–D) in terms of the largest possible whole number of gallons, quarts, and pints. Use the line plot above to fill in the blanks with the correct conversions. (The first one is done for you.)

      A. ___0__ gal __3__ qt ___0__pt

      B. _____ gal _____ qt _____pt

      C. _____ gal _____ qt _____pt

      D. _____ gal _____ qt _____pt

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b. Find the total amount of honey collected from the five hives that produced the most honey.

c. Jill collected a total of 19 gallons of honey. If she distributes all of the honey equally between 9 jars, how much honey will be in each jar?

d. Jill used $\frac{3}{4}$ of a jar of honey for baking. How much honey did she use baking?

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e. Jill's mom used $\frac{1}{4}$ of a gallon of honey to bake 3 loaves of bread. If she used an equal amount of honey in each loaf, how much honey did she use for 1 loaf?

f. Jill's mom stored some of the honey in a container that held $\frac{3}{4}$ of a gallon. She used half of this amount to sweeten tea. How much honey, in cups, was used in the tea? Write an equation, and draw a tape diagram.

g. Jill uses some of her honey to make lotion. If each bottle of lotion requires $\frac{1}{4}$ gallon, and she uses a total of 3 gallons, how many bottles of lotion does she make?

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End-of-Module Assessment Task Standards Addressed	Topics A–H

Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 +7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product of $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)*

5.NF.5 Interpret multiplication as scaling (resizing) by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

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b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students capable of multiplying fractions can generally develop strategies to divide fractions by reasoning about the relationship between multiplication and division. However, division of a fraction by a fraction is not a requirement at this grade level.)

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(\frac{1}{3}) \div 4$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(\frac{1}{3}) \div 4 = \frac{1}{12}$ *because* $(\frac{1}{12}) \times 4 = \frac{1}{3}$.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (\frac{1}{5})$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (\frac{1}{5}) = 20$ *because* $20 \times (\frac{1}{5}) = 4$.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$*-cup servings are in 2 cups of raisins?*

Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$. Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

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Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **5. NF.4** **5. NF.7**	The student draws valid models and/or arrives at the correct answer for two or more items.	The student draws valid models and/or arrives at the correct answer for three or more items.	The student draws valid models and/or arrives at the correct answer for four or more items.	The student correctly answers all eight items and draws valid models: a. $\frac{1}{12}$ b. $\frac{3}{12}$ c. $\frac{9}{20}$ d. 12 e. 20 f. $\frac{1}{20}$
2 **5.NBT.7**	The student has two or fewer correct answers.	The student has three correct answers.	The student has four correct answers.	The student correctly answers all six items: a. 48 b. 0.48 c. 400 d. 4 e. 9.6 or $\frac{384}{40}$ or any equivalent fraction f. 32

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A Progression Toward Mastery				
3 **5.OA.2**	The student has no correct answers.	The student has one correct answer.	The student has two correct answers.	The student correctly answers all three items: a. $\frac{1}{5} \times \left(\frac{1}{2} + \frac{1}{3}\right)$ b. $(9 + 12) \times 2\frac{1}{2}$ or $2\frac{1}{2} \times (9 + 12)$ c. $24 \div \left(1\frac{1}{2} - \frac{3}{4}\right)$
4 **5.NF.3** **5.NF.6** **5.MD.1**	The student has no correct answers.	The student has one correct answer.	The student has two correct answers.	The student correctly answers all three items: a. 4.8 hours b. 4 hours, 48 minutes c. 288 minutes
5 **5.NF.6** **5.NF.7**	The student gives one or fewer correct responses among Parts (a), (b), (c), and (d).	The student gives at least two correct responses among Parts (a), (b), (c), and (d).	The student gives at least three correct responses among Parts (a), (b), (c), and (d).	The student correctly answers all four items: a. $5 \div 2$ b. $5 \div \frac{1}{2}$ c. Draws a correct diagram for both expressions and solves. d. Correctly identifies $5 \div 2$ and offers solid reasoning.
6 **5.NF.5**	The student gives both a faulty example and faulty explanation.	The student gives either a faulty example or explanation.	The student gives a valid example or clear explanation.	The student is able to give a correct example and clear explanation.

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A Progression Toward Mastery				
7 **5.NF.3** **5.NF.4** **5.NF.6** **5.NF.7** **5.MD.1** **5.MD.2**	The student has two or fewer correct answers.	The student has three correct answers.	The student has five correct answers.	The student correctly answers all seven items: a. A. 0 gal, 3 qt, 0 pt B. 1 gal, 2 qt, 0 pt C. 2 gal, 0 qt, 1 pt D. 2 gal, 2 qt, 1 pt b. 13 gal, 1 pt c. $2\frac{1}{9}$ gal d. $1\frac{7}{12}$ gal e. $\frac{1}{12}$ gal f. 6 c g. 12 bottles

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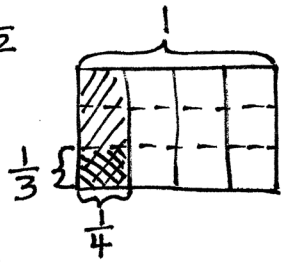
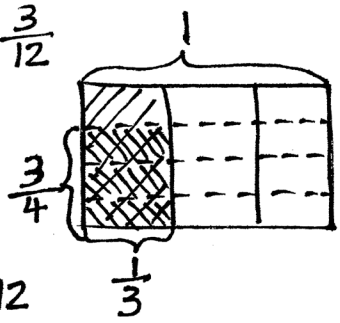
Name Seth Date

1. Multiply or divide. Draw a model to explain your thinking.

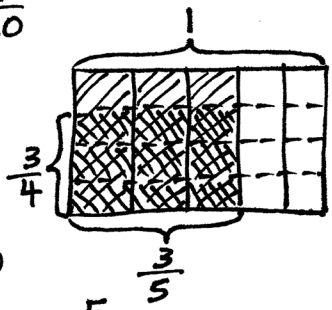
a. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



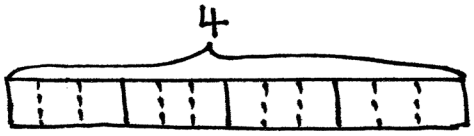
b. $\frac{3}{4}$ of $\frac{1}{3} = \frac{3}{12}$



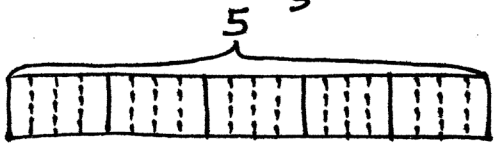
c. $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$



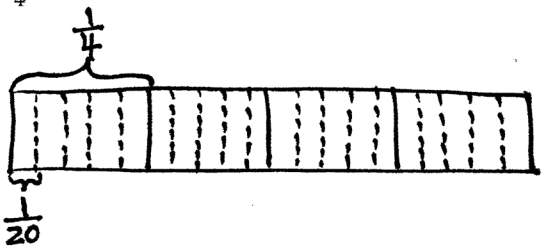
d. $4 \div \frac{1}{3} = 12$



e. $5 \div \frac{1}{4} = 20$



f. $\frac{1}{4} \div 5 = \frac{1}{20}$



2. Multiply or divide using any method.

a. $1.5 \times 32 = 48.0$

15 (tenths)
× 32
30
+ 450
480 (tenths)

b. 1.5×0.32

$= (1 \times 0.32) + (0.5 \times 0.32)$
$= 0.32 + 0.16$
$= 0.48$

c. $12 \div 0.03$

$= (12 \times 100) \div (0.03 \times 100)$
$= 1{,}200 \div 3$
$= 400$

d. $1.2 \div 0.3$

$= (1.2 \times 10) \div (0.3 \times 10)$
$= 12 \div 3$
$= 4$

e. $12.8 \times \frac{3}{4}$

$= (12 \times \frac{3}{4}) + (\frac{8}{10} \times \frac{3}{4})$
$= 9 + \frac{24}{40}$
$= 9\frac{24}{40} = 9\frac{3}{5}$

f. $102.4 \div 3.2$

$= (102.4 \times 10) \div (3.2 \times 10)$
$= 1{,}024 \div 32$
$= 32$

$$\begin{array}{r} 32 \\ 32\overline{)1{,}024} \\ -96 \\ \hline 64 \\ -64 \\ \hline 0 \end{array}$$

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3. Fill in the chart by writing an equivalent expression.

a.	One-fifth the sum of one-half and one-third	$\frac{1}{5} \times (\frac{1}{2} + \frac{1}{3})$
b.	Two and one-half times the sum of nine and twelve	$2\frac{1}{2} \times (9 + 12)$
c.	Twenty-four divided by the difference between $1\frac{1}{2}$ and $\frac{3}{4}$	$24 \div (1\frac{1}{2} - \frac{3}{4})$

4. A castle has to be guarded 24 hours a day. Five knights are ordered to split each day's guard duty equally. How long will each knight spend on guard duty in one day?

a. Record your answer in hours.

$$\begin{array}{r} 4.8 \\ 5\overline{)24.0} \\ -20 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Each knight will spend 4.8 hours on guard duty in one day.

b. Record it in hours and minutes.

$\frac{1}{10}$ of 60 min = 6 min

$\frac{8}{10}$ of 60 min = 48 min

4.8 hours = 4 hours 48 minutes

Each knight will spend 4 hours and 48 minutes on guard duty in one day.

c. Record your answer in minutes.

1 hour = 60 minutes

4.8 hour = ___ min.

= 4.8 × 1 hr

= 4.8 × 60 min

= 288.0 min

48 (tenths)
× 60
00
+2880
2,880 (tenths)

Each knight will spend 288 minutes on guard duty in one day.

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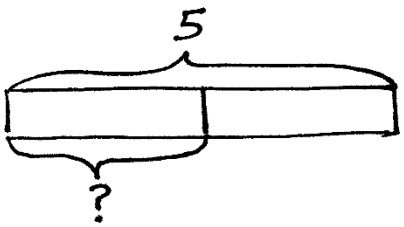
5. On the blank, write a division expression that matches the situation.

a. $5 \div 2$ Mark and Jada share 5 yards of ribbon equally. How much ribbon will each get?

b. $5 \div \frac{1}{2}$ It takes half of a yard of ribbon to make a bow. How many bows can be made with 5 yards of ribbon?

c. Draw a diagram for each problem and solve.

$5 \div 2 = 2\frac{1}{2}$

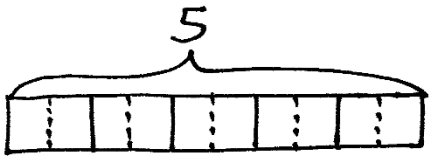


2 units = 5

1 unit = $5 \div 2$

$= \frac{5}{2} = 2\frac{1}{2}$

$5 \div \frac{1}{2} = 10$



d. Could either of the problems also be solved by using $\frac{1}{2} \times 5$? If so, which one(s)? Explain your thinking.

$5 \div 2 = 5 \times \frac{1}{2}$

Dividing by 2 is the same as taking $\frac{1}{2}$ of something, which means multiplying.

$\frac{1}{2} \times 5$ is the same as $5 \times \frac{1}{2}$.

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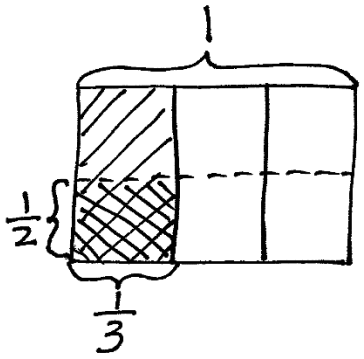
6. Jackson claims that multiplication always makes a number bigger. He gave the following examples:

   - If I take 6, and I multiply it by 4, I get 24, which is bigger than 6.
   - If I take $\frac{1}{4}$, and I multiply it by 2 (whole number), I get $\frac{2}{4}$, or $\frac{1}{2}$ which is bigger than $\frac{1}{4}$.

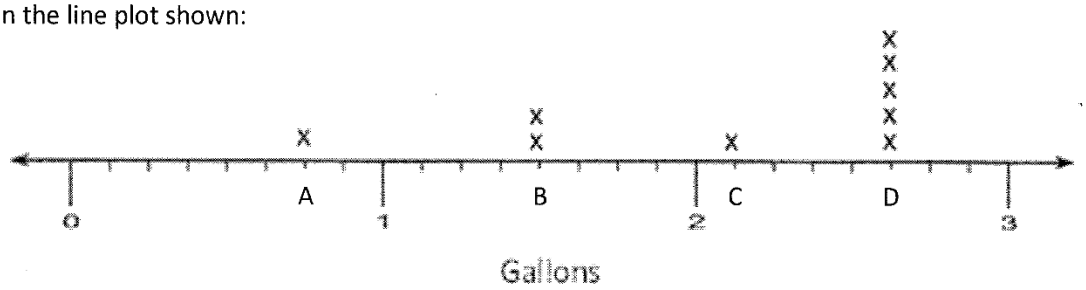
   Jackson's reasoning is incorrect. Give an example that proves he is wrong, and explain his mistake using pictures, words, or numbers.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$



$\frac{1}{6}$ is smaller than $\frac{1}{2}$ and $\frac{1}{3}$.

7. Jill collected honey from 9 different beehives, and recorded the amount collected, in gallons, from each hive in the line plot shown:



   a. She wants to write the value of each point marked on the number line above (Points A–D) in terms of the largest possible whole number of gallons, quarts, and pints. Use the line plot above to fill in the blanks with the correct conversions. (The first one is done for you.)

   A. 0 gal 3 qt 0 pt

   B. 1 gal 2 qt 0 pt

   C. 2 gal 0 qt 1 pt

   D. 2 gal 2 qt 1 pt

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b. Find the total amount of honey collected from the five hives that produced the most honey.

1 unit = $2\frac{5}{8}$ gallons

5 units = $5 \times 2\frac{5}{8}$ gallons

$= (5 \times 2) + (5 \times \frac{5}{8})$ gallons

$= 10 + \frac{25}{8}$ gallons

$= 10 + 3\frac{1}{8}$ gallons

$= 13\frac{1}{8}$ gallons

$13\frac{1}{8}$ gallons or 13 gallons and 1 pint were collected from the five hives that produced the most honey.

c. Jill collected a total of 19 gallons of honey. If she distributes all of the honey equally between 9 jars, how much honey will be in each jar?

$19 \div 9 = \frac{19}{9} = 2\frac{1}{9}$

There will be $2\frac{1}{9}$ gallons of honey in each jar.

d. Jill used $\frac{3}{4}$ of a jar for baking. How much honey did she use baking?

$\frac{3}{4}$ of $2\frac{1}{9}$ gallons

$= \frac{3}{4} \times \frac{19}{9}$ gallons

$= \frac{\overset{1}{\cancel{3}} \times 19}{4 \times \underset{3}{\cancel{9}}}$ gallons

$= \frac{19}{12} = 1\frac{7}{12}$ gallons

She used $1\frac{7}{12}$ gallons of honey for baking.

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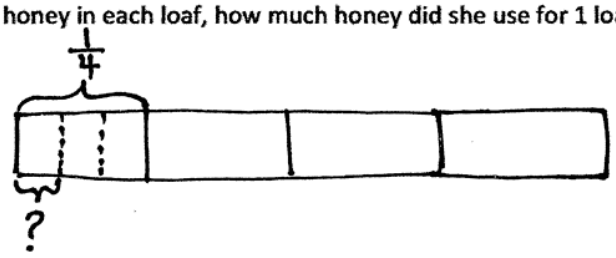
e. Jill's mom used $\frac{1}{4}$ of a gallon of honey to bake 3 loaves of bread. If she used an equal amount of honey in each loaf, how much honey did she use for 1 loaf?

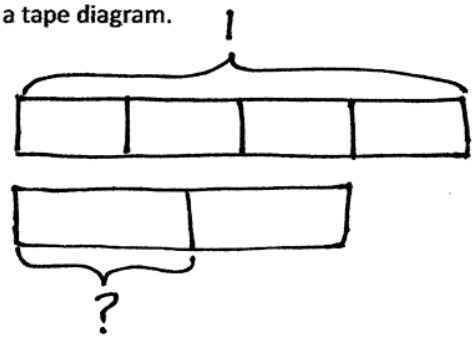


$\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

She used $\frac{1}{12}$ of a gallon of honey for 1 loaf.

f. Jill's mom stored some of the honey in a container that held $\frac{3}{4}$ of a gallon. She used half of this amount to sweeten tea. How much honey, in cups, was used in the tea? Write an equation and draw a tape diagram.



$\frac{1}{2} \times \frac{3}{4}$ gallon $= \frac{3}{8}$ gallon

$\frac{3}{8}$ gallon $= \frac{3}{8} \times 1$ gallon

$= \frac{3}{8} \times 16$ cups

$= \frac{3 \times 16^{2}}{8_{1}}$ cups

$= 6$ cups

She used 6 cups of honey in the tea.

g. Jill uses some of her honey to make lotion. If each bottle of lotion requires $\frac{1}{4}$ gallon, and she uses a total of 3 gallons, how many bottles of lotion does she make?

$3 \div \frac{1}{4} = 3 \times 4 = 12$

She makes 12 bottles of lotion.

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Answer Key

GRADE 5 • MODULE 4

Multiplication and Division of Fractions and Decimal Fractions

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Lesson 1

Problem Set

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. a. Answers will vary.
   b. Answers will vary.
   c. Answers will vary.

Exit Ticket

1. Line plot drawn correctly
2. Answers will vary.

Homework

Line plot drawn correctly

a. Location 6
b. Locations 1, 7, and 10
c. $\frac{1}{8}$ in
d. 5 in

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Lesson 2

Problem Set

1. a. Answer provided
   b. 12 fourths ÷ 4 = 3 fourths = $\frac{3}{4}$; picture representing 3 ÷ 4 drawn
   c. 24 fourths ÷ 4 = 6 fourths = $\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$; picture representing 6 ÷ 4 drawn
2. 3 ÷ 2 = $\frac{3}{2}$ or 6 halves ÷ 2 = 3 halves = $\frac{3}{2} = 1\frac{1}{2}$; picture representing 3 ÷ 2 drawn
3. a. Solutions illustrated correctly
   b. Answers will vary.
4. a. 2, 3
   b. 15, 8
   c. 11, 4
   d. 3, 2
   e. 9, 13
   f. 4, 3

Exit Ticket

1. a. 3 ÷ 9 = $\frac{3}{9} = \frac{1}{3}$ or 27 ninths ÷ 9 = 3 ninths = $\frac{3}{9} = \frac{1}{3}$; picture representing 3 ÷ 9 drawn
   b. 4 ÷ 3 = $\frac{4}{3} = 1\frac{1}{3}$ or 12 thirds ÷ 3 = 4 thirds = $\frac{4}{3} = 1\frac{1}{3}$; picture representing 4 ÷ 3 drawn
2. a. 21, 8
   b. 7, 4
   c. 4, 9
   d. 9, 7

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Homework

1. a. $\frac{1}{4}$; picture representing 1 ÷ 4 drawn
   b. $\frac{3}{5}$; picture representing 3 ÷ 5 drawn
   c. $\frac{7}{4}$ or $1\frac{3}{4}$; picture representing 7 ÷ 4 drawn
2. 4 ÷ 6 = $\frac{4}{6}$ = $\frac{2}{3}$ or 24 sixths ÷ 6 = 4 sixths = $\frac{4}{6}$ = $\frac{2}{3}$; picture representing 4 ÷ 6 drawn
3. a. 2, 7
   b. 39, 5
   c. 13, 3
   d. 9, 5
   e. 19, 28
   f. 8, 5

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Lesson 3

Problem Set

1. a. Answer provided
   b. 6, 3; $\frac{3}{2}$; algorithm completed correctly
   c. 6, 4; $\frac{6}{4}$; $1\frac{1}{2}$; algorithm completed correctly
   d. 10 halves ÷ 2 = 5 halves; algorithm completed correctly
2. a. $\frac{3}{4}$; answers will vary.
   b. 3; answers will vary.
3. a. 4; answers will vary.
   b. $\frac{4}{5}$

Exit Ticket

9 ÷ 4; 36 fourths ÷ 4 = 9 fourths; $\frac{9}{4} = 2\frac{1}{4}$; algorithm completed correctly; picture drawn representing 9 wholes, *or* 36 fourths, divided by 4

Homework

1. a. Answer provided
   b. 7, 5; 35, 7; $\frac{7}{5}$; algorithm completed correctly
   c. 7, 2; 14, 7; $\frac{7}{2}$; $3\frac{1}{2}$; algorithm completed correctly
   d. 28 fourths ÷ 4 = 7 fourths; $1\frac{3}{4}$; algorithm completed correctly
2. a. 3; explanations will vary.
   b. 7
3. a. 4; explanations will vary.
   b. $\frac{1}{2}$; 2

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Lesson 4

Problem Set

1. a. Answer provided
   b. $\frac{2}{3}$; tape diagram drawn correctly
   c. $1\frac{2}{5}$; tape diagram drawn correctly
   d. $2\frac{4}{5}$; tape diagram drawn correctly
2. a. Answer provided
   b. $\frac{6}{7}$; algorithm completed correctly
   c. 55, 10; 5 and 6; algorithm completed correctly
   d. 32, 40; 0 and 1; algorithm completed correctly
3. a. 80 cents
   b. 20 cents; explanations will vary
4. a. $\frac{1}{4}$
   b. $1\frac{1}{4}$; tape diagram drawn correctly
   c. 60 oz

Exit Ticket

$2\frac{1}{4}$; tape diagram drawn correctly

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Homework

1. a. Answer provided
   b. $\frac{4}{5}$; tape diagram drawn correctly
   c. $\frac{8}{5}$; tape diagram drawn correctly
   d. $\frac{14}{3}$; tape diagram drawn correctly
2. a. Answer provided
   b. 3, 4; algorithm completed correctly
   c. 7, 2; 3 and 4; althorithm completed correctly
   d. 81, 90; 0 and 1; algorithm completed correctly
3. a. $\frac{2}{5}$ yd; tape diagram drawn correctly
   b. $1\frac{1}{5}$ ft; tape diagram drawn correctly
4. $4\frac{2}{3}$ lb
5. $\frac{2}{3}$ lb; tape diagram drawn correctly

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Lesson 5

Problem Set

1. $\frac{2}{5}$ yd
2. $\frac{4}{6}$ or $\frac{2}{3}$ pt
3. $\frac{6}{4}$ or $1\frac{1}{2}$; tape diagram drawn showing 6 ÷ 4
4. a. $\frac{4}{8}$ or $\frac{1}{2}$
   b. $\frac{1}{8}$
5. $\frac{5}{40}$ or $\frac{1}{8}$
6. a. $\frac{4}{10}$ or $\frac{2}{5}$ L
   b. 0.4 L
   c. 400 mL
7. a. $4\frac{2}{3}$ mi; tape diagram drawn showing 14 ÷ 3
   b. 14 mi

Exit Ticket

a. $\frac{5}{9}$ yd; tape diagram drawn showing 5 ÷ 9

b. $1\frac{1}{9}$ yd

Homework

1. a. $3\frac{2}{4}$ or $3\frac{1}{2}$ gal
   b. $10\frac{1}{2}$ gal; explanations will vary.
   c. $7\frac{2}{4}$ or $7\frac{1}{2}$ sq. ft
   d. $\frac{1}{4}$
2. a. $\frac{1}{4}$; models will vary.
   b. $\frac{3}{4}$ ft
   c. 9 in
3. $7.50

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Lesson 6

Sprint

Side A

1.	$\frac{1}{2}$	12.	$1\frac{2}{5}$	23.	3	34.	$2\frac{1}{6}$
2.	$\frac{1}{3}$	13.	$1\frac{4}{5}$	24.	$3\frac{1}{2}$	35.	$4\frac{2}{7}$
3.	$\frac{1}{8}$	14.	$\frac{2}{3}$	25.	1	36.	$3\frac{3}{8}$
4.	1	15.	1	26.	$1\frac{1}{8}$	37.	$5\frac{4}{9}$
5.	$\frac{2}{3}$	16.	$1\frac{1}{4}$	27.	$1\frac{7}{8}$	38.	$4\frac{5}{6}$
6.	1	17.	$1\frac{3}{4}$	28.	2	39.	$6\frac{5}{7}$
7.	$\frac{3}{4}$	18.	2	29.	$2\frac{3}{4}$	40.	$6\frac{5}{8}$
8.	$\frac{3}{10}$	19.	$2\frac{1}{2}$	30.	$7\frac{1}{2}$	41.	$7\frac{4}{9}$
9.	$\frac{3}{5}$	20.	2	31.	$4\frac{4}{5}$	42.	$9\frac{5}{6}$
10.	1	21.	$2\frac{1}{5}$	32.	$4\frac{1}{4}$	43.	$7\frac{7}{8}$
11.	$1\frac{1}{5}$	22.	$2\frac{3}{5}$	33.	$6\frac{2}{3}$	44.	$7\frac{8}{9}$

Side B

1.	$\frac{1}{3}$	12.	1	23.	3	34.	$3\frac{1}{6}$
2.	$\frac{1}{4}$	13.	$1\frac{1}{2}$	24.	$3\frac{1}{5}$	35.	$4\frac{3}{7}$
3.	$\frac{1}{10}$	14.	$\frac{4}{5}$	25.	1	36.	$4\frac{5}{8}$
4.	1	15.	1	26.	$1\frac{1}{6}$	37.	$5\frac{5}{9}$
5.	$\frac{5}{6}$	16.	$1\frac{1}{10}$	27.	$1\frac{5}{6}$	38.	$2\frac{5}{6}$
6.	1	17.	$1\frac{3}{10}$	28.	2	39.	$6\frac{6}{7}$
7.	$\frac{3}{7}$	18.	2	29.	$2\frac{2}{3}$	40.	$6\frac{3}{8}$
8.	$\frac{3}{10}$	19.	$2\frac{1}{5}$	30.	$6\frac{1}{2}$	41.	$7\frac{5}{9}$
9.	$\frac{3}{4}$	20.	$2\frac{3}{5}$	31.	$4\frac{3}{5}$	42.	$8\frac{5}{6}$
10.	1	21.	2	32.	$3\frac{3}{4}$	43.	$7\frac{5}{8}$
11.	$1\frac{1}{4}$	22.	$2\frac{1}{2}$	33.	$4\frac{3}{4}$	44.	$7\frac{7}{9}$

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Problem Set

1. a. 3; 6; 9
   b. 5; 10; 15
   c. 4; 16; 5
   d. 3; 9; 12; 18; 21
2. 8; drawings will vary.
3. Explanations and pictures will vary.
4. 12
5. 15 ten dollar bills or $150

Exit Ticket

1. a. 4
   b. 12
2. 12

Homework

1. a. 4; 8; 12
   b. 5; 10; 15; 20
   c. 7; 14; 21; 28; 35; 42
2. 12; drawings will vary.
3. Explanations and pictures will vary.
4. 8
5. a. 2 dozen; 24 eggs
   b. $45

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Lesson 7

Problem Set

1. Tape diagrams drawn accurately
   a. 6
   b. 12
   c. 18
   d. 9
   e. 20
   f. 20
   g. $2\frac{1}{4}$
   h. $4\frac{4}{5}$
   i. 15
   j. 32

2. Tape diagrams drawn accurately
   a. 36
   b. 140 degrees
   c. $72
   d. $3\frac{1}{5}$ ounces

Exit Ticket

Tape diagrams drawn accurately

a. 18
b. 50
c. 16

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Homework

1. Tape diagrams drawn accurately
   a. 6
   b. 12
   c. 12
   d. 6
   e. 21
   f. 36
   g. $10\frac{1}{3}$
   h. 8
   i. $6\frac{1}{4}$
   j. $18\frac{3}{4}$
   k. 36
   l. 35

2. Tape diagrams drawn accurately
   a. 22
   b. 150 degrees
   c. $4\frac{2}{8}$ or $4\frac{1}{4}$ ounces
   d. 84

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Lesson 8

Problem Set

1. Explanations will vary.
2. Solution strategies will vary.
   a. $\frac{21}{4}$
   b. $\frac{28}{5}$
   c. $\frac{12}{7}$
3. Modeling will vary.
   a. 4
   b. 6
4. Solution strategies will vary.
   a. 6
   b. 27
   c. 39
   d. 36
5. a. 30
   b. 45
   c. 300
   d. 80

Exit Ticket

Solution strategies will vary.

a. 10
b. 15

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Homework

1. Solution strategies will vary.
   a. $\frac{15}{3}$ or 5
   b. $\frac{26}{5}$
   c. $\frac{27}{4}$
2. Solution strategies will vary.
   a. 12
   b. 16
   c. 44
   d. 42
   e. 15
   f. $7\frac{1}{2}$
   g. $23\frac{1}{3}$
3. a. 20
   b. 48
   c. 700
   d. 60

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Lesson 9

Problem Set

1. Explanations will vary.
   a. Answer provided
   b. 4
   c. 10
   d. 80
   e. 40
   f. 27
2. 3
3. a. 14 oz
   b. 12 oz
   c. 2 oz
   d. Mr. Paul; 2 oz
4. $12\frac{1}{2}$ in

Exit Ticket

1. $\frac{3}{5}$
2. a. 8
   b. 40
   c. 10

Homework

1. Explanations will vary.
   a. Answer provided
   b. 2
   c. 9
   d. 60
   e. 25
   f. 24
2. 9 in
3. a. 12 oz
   b. 10 oz
   c. 2 oz
   d. Mr. Phillips; 6 oz
4. $6\frac{1}{4}$ lb; $3\frac{3}{4}$ lb

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Lesson 10

Problem Set

1. $5\frac{2}{3}$; expressions will vary.
   $3\frac{3}{10}$; expressions will vary.
2. a. 6; expressions will vary.
   b. $2\frac{2}{3}$; expressions will vary.
   c. $3\frac{1}{4}$; expressions will vary.
   d. 14; expressions will vary.
   e. $26\frac{2}{3}$; expressions will vary.
   f. $10\frac{2}{3}$; expressions will vary.
3. $(4 \times 7) \div 5$, $4 \times \frac{7}{5}$, and $7 \times \frac{4}{5}$ circled; explanations will vary.
4. a. >; explanations will vary.
   b. >; explanations will vary.
   c. >; explanations will vary.
5. a. $\frac{1}{2}$ gallon; expressions will vary.
   b. $3\frac{3}{4}$ gallons; expressions will vary.
   c. $1\frac{1}{2}$ gallons; expressions will vary.
   d. Data accurately displayed on line plot
   e. 17 gallons

Exit Ticket

1. a. Expressions will vary.
   b. Expressions will vary.
2. 3; expressions will vary.

Homework

1. $5\frac{1}{4}$; expressions will vary.
   $6\frac{10}{21}$; expressions will vary.
2. $(6 \times 3) \div 8$ and $\frac{3}{8} \times 6$ circled; explanations will vary.
3. a. 5; expressions will vary.
   b. 3; expressions will vary.
   c. $7\frac{14}{15}$; expressions will vary.
   d. 4; expressions will vary.
   e. $39\frac{1}{5}$; expressions will vary.
   f. 36; expressions will vary.
4. a. >; explanations will vary.
   b. >; explanations will vary.
   c. >; explanations will vary.
5. a. $2\frac{1}{4}$; expressions will vary.
   b. $1\frac{3}{4}$; expressions will vary.
   c. $3\frac{1}{4}$; expressions will vary.
   d. Line plot accurately drawn
   e. $19\frac{3}{8}$; expressions will vary.

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Lesson 11

Problem Set

1. $\frac{3}{8}$
2. 8 pt
3. 68 oz
4. $2\frac{1}{2}$
5. Answers will vary.

Exit Ticket

$3\frac{1}{3}$; tape diagram drawn accurately

Homework

1. 25 min
2. $4\frac{3}{4}$ pt
3. $17\frac{3}{4}$ lb
4. $3\frac{1}{2}$
5. Answers will vary.

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Lesson 12

Problem Set

1. a. $\frac{3}{4}$
   b. 17
2. 16
3. Lillian; 26 min; bonus: $\frac{13}{30}$ hr
4. 2; story problems will vary.
5. 6; story problems will vary.
6. 12 in Mr. Smith's class; 10 in Mrs. Jacobs's class

Exit Ticket

6

Homework

1. 16 min
2. $\frac{21}{56}$ or $\frac{3}{8}$
3. 12
4. Jacob; bonus: $\frac{1}{6}$ min
5. 4; story problems will vary.
6. 4; story problems will vary.

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Lesson 13

Problem Set

1. Accurate area model drawn
   a. Answer provided
   b. $\frac{1}{6}; \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
   c. $\frac{1}{12}; \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$
   d. $\frac{1}{16}; \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
   e. $\frac{1}{12}; \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
2. $3 \times \frac{1}{4} = \frac{3}{4}; \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$; comparison statements will vary.
3. $\frac{1}{6}$; accurate picture drawn
4. $\frac{1}{10}$
5. $\frac{1}{12}$

Exit Ticket

1. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$; accurate area model drawn
2. $\frac{1}{12}$

Homework

1. Accurate area models drawn
   a. $\frac{1}{4}$
   b. $\frac{1}{6}$
   c. $\frac{1}{8}$
   d. $\frac{1}{10}$
   e. $\frac{1}{9}$
   f. $\frac{1}{12}$
2. $\frac{1}{10}$; accurate picture drawn
3. $\frac{1}{6}$; accurate picture drawn
4. $\frac{1}{20}$; accurate area model drawn

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Lesson 14

Sprint

Side A[1]

1.	2 fifths	12.	2	23.	60 sixths	34.	90 sixths
2.	3 fifths	13.	4 halves	24.	15 thirds	35.	24 fourths
3.	4 fifths	14.	2	25.	30 thirds	36.	72 fourths
4.	4 fifths	15.	6 thirds	26.	30 thirds	37.	32 eighths
5.	3 eighths	16.	2	27.	15 fifths	38.	96 eighths
6.	5 eighths	17.	10 fifths	28.	30 fifths	39.	160 eighths
7.	7 eighths	18.	9 thirds	29.	60 fifths	40.	224 eighths
8.	7 eighths	19.	18 thirds	30.	45 fifths	41.	270 ninths
9.	3 tenths	20.	8 fourths	31.	45 fifths	42.	441 ninths
10.	7 tenths	21.	24 fourths	32.	18 sixths	43.	168 sevenths
11.	7 tenths	22.	12 sixths	33.	90 sixths	44.	294 sevenths

Side B[1]

1.	2 sevenths	12.	2	23.	24 fourths	34.	120 sixths
2.	3 sevenths	13.	10 fifths	24.	15 fifths	35.	20 fourths
3.	4 sevenths	14.	3	25.	30 fifths	36.	60 fourths
4.	4 sevenths	15.	9 thirds	26.	60 fifths	37.	24 eighths
5.	3 tenths	16.	5	27.	45 fifths	38.	72 eighths
6.	7 tenths	17.	10 halves	28.	45 fifths	39.	120 eighths
7.	9 tenths	18.	6 thirds	29.	15 thirds	40.	168 eighths
8.	9 tenths	19.	12 thirds	30.	30 thirds	41.	315 ninths
9.	3 eighths	20.	12 sixths	31.	30 thirds	42.	378 ninths
10.	5 eighths	21.	60 sixths	32.	24 sixths	43.	147 sevenths
11.	5 eighths	22.	8 fourths	33.	120 sixths	44.	336 sevenths

[1] Note: Answers are given here in unit form for ease of reading. Students may answer in standard form.

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Problem Set

1. Accurate models drawn and number sentences provided
   a. 3; 1
   b. 4; 2
   c. $\frac{1}{2}$
   d. $\frac{1}{3}$
   e. $\frac{3}{10}$
   f. $\frac{1}{6}$
2. $\frac{1}{8}$; accurate tape diagram drawn
3. a. $\frac{1}{5}$
   b. $\frac{1}{5}$
4. Explanations may vary; accurate picture drawn

Exit Ticket

1. $\frac{1}{7}$; accurate model drawn and number sentence provided
2. $\frac{3}{8}$

Homework

1. Accurate models drawn and number sentences provided
   a. 2; 1
   b. 4; 2
   c. $\frac{1}{5}$
   d. $\frac{3}{8}$
   e. $\frac{4}{15}$
   f. $\frac{4}{15}$
2. $\frac{1}{7}$; accurate model drawn
3. a. $\frac{1}{5}$
   b. $\frac{1}{15}$
4. a. All grandchildren received the same amount; explanations may vary; accurate picture drawn
   b. $\frac{1}{5}$

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Lesson 15

Problem Set

1. a. Answer provided
   b. $\frac{3}{5}$; accurate model drawn and multiplication sentence provided
   c. $\frac{4}{15}$; accurate model drawn and multiplication sentence provided
   d. $\frac{8}{15}$; accurate model drawn and multiplication sentence provided
   e. $\frac{1}{2}$; accurate model drawn and multiplication sentence provided
2. a. $\frac{5}{8}$
   b. $\frac{1}{2}$
   c. $\frac{4}{7}$
   d. $\frac{2}{15}$
3. $\frac{4}{10}$ or $\frac{2}{5}$
4. $\frac{1}{2}$ lb
5. a. $\frac{1}{3}$
   b. 2 tons

Exit Ticket

1. a. $\frac{2}{5}$; accurate model drawn and multiplication sentence provided
   b. $\frac{1}{6}$; accurate model drawn and multiplication sentence provided
2. $\frac{3}{20}$

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Homework

1. a. $\frac{1}{2}$; accurate model drawn and multiplication sentence provided
   b. $\frac{3}{10}$; accurate model drawn and multiplication sentence provided
   c. $\frac{8}{25}$; accurate model drawn and multiplication sentence provided
   d. $\frac{3}{5}$; accurate model drawn and multiplication sentence provided
2. a. $\frac{1}{4}$
   b. $\frac{3}{5}$
   c. $\frac{25}{48}$
   d. $\frac{5}{16}$
   e. $\frac{16}{27}$
   f. $\frac{2}{21}$
3. a. $\frac{1}{2}$
   b. 250 mL
4. $\frac{1}{2}$
5. a. $\frac{1}{10}$
   b. $\frac{1}{2}$ lb

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Lesson 16

Problem Set

1. 5; accurate tape diagram drawn
2. 12; accurate tape diagram drawn
3. 90; accurate tape diagram drawn
4. 36; accurate tape diagram drawn
5. $40; accurate tape diagram drawn
6. 12; accurate tape diagram drawn
7. $12; accurate tape diagram drawn

Exit Ticket

84; accurate tape diagram drawn

Homework

1. 8; accurate tape diagram drawn
2. Accurate tape diagrams drawn
   a. 180
   b. 60
   c. 313
   d. Less than half
   e. 126

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Lesson 17

Problem Set

1. a. Answer provided
   b. $0.4 \times 0.3 = 0.12$; accurate area model
   c. $0.1 \times 1.4 = 0.14$; accurate area model
   d. $0.6 \times 1.7 = 1.02$; accurate area model
2. a. 3.5
   b. 0.35; $\frac{35}{100}$; 0.35
   c. 0.035; 5, 7; $\frac{35}{1000}$; 0.035
   d. 1.8
   e. 0.18
   f. 0.018
   g. 4.8
   h. 0.48
   i. 0.048
3. 0.14 m
4. a. 1.5 mi
   b. 1.75 mi

Exit Ticket

1. $0.1 \times 1.2 = 0.12$; accurate area model
2. a. 4.5
   b. 0.45
   c. 0.045

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Homework

1. a. Answer provided
   b. $0.6 \times 0.2 = 0.12$; accurate area model
   c. $0.1 \times 1.6 = 0.16$; accurate area model
   d. $0.6 \times 1.9 = 1.14$; accurate area model
2. a. 2.4
   b. 0.24; $\frac{24}{100}$; 0.24
   c. 0.024; 4, 6; $\frac{24}{1000}$; 0.024
   d. 2.1
   e. 0.21
   f. 0.021
   g. 6.5
   h. 0.65
   i. 0.065
3. 0.51 L
4. a. 1.44 mi
   b. 3.46 mi

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Lesson 18

Sprint

Side A[1]

1.	1 fourth	12.	4 fifteenths	23.	10 fifteenths	34.	15 twentieths
2.	1 sixth	13.	1 twelfth	24.	15 tenths	35.	18 twentieths
3.	1 eighth	14.	2 twelfths	25.	1 ninth	36.	6 twentieths
4.	1 fourteenth	15.	6 twelfths	26.	2 ninths	37.	1 forty-ninth
5.	1 fourteenth	16.	1 eighteenth	27.	4 ninths	38.	3 fortieths
6.	1 sixth	17.	5 eighteenths	28.	6 sixths	39.	5 twenty-fourths
7.	1 ninth	18.	10 eighteenths	29.	8 ninths	40.	9 sixteenths
8.	1 eighteenth	19.	10 twelfths	30.	10 ninths	41.	12 eighteenths
9.	1 fifteenth	20.	1 twenty-fifth	31.	9 tenths	42.	18 eighths
10.	1 fifteenth	21.	4 twenty-fifths	32.	3 twentieths	43.	49 seventy-seconds
11.	2 fifteenths	22.	6 twenty-fifths	33.	12 twentieths	44.	63 ninety-sixths

Side B[1]

1.	1 sixth	12.	4 fifteenths	23.	15 twentieths	34.	10 fifteenths
2.	1 eighth	13.	1 twelfth	24.	20 fifteenths	35.	12 fifteenths
3.	1 tenth	14.	3 twelfths	25.	1 sixteenth	36.	6 fifteenths
4.	1 eighteenth	15.	6 twelfths	26.	3 sixteenths	37.	1 eighty-first
5.	1 eighteenth	16.	1 eighteenth	27.	9 sixteenths	38.	3 fortieths
6.	1 tenth	17.	2 eighteenths	28.	12 twelfths	39.	3 twenty-fourths
7.	1 fifteenth	18.	10 eighteenths	29.	15 sixteenths	40.	4 ninths
8.	1 thirty-fifth	19.	9 eighths	30.	18 sixteenths	41.	24 thirty-seconds
9.	1 fifteenth	20.	1 twenty-fifth	31.	16 eighteenths	42.	12 ninths
10.	1 fifteenth	21.	9 twenty-fifths	32.	2 fifteenths	43.	48 sixty-thirds
11.	2 fifteenths	22.	12 twenty-fifths	33.	8 fifteenths	44.	56 eighty-fourths

[1]Note: Answers written in unit form for ease of reading, but students may express answers in standard form.

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Problem Set

1. a. Answer provided
   b. 2.07; 207 hundredths
   c. 18.48; 1,848 hundredths
   d. 4.62; 462 hundredths
2. a. Answer provided
   b. 2.133; 2,133 thousandths
   c. 16.968; 16,968 thousandths
   d. 0.462; 462 thousandths
3. a. Answer provided
   b. 3.84; 384 hundredths
   c. 19.944; 19,944 thousandths
   d. 26.25; 2,625 hundredths *or* 26.250; 26,250 thousandths
4. $4.44
5. a. 15.75 sq. m
   b. 39.375 sq. m

Exit Ticket

a. 4.48
b. 1.12
c. 8.484
d. 0.924

Homework

1. a. Answer provided
   b. 2.64; 264 hundredths
   c. 14.08; 1,408 hundredths
   d. 3.52; 352 hundredths
2. a. 4,704 thousandths
   b. 2.345; 2,345 thousandths
   c. 12.928; 12,928 thousandths
   d. 0.704; 704 thousandths
3. a. Answer provided
   b. 4.83; 483 hundredths
   c. 25.194; 25,194 thousandths
   d. 29.25; 2,925 hundredths *or* 29.250; 29,250 thousandths
4. $19.25
5. a. 70.2 sq. m
   b. 175.5 sq. m

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Lesson 19

Problem Set

1. a. Answer provided
   b. $1\frac{1}{3}; \frac{1}{3}; \frac{4}{3}$
   c. $\frac{7}{12}$
   d. $1\frac{1}{12}$
   e. $\frac{5}{16}$
   f. $1\frac{2}{16}$ or $1\frac{1}{8}$

2. a. $\frac{24}{36}$ or $\frac{2}{3}$ yd
   b. $4

3. $1\frac{3}{16}$ lb

4. $\frac{14}{16}$ or $\frac{7}{8}$ gal

Exit Ticket

a. $\frac{5}{12}$
b. $1\frac{1}{12}$
c. $\frac{9}{16}$
d. $1\frac{2}{16}$ or $1\frac{1}{8}$

Homework

1. a. Answer provided
   b. $2; \frac{1}{3}; \frac{6}{3}$
   c. $\frac{5}{12}$
   d. $1\frac{2}{12}$ or $1\frac{1}{6}$
   e. $\frac{7}{16}$
   f. $1\frac{4}{16}$ or $1\frac{1}{4}$
   g. $\frac{1}{2}$
   h. 2

2. a. $\frac{12}{16}$ or $\frac{3}{4}$ lb
   b. $3

3. $1\frac{5}{16}$ lb

4. 3 gal

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Lesson 20

Problem Set

1. a. Answer provided
   b. $\frac{3}{8}$
   c. 56
   d. $4\frac{3}{4}$
   e. 216
   f. $1\frac{2}{9}$
2. $4\frac{1}{2}$ t
3. 37 qt
4. $15\frac{5}{12}$ yd

Exit Ticket

a. 26
b. $1\frac{1}{4}$
c. $1\frac{1}{4}$
d. 44

Homework

1. a. Answer provided
   b. $\frac{5}{12}$
   c. 46
   d. $3\frac{3}{4}$
   e. 258
   f. $2\frac{3}{4}$
2. $2\frac{3}{4}$ min
3. $\frac{1}{4}$ lb
4. Yes, because the package weighs 15 lb

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Lesson 21

Sprint

Side A

1.	6	12.	0.15	23.	1.2	34.	21
2.	0.6	13.	14	24.	0.12	35.	0.24
3.	0.06	14.	1.4	25.	0.012	36.	24
4.	9	15.	0.14	26.	0.012	37.	4.2
5.	0.9	16.	12	27.	35	38.	0.49
6.	0.09	17.	1.2	28.	3.5	39.	0.048
7.	8	18.	1.2	29.	0.35	40.	0.054
8.	0.8	19.	0.12	30.	0.035	41.	4.8
9.	0.08	20.	0.012	31.	0.035	42.	0.63
10.	15	21.	0.012	32.	16	43.	0.064
11.	1.5	22.	12	33.	1.8	44.	0.072

Side B

1.	8	12.	0.12	23.	1.6	34.	24
2.	0.8	13.	18	24.	0.16	35.	0.27
3.	0.08	14.	1.8	25.	0.016	36.	32
4.	6	15.	0.18	26.	0.016	37.	4.2
5.	0.6	16.	15	27.	45	38.	0.36
6.	0.06	17.	1.5	28.	4.5	39.	0.048
7.	9	18.	1.5	29.	0.45	40.	0.054
8.	0.9	19.	0.15	30.	0.045	41.	4.8
9.	0.09	20.	0.015	31.	0.045	42.	0.63
10.	12	21.	0.015	32.	12	43.	0.049
11.	1.2	22.	16	33.	1.4	44.	0.072

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Problem Set

1. a. Answer provided
   b. $\frac{7}{7}$
   c. $\frac{5}{5}$
   d. Answers will vary.
2. a. $\frac{25}{100} = 0.25$
   b. $\frac{75}{100} = 0.75$
   c. $\frac{1}{5} \times \frac{20}{20} = \frac{20}{100} = 0.20$
   d. $\frac{4}{5} \times \frac{20}{20} = \frac{80}{100} = 0.80$
   e. $\frac{1}{20} \times \frac{5}{5} = \frac{5}{100} = 0.05$
   f. $\frac{27}{20} \times \frac{5}{5} = \frac{135}{100} = 1.35$
   g. $\frac{7}{4} \times \frac{25}{25} = \frac{175}{100} = 1.75$
   h. $\frac{8}{5} \times \frac{20}{20} = \frac{160}{100} = 1.60$
   i. $\frac{24}{25} \times \frac{4}{4} = \frac{96}{100} = 0.96$
   j. $\frac{93}{50} \times \frac{2}{2} = \frac{186}{100} = 1.86$
   k. $2\frac{6}{25} \times \frac{4}{4} = 2\frac{24}{100} = 2.24$
   l. $3\frac{31}{50} \times \frac{2}{2} = 3\frac{62}{100} = 3.62$
3. No; answers will vary.
4. Answers will vary.
5. $\frac{1}{8} = \frac{1}{2 \times 2 \times 2} \times \frac{5 \times 5 \times 5}{5 \times 5 \times 5} = \frac{5 \times 5 \times 5}{(2 \times 5) \times (2 \times 5) \times (2 \times 5)}$
   $= \frac{125}{1000} = 0.125$; $\frac{1}{4} = 0.25 = 0.250$
   = 250 thousandths; $\frac{1}{8}$ is half of $\frac{1}{4}$, and half of 250 thousandths is 125 thousandths, so $\frac{1}{8} = 0.125$.

Exit Ticket

1. $\frac{5}{5}$
2. a. $\frac{1}{4} \times \frac{25}{25} = \frac{25}{100} = 0.25$
   b. $\frac{2}{5} \times \frac{2}{2} = \frac{4}{10} = 0.40$
   c. $\frac{3}{25} \times \frac{4}{4} = \frac{12}{100} = 0.12$
   d. $\frac{5}{20} \times \frac{5}{5} = \frac{25}{100} = 0.25$

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Homework

1. a. 3
   b. $\frac{7}{7}$
   c. $\frac{5}{5}$; 10
   d. Answers will vary.
2. a. 75; 0.75
   b. $\frac{25}{100} = 0.25$
   c. $\frac{2}{5} \times \frac{2}{2} = \frac{4}{10} = 0.4$
   d. $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10} = 0.6$
   e. $\frac{3}{20} \times \frac{5}{5} = \frac{15}{100} = 0.15$
   f. $\frac{25}{20} \times \frac{5}{5} = \frac{125}{100} = 1.25$
   g. $\frac{23}{25} \times \frac{4}{4} = \frac{92}{100} = 0.92$
   h. $\frac{89}{50} \times \frac{2}{2} = \frac{178}{100} = 1.78$
   i. $3\frac{11}{25} \times \frac{4}{4} = 3\frac{44}{100} = 3.44$
   j. $5\frac{41}{50} \times \frac{2}{2} = 5\frac{82}{100} = 5.82$
3. $\frac{6}{8} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0.75$
4. Answers will vary.
5. \$0.31 or 31 cents

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Lesson 22

Problem Set

1. a. $\frac{1}{2} \times 8 = 4$; $\frac{1}{2}$ circled, 8 boxed; 4
   b. $8 \times \frac{1}{2} = 4$; 8 circled, $\frac{1}{2}$ boxed; 4
2. a. Accurate tape diagram shown
   b. Accurate tape diagram shown
3. a. Any number less than 4
   b. Any number less than 7
   c. 5
4. a. Any fraction greater than 1; answers will vary.
   b. Any fraction less than 1; answers will vary.
5. Answers will vary.
6. 25.5 in
7. $\frac{1}{2}$ ft by $\frac{1}{4}$ ft; $1\frac{1}{3}$ ft by $1\frac{1}{6}$ ft

Exit Ticket

a. Any number greater than 3; answers will vary.
b. Any number less than 8; answers will vary.
c. 2

Homework

1. a. $\frac{1}{3} \times 6 = 2$; $\frac{1}{3}$ circled, 6 boxed; 2
   b. $6 \times \frac{1}{3} = 2$; 6 circled, $\frac{1}{3}$ boxed; 2
2. a. Accurate tape diagram shown
   b. Accurate tape diagram shown
3. a. Any number greater than 3; answers will vary.
   b. Any number greater than 6; answers will vary.
   c. 5
4. a. Any fraction greater than 1; answers will vary
   b. Any fraction less than 1; answers will vary.
5. a. Any number less than $\frac{1}{3}$
   b. Explanations will vary
6. 17 yd
7. $\frac{1}{6}$ ft by $\frac{1}{4}$ ft; $\frac{1}{2}$ ft by $\frac{1}{3}$ ft

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Lesson 23

Problem Set

1. a. 1.00
   b. 1.021
   c. 0.989
2. a. Less: 602 × 0.489, 0.3 × 0.069, 0.2 × 0.1
   Greater: 13.89 × 1.004, 102.03 × 4.015, 0.72 × 1.24
   b. Answers will vary.
3. a. Is slightly less than; explanations will vary.
   b. Is slightly more than; explanations will vary.
   c. Is a lot less than; explanations will vary.
   d. Is slightly more than; explanations will vary.
   e. Is slightly less than; explanations will vary.
4. Dhakir's is longest; Carson's is shortest; explanations will vary.
5. Greater than 1; examples will vary.
   Less than 1; examples will vary.

Exit Ticket

1. a. 0.898
   b. 1.00
   c. 1.009
2. Slightly less; explanations will vary.

Homework

1. a. Less: 828 × 0.921, 0.05 × 0.1
   Greater: 12.5 × 1.989, 321.46 × 1.26, 0.007 × 1.02, 2.16 × 1.11
   b. Explanations will vary.
2. a. Is slightly less than; explanations will vary.
   b. Is slightly more than; explanations will vary.
   c. Is a lot less than; explanations will vary.
   d. Is slightly more than; explanations will vary.
   e. Is slightly less than; explanations will vary.
3. Kayla, Jonathan, Rachel; explanations will vary.
4. a. Greater than 1; examples will vary.
   b. Less than 1; examples will vary.

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Lesson 24

Problem Set

1. 2.5 mL
2. $\frac{21}{40}$ or 0.525 L
3. 20.25 min
4. 4,590.72 m
5. 20
6. $266

Exit Ticket

1. 3.725 kg
2. 10

Homework

1. 14.375 lb
2. 0.225 cm
3. 38 mi
4. $215,942.65
5. 108
6. $142.60

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Lesson 25

Problem Set

1. a. 8; 2; 8; 8; accurate tape diagram drawn
   b. 8; 4; 8; 8; accurate tape diagram drawn
   c. 15; 3; 15; 15; accurate tape diagram drawn
   d. 15; 5; 15; 15; accurate tape diagram drawn
2. Accurate check shown for each
   a. 10
   b. 6
   c. 20
   d. 6
   e. 16
   f. 42
   g. 24
   h. 36
3. 20
4. a. 18
   b. 60
   c. 32
5. 12 gal

Exit Ticket

1. a. 10; 2; 10; 10; accurate tape diagram drawn
   b. 16; 4; 16, 4; 16; accurate tape diagram drawn
2. Yes

Homework

1. a. 9; 3; 9, 9; accurate tape diagram drawn
   b. 12; 4; 12, 3; 12; accurate tape diagram drawn
   c. 12; 3; 12, 4; 12; accurate tape diagram drawn
   d. 20; 4; 20, 5; 20; accurate tape diagram drawn
2. Accurate check shown for each
   a. 8
   b. 12
   c. 20
   d. 40
   e. 18
   f. 18
   g. 30
   h. 60
3. 24
4. 24 bags of nuts, 20 bags of cherries, and 24 bags of dried fruit

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Lesson 26

Problem Set

1. Model or tape diagrams drawn
   a. $\frac{1}{6}$
   b. $\frac{1}{12}$
   c. $\frac{1}{8}$
   d. $\frac{1}{12}$
2. Accurate checks shown
   a. $\frac{1}{14}$
   b. $\frac{1}{18}$
   c. $\frac{1}{20}$
   d. $\frac{1}{20}$
   e. $\frac{1}{10}$
   f. $\frac{1}{18}$
   g. $\frac{1}{16}$
   h. $\frac{1}{100}$
3. $\frac{1}{4}$; picture drawn
4. a. $\frac{1}{16}$ gal
   b. 1 c
5. a. $\frac{1}{12}$
   b. $28.80

Exit Ticket

1. Model or tape diagram drawn for each
   a. $\frac{1}{8}$
   b. $\frac{1}{40}$
2. $\frac{1}{12}$

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Homework

1. Model or tape diagrams drawn
   a. $\frac{1}{8}$
   b. $\frac{1}{18}$
   c. $\frac{1}{12}$
   d. $\frac{1}{10}$
2. Accurate checks shown
   a. $\frac{1}{20}$
   b. $\frac{1}{40}$
   c. $\frac{1}{15}$
   d. $\frac{1}{15}$
   e. $\frac{1}{32}$
   f. $\frac{1}{21}$
   g. $\frac{1}{50}$
   h. $\frac{1}{100}$
3. $\frac{1}{16}$ mile
4. a. $\frac{2}{15}$
   b. 105 pages

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Lesson 27

Problem Set

1. 12; accurate model shown
2. $\frac{1}{12}$; accurate model shown
3. a. $\frac{1}{20}$ m; accurate model shown
   b. 5 cm
4. a. $\frac{1}{20}$ t; accurate model shown
   b. 100 lb
5. a. 30 sixths
   b. 6 in
6. a. $\frac{1}{16}$
   b. 48 fl oz
   c. $3\frac{15}{16}$ c

Exit Ticket

1. 12 fourths; accurate model shown
2. $\frac{1}{8}$; accurate model shown

Homework

1. 32; accurate model shown
2. $\frac{1}{24}$; accurate model shown
3. a. $\frac{1}{20}$ L; accurate model shown
   b. 50 mL
4. a. 20 fifths
   b. 20 cm
5. a. $\frac{1}{12}$
   b. 72 oz
   c. 3 lb

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Lesson 28

Problem Set

1. Answers will vary; 20
2. Answers will vary; $\frac{1}{20}$
3. a. Answers will vary; 6
   b. Answers will vary; $\frac{1}{12}$
   c. Answers will vary; $\frac{1}{12}$
   d. Answers will vary; 15

Exit Ticket

a. Answers will vary; 8

b. Answers will vary; $\frac{1}{8}$

Homework

1. Answers will vary; 14
2. Answers will vary; $\frac{1}{9}$ lb
3. a. Answers will vary; 8
   b. Answers will vary; $\frac{1}{8}$
   c. Answers will vary; $\frac{1}{15}$
   d. Answers will vary; 30

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Lesson 29

Problem Set

1. a. $5 \div \frac{1}{10} = 50$; 10; 50
   b. $8 \div \frac{1}{10} = 80$; 10; 80
   c. $5.2 \div \frac{1}{10} = 52$; 50; 2; 52
   d. $8.7 \div \frac{1}{10} = 87$; 80; 7; 87
   e. $5 \div \frac{1}{100} = 500$; 100; 500
   f. $8 \div \frac{1}{100} = 800$; 100; 800
   g. $5.2 \div \frac{1}{100} = 520$; 500; 20; 520
   h. $8.7 \div \frac{1}{100} = 870$; 800; 70; 870

2. a. 60
   b. 180
   c. 600
   d. 17
   e. 3,100
   f. 1,100
   g. 1,250
   h. 374
   i. 1,250
3. 46
4. Cheryl; answers will vary.
5. 20

Exit Ticket

1. 83; 830
2. 2,800; 280
3. 1,509
4. 2,674
5. 63,298

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Homework

1. a. $9 \div \frac{1}{10} = 90$; 10; 90
   b. $6 \div \frac{1}{10} = 60$; 10; 60
   c. $3.6 \div \frac{1}{10} = 36$; 30; 6; 36
   d. $12.8 \div \frac{1}{10} = 128$; 120; 8; 128
   e. $3 \div \frac{1}{100} = 300$; 100; 300
   f. $7 \div \frac{1}{100} = 700$; 100; 700
   g. $4.7 \div \frac{1}{100} = 470$; 400; 70; 470
   h. $11.3 \div \frac{1}{100} = 1{,}130$; 1,100; 30; 1,130

2. a. 20
   b. 230
   c. 500
   d. 72
   e. 5,100
   f. 310
   g. 2,310
   h. 437
   i. 2,450

3. 1,260
4. Geraldine; answers will vary.
5. $132.64

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Lesson 30

Sprint

Side A

1.	$\frac{1}{4}$	12.	35	23.	16	34.	18
2.	$\frac{1}{6}$	13.	$\frac{1}{35}$	24.	$\frac{1}{9}$	35.	36
3.	$\frac{1}{8}$	14.	$\frac{1}{6}$	25.	$\frac{2}{9}$	36.	49
4.	$\frac{1}{14}$	15.	6	26.	$\frac{1}{8}$	37.	64
5.	14	16.	$\frac{1}{8}$	27.	$\frac{3}{8}$	38.	81
6.	12	17.	8	28.	$\frac{1}{10}$	39.	$\frac{1}{56}$
7.	10	18.	$\frac{1}{10}$	29.	$\frac{3}{10}$	40.	72
8.	6	19.	10	30.	$\frac{1}{12}$	41.	$\frac{1}{56}$
9.	10	20.	12	31.	$\frac{5}{12}$	42.	42
10.	15	21.	$\frac{1}{12}$	32.	$\frac{5}{18}$	43.	63
11.	20	22.	$\frac{1}{16}$	33.	$\frac{1}{18}$	44.	$\frac{1}{72}$

Side B

1.	$\frac{1}{4}$	12.	14	23.	9	34.	30
2.	$\frac{1}{15}$	13.	$\frac{1}{14}$	24.	$\frac{1}{16}$	35.	24
3.	$\frac{1}{20}$	14.	$\frac{1}{8}$	25.	$\frac{3}{16}$	36.	42
4.	$\frac{1}{35}$	15.	8	26.	$\frac{1}{9}$	37.	56
5.	35	16.	$\frac{1}{6}$	27.	$\frac{2}{9}$	38.	72
6.	30	17.	6	28.	$\frac{1}{12}$	39.	$\frac{1}{64}$
7.	25	18.	$\frac{1}{4}$	29.	$\frac{5}{12}$	40.	81
8.	15	19.	4	30.	$\frac{1}{25}$	41.	$\frac{1}{72}$
9.	4	20.	12	31.	$\frac{3}{25}$	42.	49
10.	6	21.	$\frac{1}{12}$	32.	$\frac{3}{20}$	43.	54
11.	8	22.	$\frac{1}{9}$	33.	$\frac{1}{30}$	44.	$\frac{1}{48}$

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Problem Set

1. a. Answer provided

   b. 90

   c. $\frac{3.5}{0.5}$; 7

   d. $\frac{3.5}{0.05}$; 70

   e. $\frac{4.2}{0.7}$; 6

   f. $\frac{0.42}{0.07}$; 6

   g. $\frac{10.8}{0.9}$; 12

   h. $\frac{1.08}{0.09}$; 12

   i. $\frac{3.6}{1.2}$; 3

   j. $\frac{0.36}{0.12}$; 3

   k. $\frac{17.5}{2.5}$; 7

   l. $\frac{1.75}{0.25}$; 7

2. Answers will vary.

3. a. 12

   b. 6

4. 83

5. 3

Exit Ticket

a. $\frac{3.2}{0.8}$; 4

b. $\frac{3.2}{0.08}$; 40

c. $\frac{7.2}{0.9}$; 8

d. $\frac{0.72}{0.09}$; 8

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Homework

1. a. 3

   b. 30

   c. $\frac{4.8}{0.6}$; 8

   d. $\frac{0.48}{0.06}$; 8

   e. $\frac{8.4}{0.7}$; 12

   f. $\frac{0.84}{0.07}$; 12

   g. $\frac{4.5}{0.15}$; 3

   h. $\frac{0.45}{0.15}$; 3

   i. $\frac{14.4}{1.2}$; 12

   j. $\frac{1.44}{0.12}$; 12

2. Leann is incorrect; answers will vary.

3. a. 8

   b. 16

4. 15

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Lesson 31

Problem Set

1. a. $53.2 \div 0.4 \approx \frac{520}{4} = 130$; 133
   b. $1.52 \div 0.8 \approx \frac{16}{8} = 2$; 1.9
2. a. $9.42 \div 0.03 \approx \frac{900}{3} = 300$; 314
   b. $39.36 \div 0.96 \approx \frac{40}{1} = 40$; 41
3. a. 154
   b. $\frac{316}{4}$; 79
   c. $\frac{23.1}{3}$; 7.7
   d. $\frac{1560}{24}$; 65
4. a. 27
   b. 21
5. 6

Exit Ticket

1. $6.39 \div 0.09 \approx \frac{630}{9} = 70$; $639 \div 9 = 71$
2. $82.14 \div 0.6 \approx \frac{8400}{60} = 140$; $8214 \div 60 = 136.9$

Homework

1. a. $61.6 \div 0.8 \approx \frac{640}{8} = 80$; 77
   b. $5.74 \div 0.7 \approx \frac{56}{7} = 8$; 8.2
2. a. $4.74 \div 0.06 \approx \frac{480}{6} = 80$; 79
   b. $19.44 \div 0.54 \approx \frac{2000}{50} = 40$; 36
3. a. 64
   b. $\frac{752}{8}$; 94
   c. $\frac{124.5}{5}$; 24.9
   d. $\frac{560}{16}$; 35
4. 54 green; 36 purple
5. 14

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Lesson 32

Problem Set

1. $(3 + 2) \div \frac{1}{3}$ circled
2. $\frac{28}{\frac{4}{5} - \frac{7}{10}}$ and $28 \div \left(\frac{4}{5} - \frac{7}{10}\right)$ circled
3. Answers will vary.
4. 3(a); explanations will vary.
5. Answers will vary.
6. 5(a); explanations will vary.
7. a. 12
   b. $\frac{5}{6}$
   c. $\frac{1}{12}$
   d. $\frac{1}{2}$
   e. $\frac{3}{40}$
   f. 12
8. a. $\frac{2}{3} \times 20 - 5$
   b. $\frac{2}{3} \times (20 - 5)$

Exit Ticket

1. Answers will vary.
2. a. Answers will vary.
   b. Answers will vary.
3. 2(b); explanations will vary.

Homework

1. $(7 - 4) \div \frac{1}{5}$ circled
2. $42 \div \left(\frac{2}{3} + \frac{3}{4}\right)$ and $\frac{42}{\frac{2}{3} + \frac{3}{4}}$ circled
3. Answers will vary.
4. 3(a); explanations will vary.
5. a. 30
   b. $1\frac{1}{5}$
   c. $\frac{1}{100}$
   d. $\frac{2}{5}$
   e. 400
6. a. Answers will vary.
   b. $32 - 5 - \frac{1}{3}(32 - 5)$ circled

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Lesson 33

Sprint

Side A

1.	1	12.	1,000	23.	50	34.	325
2.	10	13.	2,000	24.	5	35.	5
3.	20	14.	8,000	25.	0.5	36.	5
4.	70	15.	10	26.	0.8	37.	50
5.	10	16.	100	27.	400	38.	90
6.	100	17.	200	28.	4,000	39.	400
7.	200	18.	900	29.	4,700	40.	80
8.	600	19.	500	30.	5,900	41.	70
9.	1	20.	5,000	31.	30	42.	40
10.	10	21.	6,000	32.	300	43.	12.1
11.	100	22.	2,000	33.	320	44.	321

Side B

1.	10	12.	1,000	23.	40	34.	236
2.	10	13.	2,000	24.	4	35.	3
3.	20	14.	9,000	25.	0.4	36.	3
4.	80	15.	10	26.	0.7	37.	30
5.	10	16.	100	27.	500	38.	80
6.	100	17.	200	28.	5,000	39.	400
7.	200	18.	700	29.	5,300	40.	70
8.	700	19.	400	30.	6,800	41.	80
9.	1	20.	4,000	31.	20	42.	30
10.	10	21.	5,000	32.	200	43.	12.1
11.	100	22.	8,000	33.	230	44.	211

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Problem Set

1. a. $\frac{1}{12}$ L
   b. $1\frac{1}{2}$ L
2. a. 7
   b. $\frac{1}{4}$ hr or 15 min
   c. 14
3. a. 18
   b. 6
4. a. 90
   b. 360
5. Answers will vary.
6. Answers will vary.

Exit Ticket

a. 6
b. 12

Homework

1. a. $\frac{1}{15}$ kg
   b. $1\frac{1}{3}$ kg
2. a. 19
   b. 38
3. a. 13
   b. 6
4. a. Answers will vary.
   b. Answers will vary.
5. Answers will vary.

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